Statistical Modelling and Forecasting of Mortality in India

by
Rajesh Kumar Chauhan

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of

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DECLARATION

Except where indicated otherwise
this thesis is my own work

Rajesh Kumar Chauhan
April 2002

This thesis is dedicated to
the memory of my grandparents

Mrs. Raj Kumari Chauhan
[1918-1999]
&
Th. Lakhan Singh Chauhan
[1917-1996]

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जगत्जननी, शक्तिरूपा आदिशक्ति मां जड़–चेतन में ऊर्जास्वरूपा हैं। उनकी कृपा के बिना कोई आध्यात्मिक या सांसारिक कार्य पूर्ण होना संभव नही है। अत: मै भगवती की सतत कृपा और प्रेरणा के लिये नमन करता हूं।

या देवी सर्वभूतेषु विद्यारूपेण संस्थिता।
नमस्तस्यै नमस्तस्यै नमस्तस्यै नमो नम:।।

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मेरे जीवन की अमूल्य धरोहर थे मेरे मेरी दादीजी (अम्मा) और दादाजी (बाबा)। दुर्भाग्य से दोनो ही आज हमारे मध्य नही हैं लेकिन उनकी स्मरणमातृ से प्रतिपल मुझे मेरे कर्तव्य का बोध हो जाता है। यह शोधकार्य उनदोनों महामानवों की स्मृतिओं को सादर सर्मपित है।

मेरी नानीजी और नानाजी का अद्वितीय वात्सल्य, स्नेह, प्यार और आशीर्वाद सर्वथा मेरे साथ एक कवच के रूप में रहे। इस कारण मैं कठिन बाधाओं का सामना सरलतापूर्वक करता रहा। आप दोनो की छत्रछाया हमेशा बनी रहे यही प्रार्थना ईश्वर से निरंतर करता हूं।

मैं अपने माताजी (मम्मी) और पिताजी (पापा) का हार्दिक आभार प्रकट करता हूँ जिनका अथक परिश्रम और अकथनीय त्याग मेरी प्रेरणा के परम स्रोत रहे। आपके अपार स्नेह और अमोघ अशीर्वाद के फलस्वरूप ये अनुष्ठान निर्विघ्न सम्पन्न हो सका। इन चार वर्षो में मेरे भाई योगेश और बहन राखी की हार्दिक शुभकामनाओं के परिणामस्वरूप मैं स्थिर और शांतचित्त रह अपना कार्य सुचारू रूप से संपन्न कर सका। मैं अपने मित्रों अजय कुलश्रेस्ठ, संतोष कुमार अग्रवाल, पदमेश कुमार सिंह, दुष्यंत शिशोदिआ, गोपाल गुप्ता, विशाल देव शास्त्री, अनुराग मिश्रा, बिरंची जेना, संजय मोहंती, अजय पाण्डेय, प्रियंका वर्मा, प्रणिता अच्युत, निधि सिन्हा, शाहिना और शालिनी वर्मा का आभार प्रकट करता हूँ जिनके परामर्शो ने मेरे जीवन के विभिन्न पहलुओं पर सक्रिय मार्गदर्शन का कार्य किआ। आप सभी का सहयोग और शुभकामनायें मेरे शोधकार्य में अत्यंत महत्वपूर्ण रहे।

मेरे चाणक्यसम महत्वाकांक्षी शिक्षक श्री सूरजपाल सिंह के अद्वितीय प्रयासों और अशीर्वाद के लिये मैं उनको सादर नमन करता हूं। आपका मार्गदशन मेरे जीवन में धुरी का कार्य करता रहेगा।

ଏହି କାର୍ଜ୍ୟ ଟି ର ସମ୍ପୁର୍ନତା ଜେ କେବଲ୍ଲ ମୋରୋ ସହଧର୍ମନି ସ୍ରିମତି ଏମିଲି ଦାସ ଙ୍କ ନିସ୍ଵାର୍ଥ ତ୍ୟାଗ ଓ ଅନ୍ତରିକତା ର ଫଲ ଏହା କନ ବା ମୁନ୍ ବର୍ନନା କରିବି । ଏହି ପରିପ୍ରେଖି ରେ ଯେହା ପରିପ୍ରକାଶ୍ କରିବା ନିରଥର୍ହକ ଜେ ଆମ ଦୁହିନ୍ ଙ୍କ ର ଦୁର ରେ ରହିବା ଆବେଗିକ୍କ ଜ୍ୟନ୍ତ୍ରନା ଜେ ନିର୍ଧାରନ କରିବା ଜୋ୍ୟଗ୍ୟ ବା ସମ୍ଭବ ଏଥିରେ ବା କାନ ପ୍ରଶ୍ନି ସ୍ଵିଇକାର କରାଜୟିଏ ପାରେ... ଏମି ମୁ ତୁମକୁ ଅସେସ ଧନ୍ୟବଦ୍ ଦେ ବାକୁଉ ଚାହିଁଏନ୍ ଜେ କେବଲ୍ଲ ତୁମର ସ୍ନେହ, ଶର୍ଧା, ଅନ୍ତରିକସହଜୋ୍ୟଗ ଓ ଅଦମ୍ୟ ପ୍ରେରନ ଜେ ମୋ ପିଇନ୍ ନୁତନ୍ ଜିବନ୍ ସଞ୍ଚର କରିଚି ତହ ନୁହୌନ୍.ତୁମର ସାଧାରନ ଓ ପ୍ରସୁ୍ତିତ ଜିବନ୍ ଜପନ୍ ସୌଇଲ୍ୟ ମୋ ପିଇନ୍ ପ୍ରେରନା ର ଉତ୍ସ ହୋଇଛି ଏଅହ ନିସନ୍ଧେୟ । କେବଲ ଯେତିକି ନୁହୁନ୍ ମୋର ଗାବେସନା ଗତ କାର୍ଜ୍ୟ ର ପ୍ରତି ଟି ପାହଚ ରେ ତୁମର ତତ୍ଥ୍ୟ ନିର୍ଧାରଣ ଜନିତ ଗ୍ୟନ ସତରେ ମୋ ପାଇନ୍ ଜେ କେବଲ୍ଲ ଉସାହ ଓ ଉଦ୍ଦିପନା ର ଉତ୍ସ ହୋଇଚି ଯେହା କିପରି ବା ମୁଉନ୍ ବର୍ନନନା କରିବି ।

ତୁମ ଦ୍ୱାରା ସୁସ୍ତ ଅଣୁଶମାନ୍ ରୁପକ ଅମୁଲ୍ୟ ଧନ ମୋ ଜିବନ ରେ ଜେଉନ୍ ଅନନ୍ଦ, ଉତ୍ସାହ, ଉଦ୍ଦିପନା ଓ ବଞ୍ଚି ରହିବର ନୁତନ ପ୍ରେରନା ଅନିଦୀଇଛି ଯେହା ଜେ ମୁ କେଉନ୍ ଭାସା ରେ ବର୍ନନା କରିବି ଜାନିପାରୁନହିନ୍ । ଯେହି ପରିପେଖି ରେ ମୁ ଯେତିକି କହିବକୁ ଚହୀଇନ୍ ଜେ ମୋରୋ ଗବେସନ ଗତ କାର୍ଯ୍ୟ ରେ ନୁତନତା ଓ ପୁର୍ନତା ପରିପ୍ରକଶ୍ କରି ବାର ସମ୍ପୁର୍ନ ସୁଜୋଗ୍ଯ ଜେ କେବଲ୍ଲ ତୁମ୍ଭ ପ୍ରଦ୍ଦତ୍ତ ଅମୁଲ୍ୟ ରତ୍ନ ର ସାକାର ରୁପ ରେଖ ଓ ମୁ କନ୍ ବା ଆଉ ଧନ୍ୟବଦ୍ ଜେ ତୁମ କୁ ଦେବି ।

ସେସ ରେ ମୁନ୍ କିପରି ବା ମୋର ସାସୁ, ସୌସୁର, ଭାଇ, ଭାଉଜ ଓ ଅନ୍ୟାନ୍ୟ ପରିବାର୍ ସଦଷ୍ୟ କୁ ବା ଭୁଲିଜିବି.... ଜେଉନ୍ ମାନଙ୍କର ପାଇନ୍ ଓ ଜେଉନ୍ ମନଙ୍କର ଅନ୍ତରିକ ସ୍ରଧା, ସ୍ନେହ,ସହଯୁଗ୍ ଓ ସହନୁଭୁତି ବିନା ବୋଧ ହୁଉୟେ ମୋରା ଯେହି ଗବେସନା କାର୍ଯ୍ୟ ଅଧୁରା ହୋଇଥାହନ୍ତୁ । ଏଣୁ ଯେହି ସମସ୍ତଙ୍କ ପ୍ରତି ମୁନ୍ ମୋର ଅନ୍ତରିକ କୃତଜ୍ଞ୍ୟତା ଓ ସମ୍ମାନ ଜନାଉଛି ।

जातस्य हि ध्रुवो मृत्युर्ध्रुवं जन्म मृतस्य च ।

jātasya hi dhruvo mr·tyur
dhruvam janma mrtasya ca
(Text 27, Chapter 2; Bhagavad-Gītā)

One who has taken his birth is sure to die, and after death one is sure to take birth again.

CURRICULUM VITAE

Born: Jasrana, UP, India, 19 July 1972.

Married: To Emily, Agra, India, 18 April 2000.

Children: Anshuman, born in Canberra, 1 August 2001.

Nationality: Indian.

Education: **Masters**

Master of Population Studies (MPS), International Institute for Population Sciences (IIPS), Mumbai, India, 1995.

Title of term paper: 'Graduation of infant deaths by age.'

Master of Science (M.Sc.) in Statistics, St. John's College, Agra University, Agra, India, 1993.

Title of submitted dissertation: 'Bayesian analysis of reliability and hazard rate function of a mixture model.'

Bachelors

Bachelor of Science (B.Sc.) with Physics, Mathematics & Statistics, St. John's College, Agra University, Agra, India, 1991.

Pre University

Intermediate Science (yr. 12) Narain Inter College, Shikohabad (UP), India, 1988.

High School Science (yr. 10) Government Inter College, Etah (UP), India, 1986.

Awards and Scholarships:

➊ 1997-2002, AusAID scholarship, for Ph.D., ANU, Canberra.

➋ 1995, Offered Government of India Scholarship for Ph.D., IIPS, Mumbai

➌ 1995-1997, Junior Research Scholarship (JRF), University Grants Commission (UGC) of India, for Ph.D., IIPS, Mumbai.

➍ 1994-1995, Government of India Scholarship, for MPS, IIPS, Mumbai.

➎ 1995, Awarded silver medal and Dr. J.R. Rele award for best MPS seminar, IIPS, Mumbai.

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ABSTRACT

Mortality forecasts for any society are important for policy planners for health and related planning areas other than key inputs for the population forecasts. Mortality forecasts can be based on three types of models: model life tables, relational type and laws of mortality. The most appropriate way to model human mortality is by laws of mortality, which are mathematical functions guided by parameters which themselves serve as a source of information apart from representing the mortality curve.

This thesis focuses on modelling and forecasting mortality for India. A new mortality model is developed by improving the Heligman-Pollard model, reducing the number of parameters to seven. The new mortality model is tested with data from Australia, Japan and Sweden. Then the new model is fitted to sex-specific data for 1970-1997 for India, rural India and urban India. The estimated parameters are forecasted to 2020 using advanced time series techniques. Forecasts are also obtained by the Lee-Carter method of mortality forecasting; this is the first time that this method has been applied to data from a developing country. In all cases the chosen forecasting model is more complex than a (0,1,0) model. An improvement can be seen by comparing optimal models with (0,1,0) models, in terms of the standard errors. Comparison is made between the mortality forecasts obtained by the new mortality model and the Lee-Carter method.

Not only does the new mortality model provide a mortality forecasting system for the future, but the robustness of the estimation provides stability to the process. Every model estimated with Indian data was highly statistically significant, as for individual parameters, 88 per cent of the total of 1176 parameters were significant at the 5 per cent level of significance. The new mortality model developed in this thesis is parsimonious and expected to be applied successfully to model mortality in other populations.

The expectation of life at birth in India is estimated to be 78.6 years for males and 79.2 years for females in 2020. Urban males and females have about four years longer life expectancy than all males and females in 2020. The gap between rural and urban areas is 7.7 years for males and 6.3 years for females in 2020. Urban females and rural males are likely to be the longest and shortest surviving groups, respectively, in India by 2020.

ACRONYMS

ACI	Akaike information criteria
ARIMA	Autoregressive integrated moving average
ASDR	Age specific death rates
ASMR	Age specific mortality rates
cdf	cumulative density function
CDR	Crude death rates
CRS	Civil registration rates
ESACF	Extended sample autocorrelation function
H-P	Heligman-Pollard
IMR	Infant mortality rates
L-C	Lee-Carter
MMR	Maternal mortality ratio
MP	Madhya Pradesh
pdf	Probability density function
RGI	Registrar General of India
SRS	Sample registration system
SVD	Singular value decomposition
UP	Uttar Pradesh

KEY WORDS

Age pattern of mortality, ARIMA process, ARIMA coefficients, Brass logit system, Carriere model, Critical level, Critical value, Deaths, Eigen values, Eigen vectors, Error sum of squares, Expectation of life at birth, Finite range models, Forecasting, F-test, Graduation, Heligman-Pollard model, Human mortality, India, India mortality, Laws of mortality, Life expectancy, Life table, Model life table, Mortality curve, Mortality forecast, Mortality graduation, Mortality indicators, Mortality models, Mortality pattern, Mortality schedules, Mortality smoothing, New mortality model, Non-linear regression, Parameter estimation, Parameterised forecasting, Parameters, Parametric forecasting, Parsimony, Pre-teen mortality, Pre-teen mortality model, Probability of deaths, Residual sum of squares, Relational models, Smoothing, Standard error, Statistical significance, Testing models and Weighted estimation.

Chapter 1: Introduction

The best way to predict the future is to invent it.
Alan Kay

1.1 Background

In general, mortality forecasts are necessary for planning in the field of public health, social sectors such as education, the insurance sector and other long and short-term planning needs by government and private sectors. Mortality as a negative aspect of health depicts the status of a population through various summary and detailed measures. Expectation of life at birth is not only a demographic measure of longevity but also a widely accepted measure of overall health and its comparison among different populations. Crude death rates provide a summary indication of the rate of death in a population in a calendar year, but they are very crude as no age patterns are reflected in them. The age pattern of human mortality has a unique shape with only small variations among populations. Typically, it is a mixture of three defined curves. These three curves are responsible for the young, adolescent and senescent mortalities. By using this regularity in the mathematical shape as a basis for modelling, various scholars have attempted to define mortality curves which can facilitate forecasts of mortality and other useful dimensions in mortality measurement.

This thesis attempts to model and forecast mortality in India by developing a new parametric mortality model with the help of existing models and a new model. Existing models with many parameters and tested with good quality data from industrially advanced countries may not serve as guiding rules in the Indian case. Based on parsimony and ability to forecast the estimated parameters, a new model of mortality has been developed and tested. This model is subsequently used to forecast mortality for India. As India contains wide variations in mortality on the scale of rural-urban variation, these are considered in the thesis.

1.2 India since 1947

The Democratic Republic of India came into existence on 15 August 1947; the world's largest democracy adopted its current constitution on 26 January 1950. Indian democracy has three defined pillars: the Executive, the Legislature and the Judiciary along with the important role of the press. The federal structure of a decentralised political and administrative organisation remains powerful in the shape of provinces called states, which initiate and enforce the laws at the regional level. State governments are also responsible for the economic management of resources along with the Union (central) Government of India, which is the guarantor of the constitutional rights of citizens. India adopted a public policy of integrated development through five-year development plans along with short-term plans and other schemes. Despite its political stability, and slow but sustainable development, India faced a major challenge in the alarming growth of its population. India's partly successful family planning program, the oldest official program in the world (introduced in 1952), began with the clinic-based approach, where clients themselves were supposed to approach the services. Lack of literacy and awareness remained the major reasons why demand for family planning services was generated slowly.

High birth rates and declining death rates caused growth rates to increase rapidly in most parts of the country. Public health policy brought about a drastic reduction in infant, child and maternal mortality through control over some major killer diseases.

1.3 India and its major states

At present India has 28 states and seven union territories for political and administrative reasons. Table 1.1 presents the distribution of population for the four most recent census time points . For comparison purposes, data for Goa and Daman & Diu are combined, though Goa is a state and Daman & Diu comes under the union territory. There are three states, Chhattisgarh, Jharkhand and Uttaranchal, which were created in November 2000 by dividing Madhya Pradesh, Bihar and Uttar Pradesh respectively. These states hold 2.02, 2.62 and 0.83 per cent respectively of the total Indian population, according to the 2001 provisional census totals. The population of India was 1.03 billion for the reference period March 2001.

Table 1.2 provides the percentage distribution of Indian population according to the states and Union Territories for the four most recent censuses. It is clear that the 15 major states contribute over 90 per cent of population in the most recent population censuses.

Table 1.1: Population in India by states for 1971 to 2001 censuses (in thousands)

S. No.	India/ States/ Union Territories	1971	1981	1991	2001
	India[c]	**548,160**	**683,329**	**846,388**	**1,027,015**
1	Jammu & Kashmir[b]	4,617	5,987	7,804	10,070
2	Himachal Pradesh[d]	3,460	4,281	5,171	6,077
3	Punjab	13,551	16,789	20,282	24,289
4	Chandigarh	257	458	642	901
5	Uttaranchal[f]				8,480
6	Haryana	10,036	12,922	16,464	21,083
7	Delhi	4,066	6,220	9,421	13,783
8	Rajasthan	25,766	34,262	44,006	56,473
9	Uttar Pradesh	88,342	110,863	139,112	166,053
10	Bihar	56,353	69,915	86,375	82,879
11	Sikkim	210	316	407	540
12	Arunachal Pradesh	468	632	865	1,091
13	Nagaland	516	775	1,210	1,989
14	Manipur	1,073	1,421	1,837	2,389
15	Mizoram		2,053	690	891
16	Tripura	1,556	1,336	2,757	3,191
17	Meghalaya			1,775	2,306
18	Assam[a]	15,969	18,535	22,414	26,638
19	West Bengal	44,312	54,581	68,078	80,221
20	Jharkhand[f]				26,909
21	Orissa	21,945	26,370	31,660	36,707
22	Chhattisgarh[f]				20,796
23	Madhya Pradesh	41,654	52,179	66,181	60,385
24	Gujarat[e]	26,697	34,086	41,310	50,597
25	Dadra & Nagar Haveli	74	104	139	220
26	Maharashtra	50,412	62,783	78,937	96,752
27	Andhra Pradesh	43,503	53,551	66,508	75,728
28	Karnataka	29,299	37,136	44,977	52,734
29	Goa, Daman & Diu	858	1,087	1,271	1,502
30	Lakshadweep	32	40	52	61
31	Kerala	21,347	25,454	29,099	31,839
32	Tamil Nadu	41,199	48,408	55,859	62,111
33	Pondicherry	472	604	808	974
34	Andaman & Nicobar Islands	115	189	281	356
		548,159	683,337	846,392	1,027,015

Notes:

a) Assam included Mizoram and Meghalaya in 1971; Mizoram was included in Assam in 1981; the 1981 Census could not be held owing to disturbed conditions prevailing in Assam, so the population figures for 1981 have been worked out by 'interpolation'.

b) The 1991 Census could not be held owing to disturbed conditions prevailing in Jammu and Kashmir, so the population figures for 1991 have been worked out by interpolation.

c) The population of India includes the estimated population of the entire Kachchh district, Morvi, Maliya-Miyana and Wankaner talukas of Rajkot district, Jodiya taluka of Jamanagar district of Gujarat State and the entire Kinnaur district of Himachal Pradesh where population enumeration of the Census of India 2001 could not be conducted owing to natural calamities.

d) Figures shown for Himachal Pradesh have been arrived at after including the estimated figures of the entire Kinnaur district of Himachal Pradesh where the population enumeration of the Census of India 2001 could not be conducted owing to a natural calamity.

e) Figures shown for Gujarat have been arrived at after including the estimated figures of the entire Kachchh district, Morvi, Maliya-Miyana and Wankaner talukas of Rajkot district, and Jodiya taluka of Jamnagar district of Gujarat State where the population enumeration of the Census of India 2001 could not be conducted owing to natural calamities.

f) These three states came into existence in November 2000.

Source: India. ORG, 2001a, b.

Table 1.2: Per cent population distribution in India by states and Union territories in descending order for 1971 to 2001 censuses

State No.	India/ States/ Union Territories	1971	1981	1991	2001
	India	100	100	100	100
1	Uttar Pradesh	16.12	16.22	16.44	16.17
2	Maharashtra	9.20	9.19	9.33	9.42
3	Bihar	10.28	10.23	10.21	8.07
4	West Bengal	8.08	7.99	8.04	7.81
5	Andhra Pradesh	7.94	7.84	7.86	7.37
6	Tamil Nadu (Madras)	7.52	7.08	6.60	6.05
7	Madhya Pradesh	7.60	7.64	7.82	5.88
8	Rajasthan	4.70	5.01	5.20	5.50
9	Karnataka	5.34	5.43	5.31	5.13
10	Gujarat	4.87	4.99	4.88	4.93
11	Orissa	4.00	3.86	3.74	3.57
12	Kerala	3.89	3.72	3.44	3.10
13	Jharkhand	0.00	0.00	0.00	2.62
14	Assam	2.91	2.71	2.65	2.59
15	Punjab	2.47	2.46	2.40	2.37
16	Haryana	1.83	1.89	1.95	2.05
17	Chhattisgarh	0.00	0.00	0.00	2.02
18	Delhi	0.74	0.91	1.11	1.34
19	Jammu & Kashmir	0.84	0.88	0.92	0.98
20	Uttaranchal	0.00	0.00	0.00	0.83
21	Himachal Pradesh	0.63	0.63	0.61	0.59
22	Tripura	0.28	0.20	0.33	0.31
23	Manipur	0.20	0.21	0.22	0.23
24	Meghalaya	0.00	0.00	0.21	0.22
25	Nagaland	0.09	0.11	0.14	0.19
26	Goa, Daman & Diu	0.16	0.16	0.15	0.15
27	Arunachal Pradesh	0.09	0.09	0.10	0.11
28	Pondicherry	0.09	0.09	0.10	0.09
29	Chandigarh	0.05	0.07	0.08	0.09
30	Mizoram	0.00	0.30	0.08	0.09
31	Sikkim	0.04	0.05	0.05	0.05
32	Andaman & Nicobar Islands	0.02	0.03	0.03	0.03
33	Dadra & Nagar Haveli	0.01	0.02	0.02	0.02
34	Lakshadweep	0.01	0.01	0.01	0.01

Source: Same as Table 1.1.

1.4 The parameters

Modelling of mortality typically proceeds by estimating parameters that define the shape of the mortality curve. The specific aim in parameterising any phenomenon is to derive a few characteristic statistical values that summarise a greater number of numerical values. Parameters are the key values obtained from data under the restriction (discipline) of a curve. If a curve or a model explains the nature of the variability of any process, then the qualifiers for the model are termed parameters. The parameterisation can be done in two ways: one is empirical, mainly by using data sets to try to fit the model by using the data's graphically obvious or self-explanatory features; the other way is to have a specified model with predefined fitting procedures. In the latter case one can replicate the procedure for any new relevant data set to obtain the parameters.

The strategy is to reduce the amount of information from a specific form of data (raw data) to a designated number of parameters. Since the mortality process by age exhibits a defined path in the typical shape as a mixture of three curves, if plotted on graph paper, attempts to model this defined shape by various methods have been made in the past. Age-specific death rates (ASDR), probabilities of death for age x to x+1 (q_x) and force of mortality (μ_x) are the indicators of mortality against age commonly used to measure mortality events in a statistically standardised way. Either one of the three, or a derivative of these mortality measures, is used as input for the estimation of the parameters of a model after a logical fitting of that specific model. The process of fitting a curve for a specific phenomenon is defined as modelling.

Mortality modelling started with De Moivre's pioneer work in 1725 to model the force of mortality using just one parameter. Since then many attempts in the same direction have been made by demographers, actuarial scientists and statisticians.

1.5 Other important concepts

Reference will often be made in this thesis to certain concepts. Most of these will be in the form of notations of mortality; they will be defined at the time of their appearance in the discussion. However, some important concepts are presented in this section.

1.5.1 Highest attained age

De Moivre, and Petrioli (1981), had an idea of an upper age limit in their minds beyond which no person of the community can live. In extensive investigation on the topic Thatcher (1999: 6) summarised various viewpoints as follows

> The French demographer Vincent (1951) thought that there were enormous odds against exceeding the age of 110. Fries (1980) and the biologist Hayflick (1994) placed the upper limit at 115 years, and indeed Hayflick used this supposed limit as a reason for rejecting various medical theories. Most recently the mathematical statisticians Aarssen and de Haan (1994), have inferred (through very limited data) that there is an upper limit of life which lies, with 95 % confidence, between 113 and 124 years.

1.5.2 Number of parameters

The whole age range of human mortality follows a definitive pattern, which can be represented through certain functional shapes involving parameters to represent them. The number of parameters in the model varies according to the type of input values of mortality that are used and the philosophical basis for a function to be a mortality model.

The number of parameters used in mortality modelling to date ranges from one to eleven. Keyfitz (1982) expressed the concern that a minimum number of parameters is recommended for effective forecasting and sharp comparison; this is the main objective of mathematical representation of mortality. As human mortality takes three definite turns in its course, at least six parameters would normally be required to represent it without error.

1.5.3 Mathematical graduation

To represent mortality with mathematical curves is called the graduation of mortality. When different sections of the mortality curve are derived separately, they must be 'spliced' together in a smooth transition from one section to the next. This smoothing process is also referred to as graduation. Heligman & Pollard (1980) have argued that good graduation must go smoothly from one age to the next without changing the actual pattern of mortality. Keyfitz (1982) gave six separate uses of graduation which may be summarised as:

1. To remove awkward irregularities and inconsistencies;
2. To make results more precise;
3. To construct a life table;
4. To aid inferences from incomplete data;
5. To facilitate comparison;
6. To aid forecasting.

These uses are best viewed in mortality forecasting and comparisons. In his thesis, Mathew (1997:7) attributes the analysis of age patterns of mortality as a tool for statistically sound projections. So the major thrust of graduation is to bridge the gap between theoretical models and practical application in order to support policy makers in various fields requiring mortality indicators.

Mortality graduation becomes more contextual for demographers because they use key indicators as the explanatory basis of several interrelated issues. For example, in health policy formation, mortality indicators themselves are very significant. Especially for populous countries like India, the question is how to limit the rapid growth of population caused by mortality declines together with persistent high fertility. To make comparisons among various relatively homogeneous and significantly large ethnic or administrative populations, a more sophisticated mathematically comparable system is needed.

1.5.4 Recent patterns of old-age mortality

While past mortality models have assumed three phases of mortality by age, a fourth has recently attracted attention. In a recent paper by Thatcher (1999), a logistic model has been introduced for the representation of the Gompertz factor of human mortality in developed societies. After age 80 or similar higher ages of life, the probability of death stabilises and then starts declining slightly; these trends are persisting in almost all developed countries, and have attracted the attention of policy makers and researchers wishing to know more about the selectivity of survival at those higher ages. The model reads:

$$\mu_x = \frac{\kappa z}{1+z} + \gamma \text{........}(1.1)$$

where

$$z = \alpha \, e^{\beta x}$$

Deceleration in the age pattern of old age mortality has also been observed and explained using cause-of-death data by Horiuchi & Wilmoth (1998).

1.6 Present research

This research aims at modelling the age pattern of mortality for India. The proposed model to be used for human mortality is parametric in nature. Modifications of a model developed by Heligman & Pollard (1980) using a new reliability model will be tested with various data sets. Parameters will be used for the basis of forecasts for the near future. Forecasted parameters will be then transformed to standard mortality indicators.

Inherent errors in available data in developing countries prevent the adoption of models conventionally used in industrially advanced countries with reliable data. In the absence of reliable data on mortality, Ishak (1992) advocated using advanced statistical techniques like Bayesian approaches for infant mortality graduation. Though mortality can be graduated by existing formulae, there seems a need to develop a system with fewer parameters (improved reliability) than the existing ones. The use of a reliability model in modification of the Heligman & Pollard (1980) model will reduce the number of parameters.

To meet the requirements of developmental planning, the rapidly growing population of India requires much attention to be paid to health, through its negative aspect, mortality, to know its size and structure at future time points. The overall need is to project future population. This requires actual mortality patterns to be taken into account, as the major factor after fertility to decide population size and structure.

Mortality analysis by different techniques will enable us to have a better idea of future Indian mortality, along with methodological implications to model the mortality in a single platform. A further review of various mortality models is presented in Chapter 3.

1.7 Objectives

The general objectives of this thesis are to model and forecast the Indian mortality by age and sex. It involves finding the appropriate model among the model life tables, relation models and laws of mortality, which can model the age pattern of mortality and subsequently be used for forecasting the mortality. A search is required for the right model among the existing models such as the Brass relational model, the Carriere model, the Heligman-Pollard model and the Lee-Carter model in the Indian situation. In order to model Indian mortality in parametric form, the specific objectives are:

(1) To model the age pattern of Indian mortality

- (a) By fitting a finite range reliability model at pre-teen ages;
- (b) By modifying the parametric model of mortality, which is a mixture of three curves, such as the Heligman & Pollard (1980) formula with the help of a finite-range reliability model;

(2) To forecast Indian mortality

- (a) By projecting the estimated parameters of the new mortality model;
- (b) By the Lee-Carter method.

1.8 Methodology

The Heligman & Pollard (1980) formula is represented as a mixture of three continuous mathematical curves. The first is responsible for the mortality corresponding to the ages from birth to adolescence, capturing the continuous decline in mortality. The second curve represents the mortality pattern with increased rates for the young adult population. These increased rates are sometimes called 'accident hump' for males and 'excess female mortality' for maternal deaths. The third curve represents senescent mortality with increasing rates at later (older) ages. Modifications to the formula of Heligman & Pollard (1980) are proposed by replacing the first curve with a reliability model (Mukherjee & Islam, 1983). This new reliability model will serve as the probability density function (pdf) for the distribution of deaths up to the age 10-14 years. This model gives a very good fit during early childhood. Also, fitting this model is very simple, since it involves estimation of effectively one parameter only (by the minimum Chi-square method). Hence a modified form with fewer parameters for the parametric model such as Heligman-Pollard model will be obtained.

The reliability model proposed here has the finite range (in terms of age) and characteristics to capture the regularity in the age pattern of mortality. It has two parameters, but one is a scale parameter, the fitting can be done by minimising the errors.

The proposed model is given as

$$q_x = \frac{p}{x}\left(\frac{x}{\theta}\right)^p \text{...............}(1.2)$$

$$0 < x < \theta;\ \theta, p > 0;$$

The second curve has three parameters required to represent the unimodal patterns of the accident hump: the location, spread and severity of the mode. Hence the second curve is parsimonious to the needs of the mortality curve. The final curve responsible for ages 35 and above (Gompertz curve), seems suitable to remain as it is in the age pattern of mortality.

The projection of estimated parameters will be done by employing autoregressive integrated moving average (ARIMA) techniques of forecasting time series. This method has many advantages over ordinary least square estimation (linear regressions). ARIMA takes into account the fact that recent points of a time series get more weight for forecasting. Also ARIMA takes account of the violation of the assumption, which is rigidly observed during the estimation of linear regression parameters, that errors are randomly distributed. In an ARIMA process the dependence of errors affecting the time series is measured statistically and reported as an integral part of the analysis. In the ARIMA process,

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-3}, \ldots, U_t) \text{..................}(1.3)$$

where X_t are the values of a parameter X at time point 't' and U_t is the error component. The ARIMA technique provides workable arrangements to estimate the connecting coefficients of the X values to its previous values and the error structure.

Forecasted parameters of the mortality models can then be used to re-estimate the probability of death. Simple life table techniques can be used to get the summary measures such as life expectancy for reporting and comparison purposes.

A second method by Lee & Carter (1992) will also be used to forecast mortality in India. This method has been used extensively for developed countries and this will be the first time this technique has been used for a developing country. A brief methodological note on the technique is given here.

Age-specific death rates ($m_{x,t}$: ASDRs) are decomposed into age and time effects by the singular value decomposition (SVD) method using a two-parametric model (Lee & Carter, 1992). The time effect in this model is called the mortality index. This method ensures that total variance of ASMRs has been taken into account and minimised for a range of time periods taken as a group. The ARIMA model of time series analysis will be applied to the mortality index obtained from the SVD, and will provide the forecasted values for the future. The model is given as

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \text{.........................(1.4)}$$

or

$$m_{x,t} = e^{(a_x + b_x k_t + \varepsilon_{x,t})} \text{...............................(1.5)}$$

where $m_{x,t}$ is the age specific death rate for age x in the year t.

a_x is the overall age pattern of mortality

b_x are the age specific constants; refers to rate of change in specific age groups

k_t is the mortality index

$\varepsilon_{i,t}$ is the error term

This method and other technical issues are discussed in detail in Chapter 4.

1.9 Data sources

Data obtained from the Sample Registration System (SRS) of India for the time period 1970-1997 will be used for various analyses in this thesis. In particular, age-specific death rates for the annual time series will be analysed. Data sets for males and females and for persons are available in SRS. Further data are available for rural and urban residential areas. In this thesis the main analysis will be carried out for the male and female populations of India. A discussion of quality of data, coverage and use of data for India in representing most likely and least likely patterns of mortality is given in Chapter 2. Pre-independence discussions on Indian mortality data from Davis (1951) and the country monograph of India (United Nations, 1982b) have also been used. Life tables derived from Indian census data will also be used in this thesis.

1.10 Organisation of the thesis

The thesis is divided into eight chapters. Chapter 1 provides the brief introduction and background including specific objectives of the research. Chapter 2 discusses basic details of mortality levels and trends for pre-Independence and post-Independence India and includes a brief discussion on data sources and their quality.

Chapter 3 provides a review of important outcomes in the field of mortality modelling. To start, a few important concepts are defined. There is a categorisation of the different models according to three types: model life tables, relational models and mortality laws. The viability and usefulness of these models are determined in the methodological and Indian contexts. This chapter ends with the selection of four models to be tested with Indian data: two models from 'mathematical transformation' and two from the 'mortality laws. In Chapter 4, the four selected models are fitted to Indian data. Two models are selected for further consideration: the Heligman-Pollard model and the Lee-Carter model.

In Chapter 5 a reliability model is used as the first curve of the Heligman-Pollard model. This chapter describes the statistical characteristics of the model and the fitting methodology; it then provides details of testing of the new segment of mortality with data from Australia, Japan and Sweden. These data are then used to test the new modified Heligman-Pollard model over the entire age range of mortality. Chapter 6 describes fitting the new mortality model to Indian mortality data by sex and urban-rural residence. This chapter shows that the model performs well in this situation.

Chapter 7 presents the results of mortality forecasts for India. To start with, it gives a description of the ARIMA technique of time series forecasting. Forecasts of the time series of the parameters generated in Chapter 6 are undertaken to 2020. These forecasted parameters are then used to estimate future mortality rates, and by life table techniques, estimates of future expectation of life at birth are reported. The chapter then presents mortality forecasts obtained by the Lee-Carter method. It concludes with a comparison of the performance of the two forecasts. The mortality forecasts are also compared with those of the UN and the Registrar General of India. Chapter 8 discusses what has been achieved by the parameter reduction and forecasting as well as the numerical findings of the research. Further research areas are outlined at the conclusion of this chapter.

1.11 Computer programs or software used

For data management, simple analysis and advanced model fitting, various software have been used. Other software for creating and presenting the output are as follows. The thesis is written in Microsoft Word 97 with the tables either created in Microsoft Word or taken from Microsoft Excel 1997. The equations are typed in Microsoft's equation editor 3.0. Data were entered and maintained in Microsoft Excel. To use the data for analysis in various packages, they were also converted to ***txt*** and ***dat*** formats by using 'Note Pad'. The thesis is based on data analysis in SAS 8.02 and SPSS 10.0. Some calculations in 'R' (Ihaka & Gentleman, 1996) were also done to analyse some fitting processes, and dropped later. An Excel plug-in called 'Pop Tools 2.35' (Hood, 2001) was used for some computations.

Most of the graphs were created in Microsoft Excel 1997; a few were done in 'Lexis 1.1' (Andreev, 1999). A graphic software 'Paint Shop Pro 5.0' was used for editing some graphical images. References were managed through the software 'Endnote 5'. The references were also taken from the world-wide web as well as conventionally from printed and other electronic sources.

Chapter 2: Trends and Variations - Mortality in India

Because I could not stop for Death --
He kindly stopped for me --
The carriage held but just ourselves
And immortality. Emily Dickinson

2.1 Introduction

This chapter describes the levels and trends in mortality in India, at the national level, by rural-urban residential status and by state. In all cases males and females are discussed separately.

India has wide variations in mortality levels according to its states; there are northern states with expectation of life below 60 and southern states with expectation about 70 years. When the sexes compared, females remained disadvantaged in longevity until the mid-1980s, and there is a wide gap exists in overall mortality indicators for rural and urban India. With the variations according to the factors described, a proper and careful selection of units for the analysis must be made in order to model the more likely and unlikely examples of the mortality levels and patterns in India.

This chapter also describes the quality and other aspects of available data, in particular Sample Registration System (SRS) data. Some data used in this chapter to describe quality of data and cause-specific data sets do not correspond to the latest time periods, because up-to date comparable data sets are not available. However, the analysis of these data indicate that they are selectively presented, incomplete and unreliable thus providing a basis for not using them in the later analysis in the thesis.

2.2 Quality of data and other issues related to data

There are three sources of data on mortality in India: the census, the vital registration system (also known as the civil registration system) and the sample registration system. In this thesis, data from the census and SRS are used. The quality of the three sources of data is discussed in this section.

2.2.1 The census

Censuses started as early as 1872 in India when the first census was conducted; they are held at ten-year intervals to collect data on demographic and socio-economic aspects of Indian society. The modern Indian census is considered to be of good overall quality except for some age heaping because of digit preference. The papers published after the census operation serve as sources of some analysed information; in the case of mortality data, the relevant data sets are available in the form of intercensal life tables. Despite the disadvantage of being available only for relatively long periods (10 years), an advantage is that they are available at the state level. Ten-year periods are not ideal for analysing the levels and trends of mortality because short-term changes may be smoothed out by averaging. Census based life tables are used to fit the Heligman-Pollard model for India and results are presented in Chapter 4. These data are available for single years.

2.2.2 Civil registration system

Civil registration is in theory a solution to the problem of data being available only for intercensal periods, but the poor and incomplete returns make data almost unusable for any research and planning purposes (Pathak & Ram, 1994).

Table 2.1 provides an example of the vital statistics reporting and recording in percentages of births and deaths recorded through the civil registration system in 1984. Reporting efficiencies are self-assessments of the completeness of vital events actually registered; recording efficiencies are estimates of the proportion of events actually registered. For example, Bihar reported that in 1984 it had perfect (100 per cent) registration of vital events but when estimated it was found that only 20 per cent of births and 23 per cent of deaths were registered. Many states claimed to report all the vital events, but their varying efficiency in recording births and deaths can be seen from the table. The majority of the 13 major states of India covered less than half of deaths and births; a few did better than others but no reliable state-level representative estimates can be derived from the disproportionate returns available. The poor vital registration system is a cause of concern for planners and policy makers. The main obstacles to improving the registration system are illiteracy and poverty; the public interest messages to strengthen vital registration do not reach the poor rural mass of the country.

Table 2.1: Reporting and recording efficiencies together with associated vital rates for major states in India, 1984 (per cent)

State	Efficiencies of			Birth rate[c]	Death rate[c]
	Reporting[a]	Recording[b]			
		Births	Deaths		
Andhra Pradesh	27.3	29.5	23.2	8.5	2.3
Bihar	100.0	20.1	22.6	7.6	3.1
Gujarat	72.4	63.9	40.1	20.8	4.2
Haryana	100.0	64.5	60.8	21.8	6.0
Kerala	100.0	95.6	75.2	20.3	4.5
Madhya Pradesh	100.0	46.2	45.8	17.3	6.6
Maharashtra	89.3	65.6	63.3	19.1	5.6
Orissa	100.0	49.1	40.8	15.3	5.6
Punjab	100.0	85.2	91.0	22.6	7.1
Rajasthan	63.4	14.7	16.4	5.6	2.3
Tamil Nadu	82.9	67.5	55.7	18.0	5.7
Uttar Pradesh	N.R.	14.1	7.7	5.5	1.4
West Bengal	7.4	10.8	11.5	3.4	1.3

a. Percentage of total units reported
b. Percentage of total expected events recorded
c. Per thousand population
Source: Srivastava, 1989: 22.

Table 2.2 gives the registered and officially estimated birth and death rates for India for three decades. It is clear that the extent of omission of births and deaths recorded by the vital registration system is substantial. Table 2.3 provides completeness of civil registration system when compared to the sample registration system for the years 1971 to 1987 (Pathak & Ram, 1994). It can be noticed from the table that the coverage of CRS has worsened over the years for India and shows no significant improvement in any of the states. No reliable and representative estimates of mortality rates could have been derived on the basis of the vital registration system.

Table 2.2: Birth and death rates from civil registration, India, 1941-1971

Period	Registered		Official estimates		Percentage omission	
	Birth rate	Death rate	Birth rate	Death rate	Birth rate	Death rate
1941-1951	27.5	19.7	39.9	27.4	28.1	31.3
1951-1961	22.1	11.3	41.7	22.8	47.0	50.4
1961-1971	20.7	8.7	41.1	18.9	49.6	56.6

Source: UN, 1982b: 406.

Table 2.3: Completeness of death registration (%) when deaths rates from CRS are compared with SRS Rates

India/ states	Years					
	1971	1975	1979	1981	1985	1987
India	49.7	46.5	53.1	45.6	33 .1	35.8
Andhra Pradesh	56.2	54.6	56.3	55.9	21.4	24.2
Gujarat	49.4	48.7	55.8	75.8	39.8	46.9
Haryana	71.7	62.6	62.9	60.2	60.4	61.4
Madhya Pradesh	53.8	64.9	56.9	53.0	44.4	48.9
Maharashtra	83.7	79.8	79.4	67.7	70.2	67.5
Orissa	53.5	44.6	43.2	36.6	40.0	48.9
Punjab	71.2	69.4	72.6	68.1	74.2	79.0
Tamil Nadu	61.8	60.0	58.7	58.5	56.8	58.6

Source: Pathak and Ram, 1994: 20.

2.2.3 Sample registration system

The SRS was begun to meet the need for data for planning at lower levels and shorter periods. With the need to get reliable estimates of birth rates, death rates and other vital events, a dual record system of registering vital events was introduced in 1964; it is based on two basic principles of data collection: selecting a sample area, and conducting a complete vital registration in the selected sample area. The SRS collects data on births and deaths and other vital events in India for a selected sample. Results from SRS are available at state level with rural and urban categories of residence.

The births and deaths to the usual residents are recorded by a part-time local resident enumerator, often a teacher, at the time of the event. A survey conducted by a supervisor confirms the records each six months. Any discrepancies between the enumerator's record and the results of the survey are verified in the field to match the births and deaths. SRS provides a reasonably representative record of births and deaths to calculate national and subnational estimates.

SRS provides data for urban and rural areas separately. All the states and union territories of India are divided into sets of natural divisions formed on the basis of similarities in physical and other related features. SRS uses a stratified simple random sampling design. Strata for the rural areas are defined according to village population size: 2,000 and over; 1,000 to 1,999; 500 to 999 and less than 500. Villages with populations over 2,000 are considered in segments to ensure a maximum of 2,000 population size in sample areas. Simple random samples are drawn from each stratum so that villages in the sample with the maximum of 2,000 population provide a manageable workload for a part-time enumerator. The sampling units in urban areas came directly from the census frame, called census enumeration blocks. Stratification is done on the following cut-off limits of urban population sizes; 1,000,000 and above; 100,000 to 999,999; 50,000 to 99,999; 20,000 to 49,999; and below 20,000. After each census the sampling frame for SRS is revised. The population covered in SRS is roughly one per cent of the total Indian population with minor variations in coverage for different years. Estimation of births and deaths is first done at the stratum level; weighted estimation is used to obtain subnational and national estimates for rural and urban areas separately.

Table 2.4 provides SRS coverage for rural areas of 14 major states of India and all India for the 1984 registration (Srivastava, 1989). Noticeably the rural sample covered 0.77 per cent of the total rural population of India. Coverage varied in different states because of variations in the composition of strata and their numbers in different states. The coverage of SRS in 1990 for rural and urban areas was 0.77 and 0.69 per cent of the population respectively. More recently, in 1997, 0.64 and 0.88 per cent of the rural and urban populations respectively were selected as samples for SRS.

SRS was started in 1964 on a pilot basis and first gave national and subnational estimates for 1970. Since then SRS has been an annual source of data on births and deaths in India. This thesis mainly uses data from SRS, which are available as age-specific death rates for five-year age groups for total, rural and urban areas separately. Also, they are provided for males, females and persons. The SRS data are considered of good quality and provide a suitable database to model mortality in India.

Table 2.4: Average size of SRS sample units and proportion of population covered in major states, 1984 (rural)

State	Universe			Sample			Coverage	
	No. of units	Population (1981 census)	Average unit size	No. of units	Population (1981 census)	Average unit size	Units	Population
Andhra Pradesh	35,931	40,566,923	1,129	190	240,498	1,266	0.0053	0.0059
Bihar	76,450	63,877,524	836	400	398,352	996	0.0052	0.0062
Gujarat	22,206	23,231,174	1,046	200	243,483	1,217	0.0090	0.0105
Haryana	8,927	10,195,624	1,142	100	120,412	1,204	0.0112	0.0118
Karnataka	30,188	25,979,856	661	250	260,050	1,040	0.0083	0.0100
Kerala	12,983	20,142,539	1,551	150	228,988	1,527	0.0116	0.0114
Madhya Pradesh	70,927	30,011,242	547	300	223,547	745	0.0042	0.0074
Maharashtra	44,035	40,121,722	911	190	199,132	995	0.0043	0.0050
Orissa	44,155	23,565,655	536	250	176,600	787	0.0057	0.0075
Punjab	13,561	12,581,620	929	100	97,051	979	0.0074	0.0077
Rajasthan	37,178	27,657,610	714	230	183,284	797	0.0062	0.0066
Tamil Nadu	24,749	31,212,343	1,261	190	260,593	1,372	0.0077	0.0083
Uttar Pradesh	115,215	9,081,318	773	450	435,400	960	0.0039	0.0479
West Bengal	44,938	40,282,114	917	300	307,667	1,025	0.0067	0.0076
India	614,175	502,846,467	819	3947	3,864,133	979	0.0064	0.0077

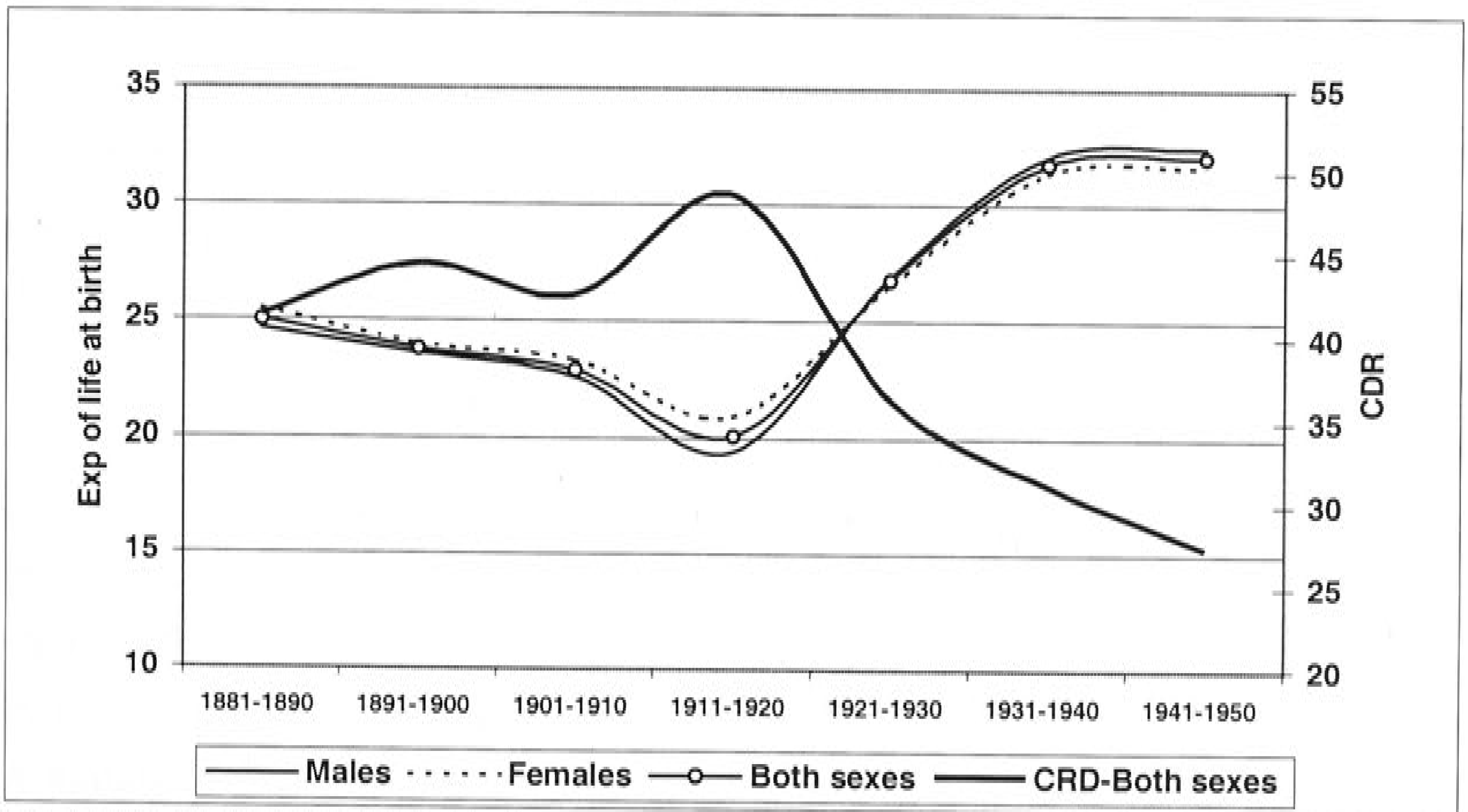
Source: Srivastava, 1989: 23.

SRS data are the prime source of national mortality statistics. Limitations to these data are that they are available on hard copies only and their first age group is 0-4; nonetheless the data provide consistent trends and patterns of declining mortality in India.

2.3 Mortality trends in pre-Independence India

The mortality levels in India up till 1921 were very high, as in other developing parts of the world: the crude death rate increased to as high as 48.6 deaths per 1000 population in the decade 1911-20. The expectation of life at birth for both sexes was 25 years for India during the decade 1881-90 and continued to decline for another 20 years. As Figure 2.1 shows, the expectation of life at birth was falling slowly but at a steady pace from 1881-90 to 1901-10. The steep decline to 1911-1920 was due to the influenza epidemic of 1918 (United Nations, 1982b), which took a heavy toll on the population in that year; 1921 was the turning point for Indian mortality as improvements started to be seen from that date (Davis, 1951: 34). Death rates fell to 27.4 in the decade 1941-50, an almost 48 per cent decline on the base of 1921. Females had a marginal advantage in the expectation of life at birth in the 1881-1920 period but this marginal advantage became a marginal disadvantage compared with the expectation of life for males during the period 1921-50. The estimated sex difference in the expectation of life at birth was less than one year during this period. However the advantage was partly maintained as an artefact generated by adjustment of intercensal survival ratios by the British Actuaries with an inherent assumption that biologically female have higher longevity than male.

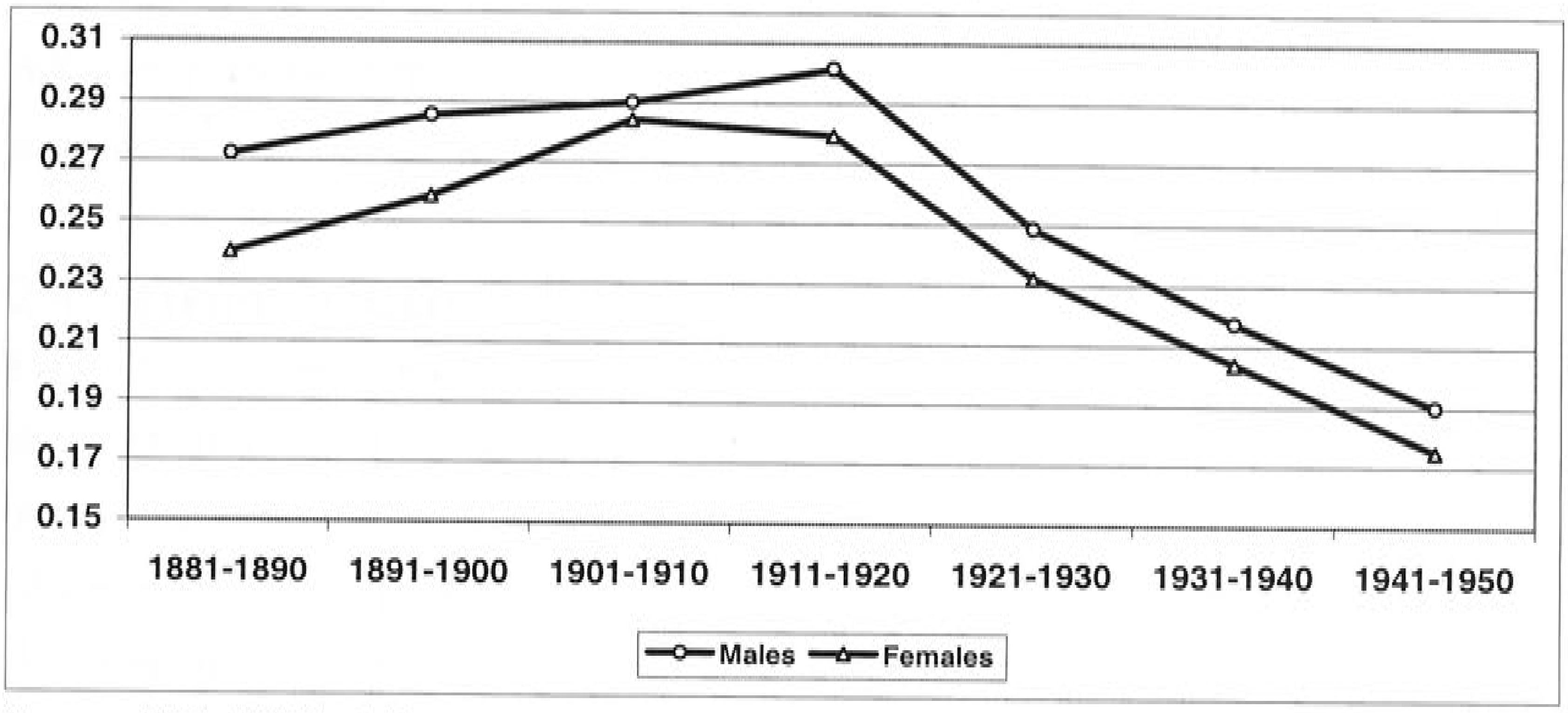
Figure 2.1: CDR and expectation of life at birth, India, 1881-1950



Note: Data relate to undivided India.
Source: UN, 1982b: 137.

Figure 2.2 gives the probability of death at under age one by sex for the time period 1881-1950 based on the available intercensal estimates (United Nations, 1982b: 143); it is evident that the probability of death before completing infancy was very high. As with the overall mortality indicators, a steady decline was observed in these probabilities from 1911-21 to 1941-50. These rates are estimated from intercensal survival ratios and based on models and are not very reliable; however, they provide a rough indication of the situation. There were undoubtedly regional variations in mortality but data limitations make it impossible to discuss them.

Figure 2.2: Probability of death before one year of age, India, 1881-1950



Source: UN, 1982b: 143.

The data presented in this section have been mainly taken from the United Nations country monograph on India (United Nations, 1982b). All the major indicators showed a decline in mortality in India after the 1911-21 period. It has been claimed on the basis of the limited data available that the decline in mortality was due to the elimination of war and banditry, the control of famine and the control of epidemic diseases (Davis, 1951). In the early years, the causes of death were very ill defined and very few people went to doctors for treatment; the causes of death were reported by the village headman or his nominated person. In many cases the reporters were illiterate so there tends to be a bias towards reporting fever in cases where the cause was not known or not clear (Davis, 1951). On this basis around 60 per cent of deaths were said to be due to fever for the period 1896 to 1945. A mixture of cholera, smallpox and plague was another large component accounting for about 10 per cent of the total deaths in 1896. This percentage touched 20 per cent in 1905 and fluctuated around 8 per cent in 1920. After 1920 the percentage of deaths from these three causes went down to around 6 per cent and then fluctuated around that value for another three decades. A combination of dysentery and diarrhoea accounted for four per cent of deaths throughout the period 1901-1945. Deaths from respiratory diseases increased gradually from about 2 per cent in 1902 to 7 per cent in 1945. Other causes remained at around 20 per cent during the whole period. Reported causes of death, however, must be considered to have a low degree of reliability.

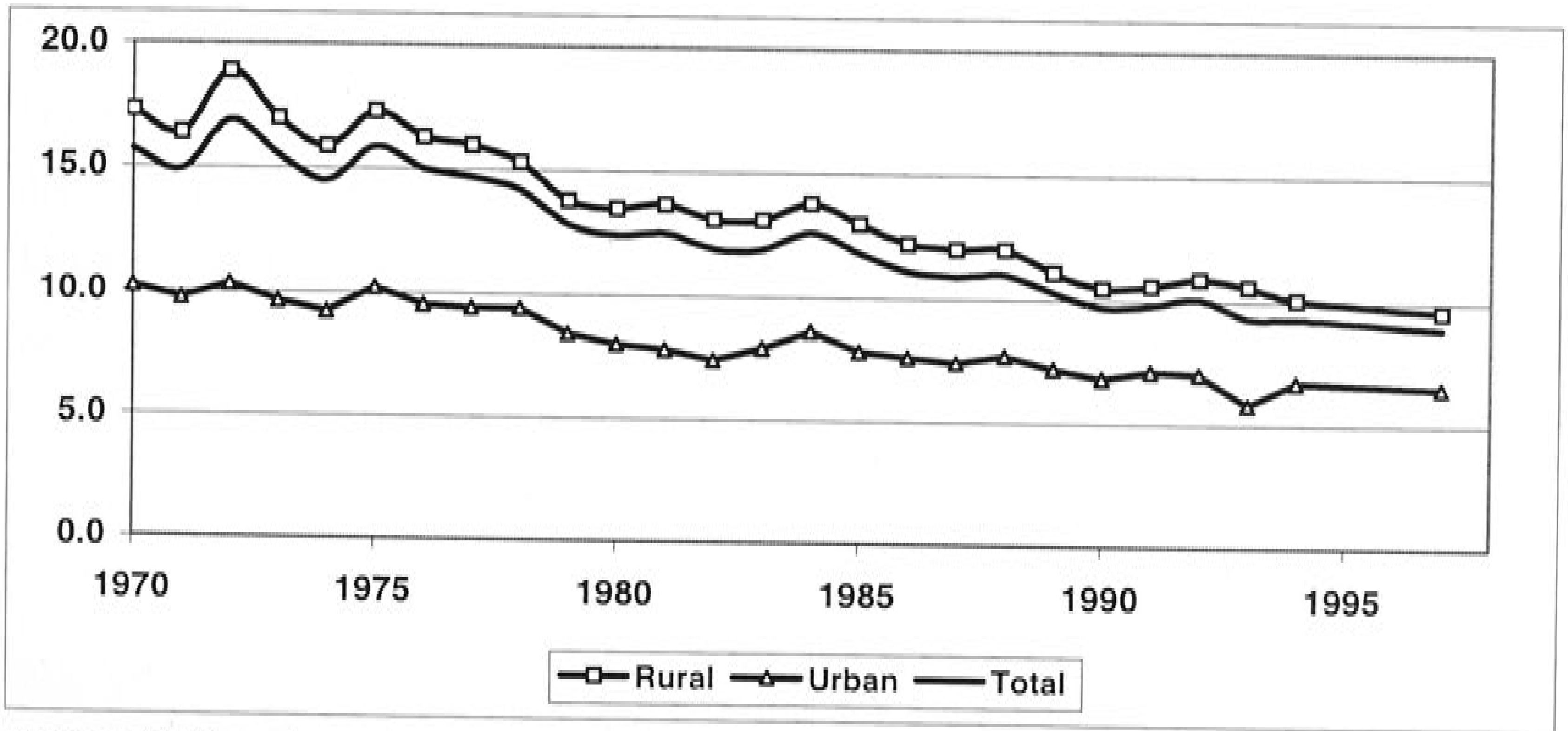
2.4 Mortality trends in post-Independence India

After Independence in 1947, mortality continued to decline slowly in India. Data by urban-rural status show that differentials between these areas were very large. The data used in this section are taken from the SRS, concentrating on the period for 1970 onwards in the majority of the analysis.

2.4.1 Trends in CDR

Figure 2.3 provides trends in crude death rates for the period of 1970 to 1997. It is evident that there is a huge rural-urban gap for the entire period of analysis. Mortality declined slowly with only relatively minor fluctuation throughout the period of 27 years of observation. In 1970 the CDR was 15.7, and it declined by 43 per cent to the level of 8.9 deaths per thousand population by 1997. The declines were 45 and 36 per cent of the base of 1970 values for rural and urban India respectively. The wide variation in rural-urban mortality makes it important to consider both categories for analysis in this thesis. The gap between rural and urban areas is narrowing over time.

Figure 2.3: CDR, both sexes, India, 1970-97



Source: SRS; various years.

2.4.2 Trends in expectation of life at birth

Table 2.5 shows the expectation of life at birth for males and females. Females were disadvantaged compared to males until 1983. From 1988, a marginal advantage in expectation of life at birth for females was observed: for rural females the convergence was as late as the 1989; urban females have had a marginal advantage since the 1970s. However in four northern states of India namely, Bihar, Madhya Pradesh, Orissa and Uttar Pradesh higher female mortality still persisted for 1992-96 (India.ORG, 2000b). The over all gap between rural and urban females in 1993 was as high as seven years. For males, the gap in life expectancy is about five years in 1993. For males, the gap in life expectancy is about five years in 1993. The urban-rural gap is due to better health care and a better quality of life in urban than rural areas. The gap is narrowing with time at a slow pace.

Table 2.5: Expectation of life at birth, India 1973-1993 (years)

Period	Males			Females		
	Total	Rural	Urban	Total	Rural	Urban
1970-75	50.5	48.9	58.8	49.0	47.1	59.2
1976-80	52.5	51.0	59.6	52.1	50.3	60.8
1981-85	55.4	54.0	61.6	55.7	53.6	64.1
1986-90	57.7	56.1	62.0	58.1	56.2	64.9
1991-95	59.7	58.5	64.5	60.9	59.3	67.3
1992-96	60.1	58.9	64.9	61.4	59.8	67.7

Source: SRS, various years.

2.4.3 Age pattern of mortality and trends in IMR

Figures 2.4 and 2.5 give the lexis map of the age-specific death rates for males and females in India from 1970 to 1997. The scale is defined so that darker colours represent high death rates and lighter colours represent low death rates. As time advances, clear patterns of mortality decline are seen for most of the age groups for males and females.

For males the lowest level of mortality in 1970 was for the age group 10-14. Gradual declines in mortality suggest that a similar level was attained by age band 5-24 by 1996. Decline is evident and there is not much excess mortality evident for the accident-prone ages.

Figure 2.4: Age-specific death rates for India, males, 1970-97 (per thousand)

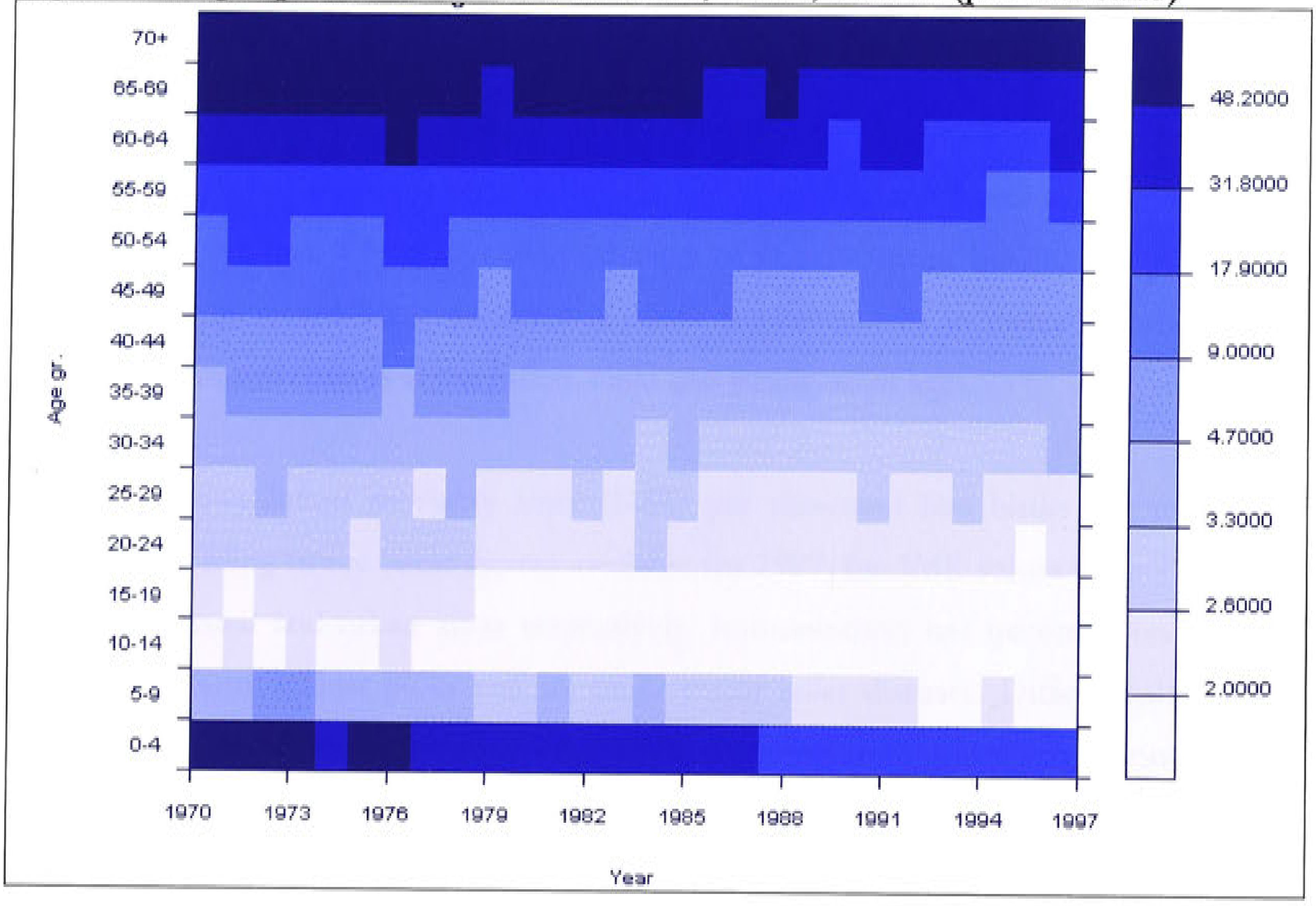
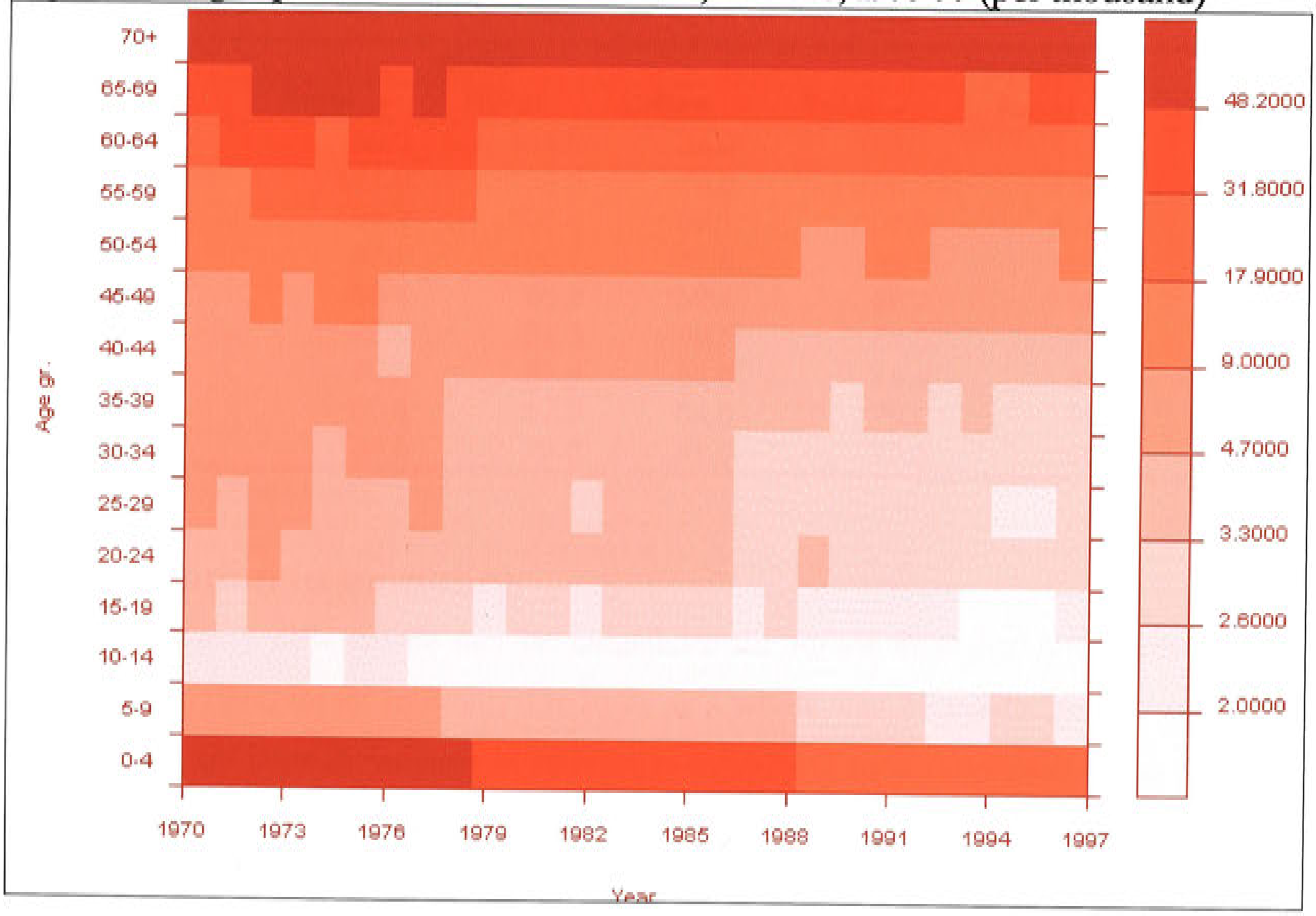


Figure 2.5: Age-specific death rates for India, females, 1970-97 (per thousand)



For females, the lowest mortality was for age group 10-14 in 1970 and had expanded to age range 5-19 by 1997. Under-five (child) mortality shows a systematic pattern of improvement with time, probably because of the increasing level of infant and child immunisation against major diseases. Mortality decline in the childbearing ages is also observed. Old-adult mortality declined in the 1970s mostly and remained constant for a long period after that. Older ages showed signs of improvement, but the contribution at these ages to increased life expectancy is small. Improvements in overall mortality arise mainly from improvements at the infant, child and young-adult ages.

Table 2.6 shows infant mortality rates (IMR) per thousand live births for males and females. According to the latest figures available for 1997, the IMR values were 71, 77 and 45 for total, rural and urban areas respectively. Immunisation has become prevalent for protecting infants against polio and six other major killer diseases. Differentials in IMR persist at a high level between rural and urban areas, suggesting that apart from medical facilities, the socio-economic condition of the household should also be considered in infant survival.

Table 2.6: Infant mortality rate, India 1973-1997 (deaths per 1000 live births)

Year	Males			Females		
	Total	Rural	Urban	Total	Rural	Urban
1973	130.0	139.3	84.6	121.1	144.8	82.8
1978	121.0	130.6	74.0	126.7	136.1	74.8
1983	104.2	103.0	65.1	104.9	112.8	62.1
1988	87.2	100.7	61.6	86.5	100.0	54.1
1989	89.4	96.2	59.4	86.7	95.2	52.4
1990	85.0	91.3	56.7	82.5	90.3	50.5
1992	80.0	85.9	53.8	78.3	84.5	50.9
1993	80.8	87.0	54.8	79.2	85.7	51.7
1997	69.8	76.0	41.5	73.5	78.9	48.7

Source: SRS, various years.

2.4.4 Variation in mortality

Current mortality differentials can be judged by the infant mortality rates presented in Table 2.7. In 1997, infant mortality for India as a whole was 71 deaths per thousand live births. There are large differences in relation to urban and rural residential status. The rate for urban India is 45 per thousand, almost 37 per cent lower than the rural rate, which was 77 per thousand live births. The very low rates in Kerala, especially in rural areas, are prominent. Uttar Pradesh and other northern Indian states on the other hand had rates that were well above the national level.

Table 2.7: Infant mortality rate, India and major states, 1997 (deaths per 1000 live births)

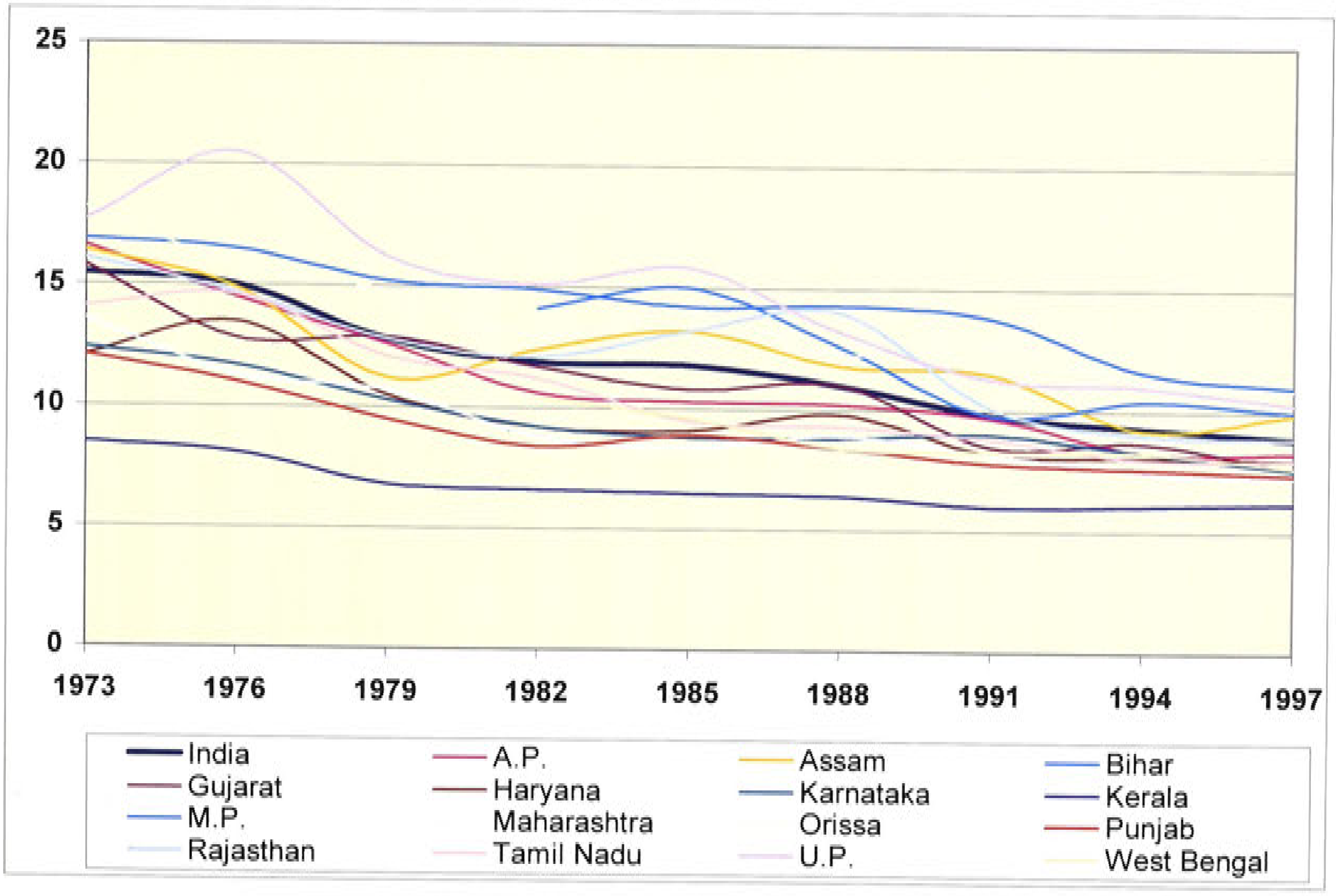
India/ major states	Total	Rural	Urban
India	71	77	45
Andhra Pradesh	63	70	37
Assam	76	79	37
Bihar	71	73	53
Gujarat	62	69	46
Haryana	68	70	59
Karnataka	53	63	24
Kerala	12	11	15
Madhya Pradesh	94	99	57
Maharashtra	47	56	31
Orissa	96	100	65
Punjab	51	54	38
Rajasthan	85	89	61
Tamil Nadu	53	58	40
Uttar Pradesh	85	89	66
West Bengal	55	58	43

Source: India ORG, 1998a.

2.4.4.1 Trends in CDR for India and its major states

Figure 2.6 shows trends in CDR for India and its major states from 1973 to 1997. It is seen that mortality has declined overall in each state presented in the figure. It is clear that Kerala remained well below the rest of the country for the entire period of 24 years, while Bihar, Orissa and UP remained in the highest group.

Figure 2.6: CDR, India and major states, 1973-97



2.4.4.2 Trends in life expectancy for India and its major states

Figures 2.7 and 2.8 show expectations of life at birth for India and major states for 1970 and 1994 respectively. In 1970 the states of UP, Gujarat and Orissa had the lowest expectations of life at birth among all the states. Haryana had the highest expectation of life followed by the state of Kerala. The evidence provided supports regional variations in the life expectancies over the period of analysis.

Figure 2.7: Expectation of life at birth, India and major states, persons, 1970

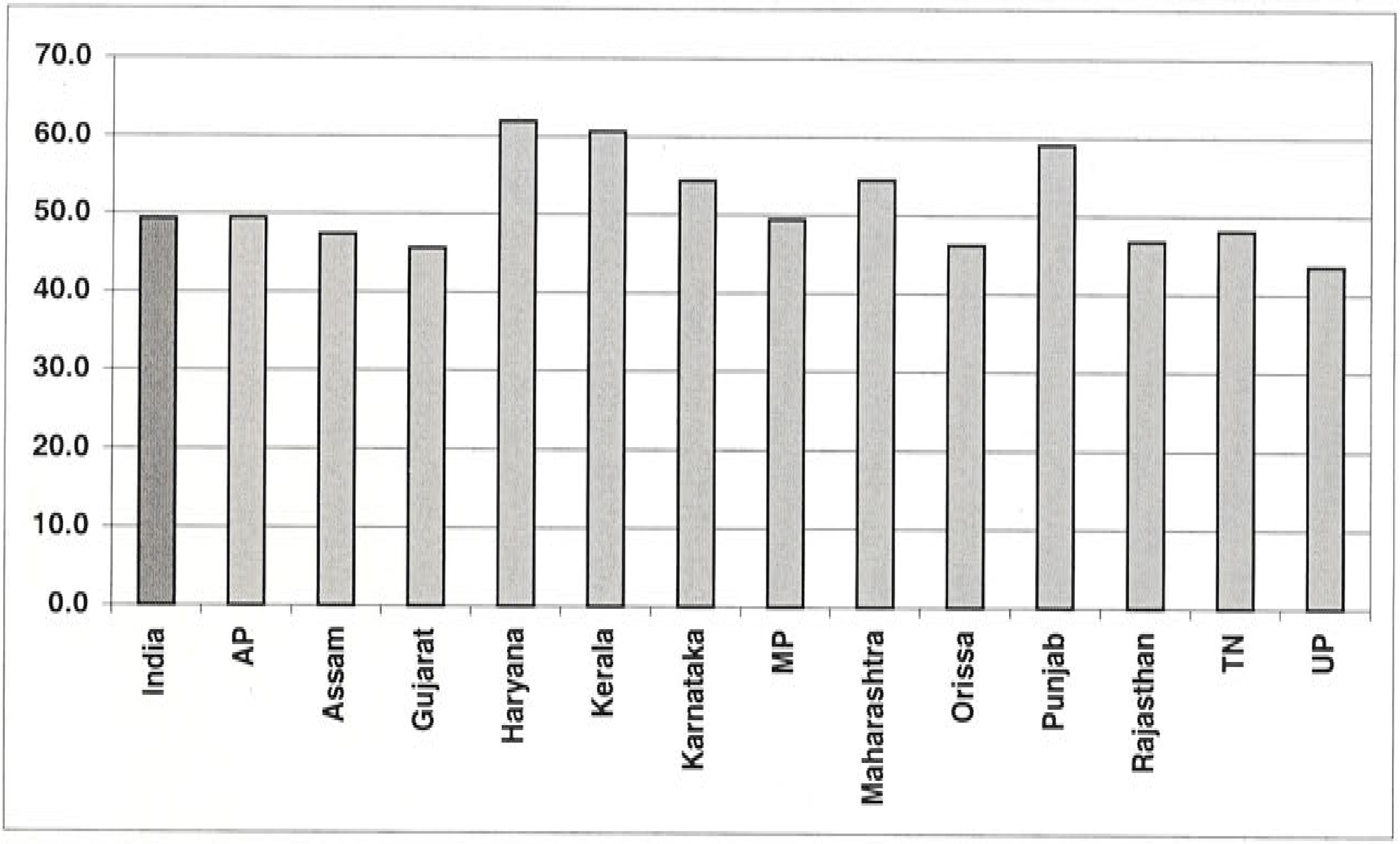
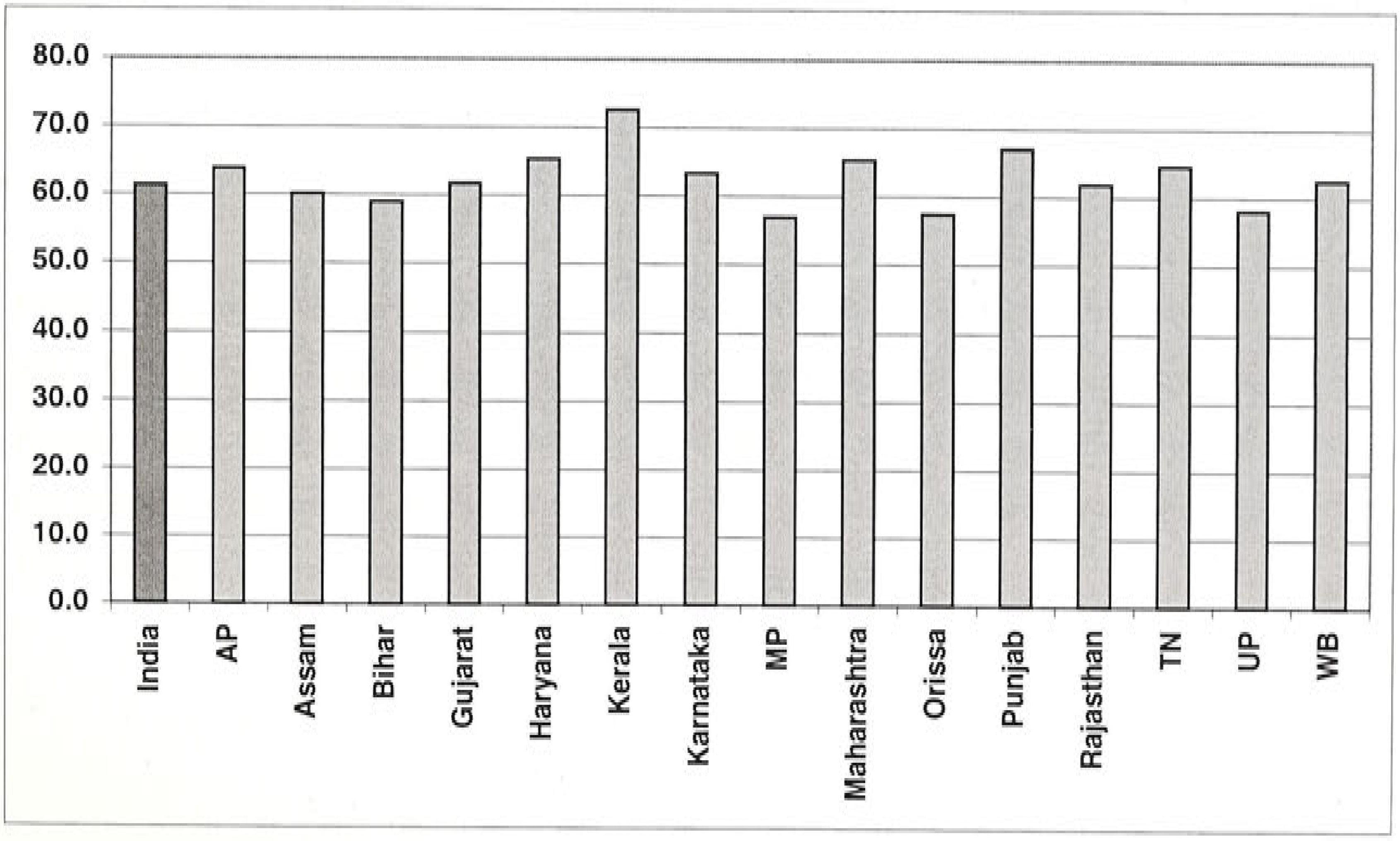


Figure 2.8: Expectation of life at birth, India and major states, persons, 1994



Kerala, having the lowest level of mortality and an exceptional performance in Indian demography, has always been a demographic unit for attention. On the other hand UP, the largest state in Northern India, has shown only slow and gradual improvements in demographic indicators.

2.4.4.3 Age pattern of mortality for India and major states

Figure 2.9 provides the plot of age-specific death rates for India and major states for the year 1970. Haryana and Punjab show irregular variations, possibly because of a smaller number of deaths for analysis, as they are relatively small states.

Figure 2.9: ASDR for India and states, 1970

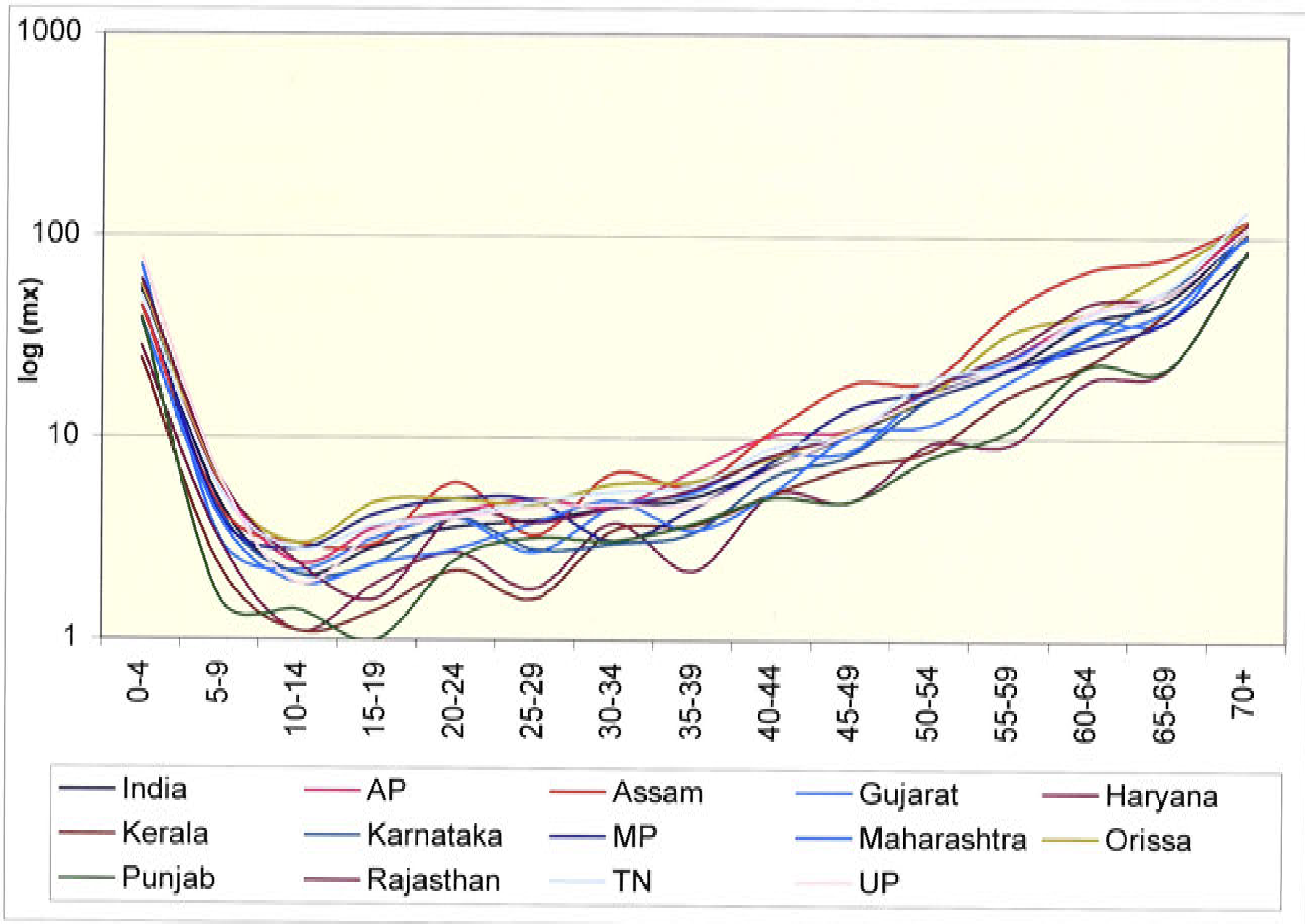


Figure 2.10 provides the plot of age specific death rates for India and major states for the year 1994; UP and Orissa have child mortality rates among the highest in this year as well. Kerala shows the lowest child mortality in 1994. The various states demonstrate varying age patterns of mortality. These patterns may be captured by the urban-rural data for 1970-1997. Figure 2.11 presents the four variants under consideration.

Figure 2.10: ASDR India and states, 1994

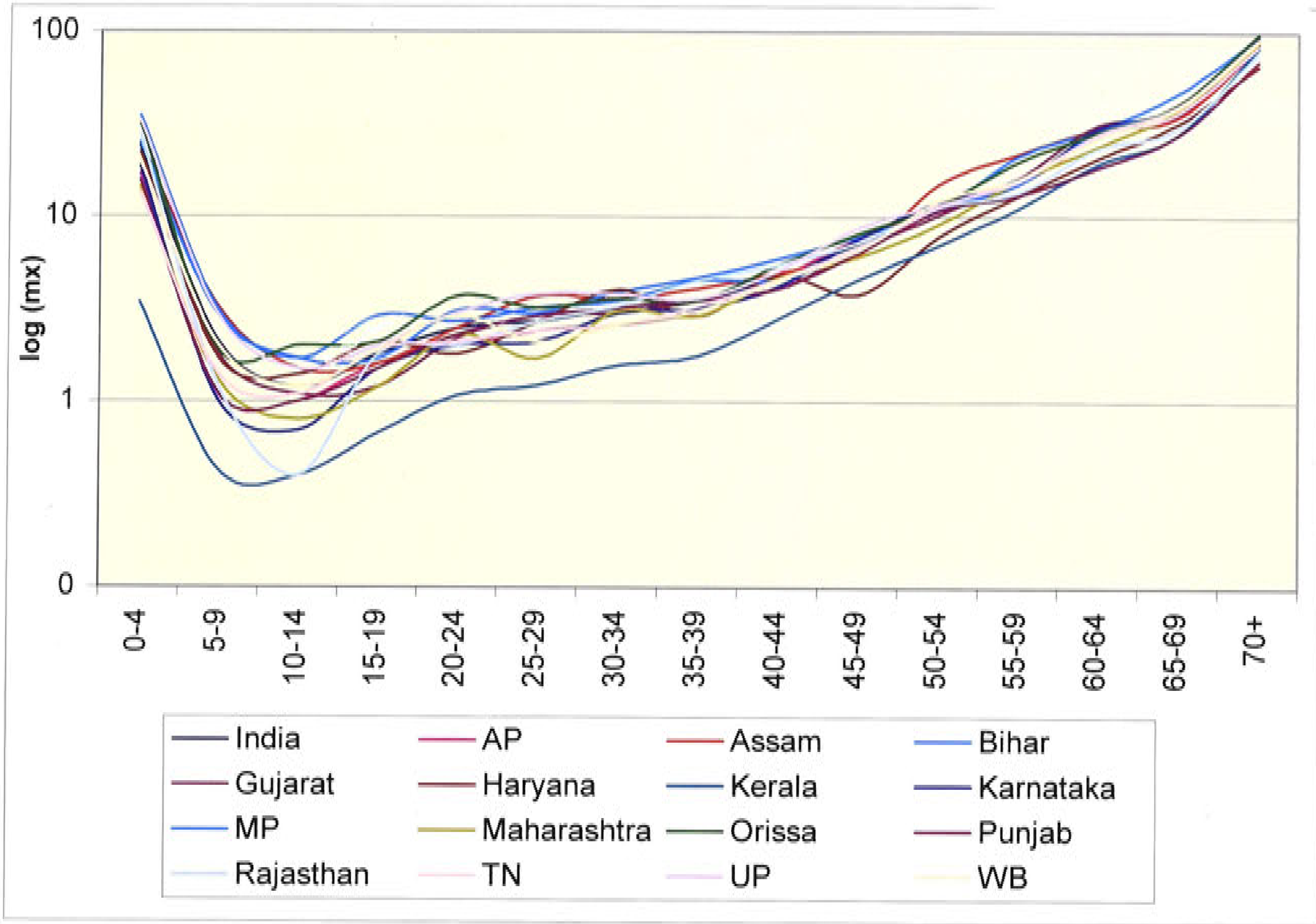
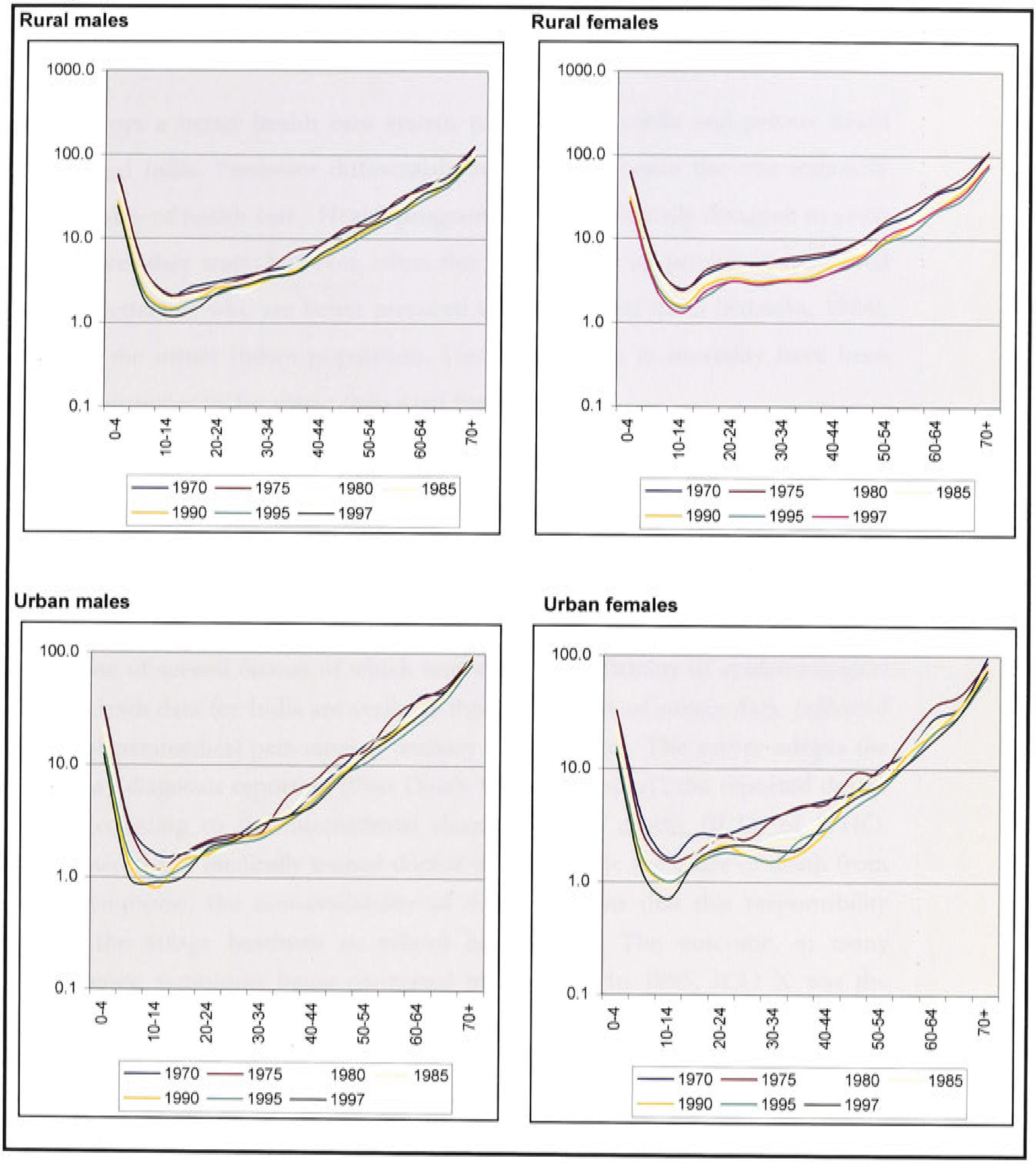


Figure 2.11: ASDR for rural-urban males and females, India 1970-97 (m_x on log scale)



Accelerated urban growth in economic development has been instrumental in bringing better health care infrastructure and quality of life. In India, like other developing countries, urban centres have been major tax paying areas and hence have been rewarded with better infrastructure and healthcare facilities. This facet of non-integrated development left rural India with very limited and inadequate health services along with other low quality of life aspects like lower literacy and very low per capita income. It can be safely assumed that rural parts are likely to experience higher mortality levels for these simple reasons. This gap widens in states like Uttar Pradesh which has low overall development levels compared with other states. For India this gap has been narrowing at a very slow pace.

Urban India enjoys a better health care system provided by public and private heath systems than rural India. Persistent differentials in survival between the two indirectly reaffirms the quality of health care. Health programmes were originally designed to assist the needy whoever they were; however, often they have ended up providing subsidised services to the better-off who are better prepared to make use of them (Ruzicka, 1984). In present case the urban Indian population. Declining trends in mortality have been observed more prominently for urban than rural India.

2.5 Cause-of-death data for India

Age patterns of mortality are related to causes of death. It is therefore of interest to examine the available data on causes of death, particularly with a view to discussing trends. Ruzicka (1986) described the difficulties of mortality projections in developing countries because of several factors of which one was non-availability of epidemiological data. Cause-of-death data for India are available through a series of survey data collected with the help of paramedical personnel at primary heath centres. The survey adopts the technique of 'Lay diagnosis reporting (Post Death Verbal Autopsy)'; the reported deaths are classified according to the international classification of deaths (ICD) of WHO. While it is claimed that a medically trained doctor will determine the cause of death from the reported symptoms, the non-availability of doctors means that this responsibility often falls to the village headmen or school headmasters. The outcome, in many instances, becomes, symptoms being converted into causes. In 1995, ICD X was the basis for analysing cause of death in India. In 1995, respiratory diseases were the major killer with 11.6 per cent of deaths. Heart disease, TB, prematurity, pneumonia and cancer were responsible for 8.2, 6.2, 6.1, 5.7 and 4.9 per cent of deaths respectively. Paralysis, anaemia, vehicular accidents and suicides were each accountable for three to four per cent of deaths. Examining the age composition of deaths in India shows that over three-quarters of the deaths due to bronchitis and asthma were concentrated in ages sixty and over. More than 80 per cent of deaths from heart disease applied to aged 45 and above. Deaths from tuberculosis of the lungs were also spread skewedly, with a concentration at higher ages. Over three-quarters of the deaths due to pneumonia were concentrated at ages below five; paralysis attacked more older people. Anaemia was a cause of death at both younger and older ages. As expected, vehicular accidents were highly concentrated at ages 15 to 44; over 80 per cent of the suicides were also in this age group.

Figure 2.12 shows trends in the ten leading causes of deaths for the period 1970 to 1991. Circulatory system diseases are increasing, owing to the changing age structure. Accidents and injuries have also increased over time, perhaps because of the increased number of vehicles.

Figure 2.12: Ten leading causes of death, India, 1970-91

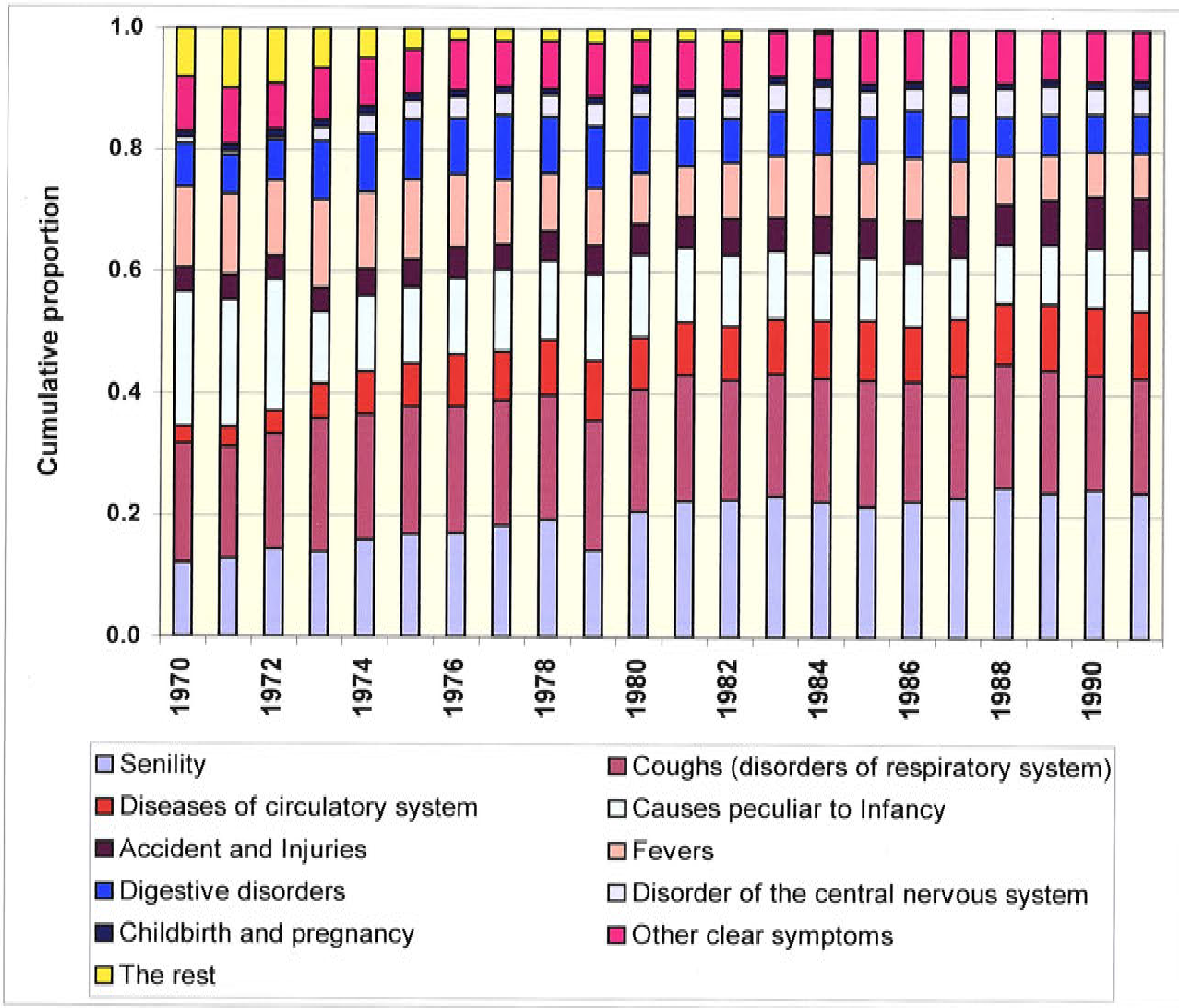


Table 2.8 presents registered deaths and their distribution according to medical certification in India and a few selected states for the years 1986 to 1988 . This table provides data for states by level of medical certification of registered deaths. Rates are shown for the two states with the highest level of certification, Goa and Manipur, and the two states with the lowest levels of certification, Uttar Pradesh (UP) and Madhya Pradesh (MP). Kerala has surprisingly low rates of medical certification.

Such disproportionate completeness of medical certification will lead to bias in cause-of-death data available in this database for India. Also the availability and proximity of heath facilities may have affected certification. In a data regime where vital registration itself is very poor and disproportionate around the country, the high proportion of uncertified deaths makes cause-of-death data unusable for the purposes of analysis. The unexpectedly high value for the certification of deaths for 1988 reported in the table seems an error, because not much improvement in bigger states like UP and MP has been reported

Table 2.8: Registered and medically certified deaths in India and a few selected states

India/ States	Registered Deaths			Medically certified deaths		
	1986	1987	1988	1986	1987	1988
India	2,272,794	1,821,940	2,417,865	330,239 (14.53 %)	338,323 (18.57 %)	353,593 (14.62 %)
UP	260,313	266,492	269,985	2,462 (0.95 %)	NA NA	2,379 (0.88 %)
MP	374,242	389,170	422,133	10,941 (2.92 %)	13,617 (3.50 %)	18,471 (4.38 %)
Kerala	133,654	141,047	NA	9,438 (7.06 %)	9,493 (6.73 %)	10,549 NA
Goa	7,453	7,682	8,839	6,686 (88.64 %)	6,745 (87.80 %)	7,691 (87.01 %)
Manipur	884	882	1,041	598 (67.65 %)	531 (60.20 %)	803 (77.14 %)

India ORG, 1992: 10-11.

2.6 HIV/AIDS and India

India has the second largest number of HIV/AIDS-infected people after South Africa: at the end of 1999, India had 3.7 million people with HIV or AIDS. However, the HIV prevalence rates are considered 'low' as only seven adults per 1000 are infected with HIV. India's HIV/AIDS prevalence is highly diverse among the states: some states reach adult rates of two per cent but a few states register almost no HIV infection (UNAIDS, 2000). At the end of 1999, the total number of AIDS deaths reported was 310,000 in a total population of 998 million: an average of one death per 3,218 persons. Not much is known about the age structure of deaths due to causes related to AIDS, but the low level compared with overall mortality is highly unlikely to affect significantly the age pattern of mortality in the near future. The technical group on population projections in India considered the effect of deaths due to AIDS and concluded: the age specific death rates were found to be unaffected up to second place of decimal after incorporating likely deaths due to AIDS. As such the future levels of the expectation of life at birth will have no significant impact of AIDS in the next 25 years (India ORG, 1996: 13).

2.7 Selection of data sets for analysis

The data used for the analysis are for India for the six variants; both sexes by total, rural and urban areas. Each of the six data series is available for 28 years on an annual basis, from 1970 to 1997. In total this adds to 162 sets of age specific death rates covering all the patterns of Indian mortality.

The states vary in socio-economic development and there are remarkable differences in demographic indicators, including mortality indicators. There remains a clear divide between the northern and southern states of India in demographic levels and patterns since the 1970s. While modelling mortality for Indian males and females with rural and urban variants for the period of 1970-1997, the variations persistent in various geographical regions and states have been included indirectly. One hundred and sixty two data sets for India are enough to cover most patterns in the states. Data from SRS volumes for various years have been used in this thesis for analysis (India ORG, 1982, 1984a, 1985a, b, 1986, 1987, 1988a, b, 1989a, 1991, 1992a, b, 1993a, b, 1994a, 1995a, 1997, 1998b, 1999, 2000a).

2.8 Conclusion

Mortality decline in India has been slow in the last three decades, but the striking variations in the levels between the states make it more interesting. There exists a clear divide between northern and southern India in mortality levels and patterns, as with other demographic indicators. However, rural-urban differentials are likely to cover the possible variations in patterns of mortality for individual years during 1970-97. As the civil registration system in India is still grossly incomplete, the SRS emerged as a useful data source on key vital events. Data from SRS will be used to analyse the mortality in India in this thesis. A concern about SRS data is that it provides age specific death rates for age 0-4 rather than the conventional 0-1 and 1-4; this limits the analysis to a certain extent. While it would have been possible to separate 0-4 mortality into 0-1 and 1-4, this would have involved the use of an existing and arbitrarily chosen model, which is inappropriate in a modelling exercise.

Cause-specific death data suffer from two major problems: massive under registration of deaths and insignificant levels of medical certification of registered deaths. Added to this, there is a wide variation in reporting among the different states giving a disproportionate representation to some causes of death. In this situation, cause of death data reveal very little of the true epidemiology of India. HIV/AIDS may emerge as a big health concern for Indian health care providers and other related agencies in future but its effect on the death patterns in India is likely to be negligible in the near future.

Chapter 3: Mortality Models - A Review

I have not failed. I've just found 10,000 ways that won't work.

Thomas Alva Edison

3.1 Introduction

Efforts to model human mortality started over 275 years ago when De Moivre discussed a one-parameter model in 1725. Mathematicians have long given attention to modelling this phenomenon; demographers, of course, made a larger contribution to mortality modelling, and actuarial scientists also contributed to developing suitable calculation procedures.

A range of models have been developed by various researchers; these models fit well in certain situations, such as for specific age segments or for developed-country data. The models are used in the graduation or smoothing of different functions of life tables or aggregated sets of mortality values. There are three major classes of mortality graduation: model life tables; relational models; and laws of mortality (mathematical equations). Each has several variations as described in the following sections. This chapter examines the major attempts at modelling human mortality. In the Indian case, adequate representation of the mortality patterns and availability of data are the guiding rules for the selection of the models for analysis. This chapter evaluates these models and considers their appropriateness for India.

3.2 Important Definitions

The following are the definitions and notation used in this thesis.

3.2.1 Life table

A life table is a statistical device used by actuaries, demographers, public health workers and others to present the mortality experience of a population aggregate in a form that permits answering many related questions on mortality (Namboodiri & Suchindran, 1987). The life table is also referred to as a mortality table; it is represented in a matrix form with the age column as the reference column. Despite the redundancy (Anson, 1988), it carries seven columns altogether. The first is the reference column, age or age group. Of the remaining six columns, three refer to exact age and three to age groups. The specific columns are presented as:

1. *Age interval x to x+n:* Age group, defining the lower and upper limit for the age interval. The final age interval is an open-ended interval.
2. *Survivorship lx:* this presents the probability of survival (or number of persons surviving) to the beginning of the age interval. The first value is the radix denoted l_0. If the radix is unity, l_x will correspond to the probability of survival to exact age x. If the radix is taken as other than unity (conventionally 100,000), l_x will correspond to the number of survivors at exact age x. in this thesis l_x is also referred to S_x, for the radix unity.
3. *Deaths $_nd_x$:* the number of deaths occurring between age x to x+n is presented in this column. The total of this column equals l_0.

$$_n d_x = l_x - l_{x+n} \text{....................}(3.1)$$

4. *Probability of death $_nq_x$:* this gives the probability of dying in the age interval x to x+n experienced by the l_x number of persons. The values in this column are derived by using age-specific death rates $_nm_x$. A basic formula to derive $_nq_x$ from age-specific death rates, $_nm_x$ is:

$$_n q_x = \frac{2n(_n m_x)}{2 + n(_n m_x)} \text{...........}(3.2)$$

Another formula (Chiang, 1968, 1984) has been used in this thesis to derive the $_nq_x$ values for Indian mortality tables. The formula is

$$_n q_x = \frac{n(_n m_x)}{1 + n(1 -_n a_x) n\, _n m_x} \text{...........}(3.3)$$

where $_na_x$ is the average fraction lived by persons dying in the interval x to x+n. So $_na_x$ becomes crucial for the derivations of $_nq_x$. Chiang's empirical investigation found that $_na_x$ is invariant with respect to sex, race, cause of death, geographical location, and other demographic variables (Namboodiri & Suchindran, 1987: 25). A probability function of survival termed as $_np_x$ is also used and is defined as $1 -_n q_x$.

5. *Person-years lived $_nL_x$:* The average number of person-years lived by l_x persons during age interval x to x+n is represented in this column, as not everyone starting at the beginning of each age group survives till the end of it. L_x is calculated as follows

$$ {}_nL_x = \frac{n}{2}[l_x + l_{x+n}] \text{.................}(3.4) $$

6. *Person-years lived beyond age x, Tx:* this gives the number of person-years lived by *lx* persons beyond age x. The expression is

$$ T_x = {}_nL_x + {}_nL_{x+n} + ... + L_w \text{..........}(3.5) $$

where w is the last age group. The relationship between two consecutive values of $\mathbf{T_x}$ is:

$$ T_x = {}_nL_x + T_{x+n} \text{..........}(3.6) $$

7. *Life expectancy e_x:* the expected (average) number of years to be lived by the $\mathbf{l_x}$ persons at exact age x. Since a total of $\mathbf{T_x}$ number of years are to be lived by $\mathbf{l_x}$ persons, the expectation of life at age x is computed as

$$ e_x = \frac{T_x}{l_x} \text{........}(3.7) $$

3.2.2 The force of mortality

This refers to the instantaneous force of mortality or hazard rate, which means that $\mu_x dx$ is the conditional probability of dying between x and x+dx given that the person is surviving at age x (Biswas, 1995: 194). When mathematical models are used, most of the formulae (Table 3.1) model μ_x, the force of mortality, which is defined by the equation:

$$ \mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} \quad \text{or} $$

$$ l_x = e^{-\int \mu(x)dx} \text{..............}(3.8) $$

3.3 Mortality graduation by model life tables

Through the study of many life tables for different populations, some regular patterns have been observed and classified as 'families' of life tables. Within a family, tables are classified according to levels, from high mortality to low mortality. Model life tables compensate for the absence of good-quality data, mainly for developing countries.

To use model life tables, levels above and below are selected according to the mortality indicator, like expectation of life at birth. By interpolation between the two selected levels, the graduated mortality corresponding to the desired level of expectation of life is obtained. Thus in the absence of comprehensive data on mortality, graduation can be based on available indicators of mortality. There have been five sets of model life tables in common use (United Nations, 1955, 1982a; Coale & Demeny, 1967; Coale, Demeny & Vaughan, 1983; Ledermann, 1969; Brass, 1971).

3.3.1 United Nations model life tables

By using 158 life tables collected from a wide selection of countries representing different time periods, the United Nations calculated model life tables in 1955 (United Nations, 1955). The relationship among q_x values was assumed to be chain parabolic between the probabilities of death for successive ages or

$$q_x = a + b q_{x-n} + c q_{x-n}^2 \text{..........}(3.9)$$

So each successor was estimated from the predecessor q_x value. Here q_0 becomes the core input and thereafter successive q_x values serve as input for further estimations. So a model system was created for the values of probability of infant death corresponding to values

q(0) = 20, 25, ...100 and thereafter

q(0) = 110,120,..................330.

To graduate mortality using the UN tables, q_0 is the required information for the study population. A level can be chosen according to q_0, thereafter the relevant mortality can be calculated.

3.3.2 Coale and Demeny's regional model life tables

Four different sets of regional life tables were given the names 'West', 'East', 'North' and 'South' in 1967 by Coale & Demeny (1967). These divided all possible describable mortality patterns into four distinct groups, popularly known as patterns. The East, North and South models were derived from small but homogeneous groups of life tables: East was based upon the life tables from Central European countries, North represented Scandinavian experience while the South pattern was from Southern European countries. North has low old-age mortality with low infant mortality relative to mortality at age 1-4; East has high infant and old-age mortalities; South has higher rates for age 1-4 and lower rates in late middle age. The West model was represented by 125 life tables from more than 20 countries including Canada, the USA, South Africa, Israel, Japan, Taiwan and western European countries. The West mortality pattern is close to the pattern for the whole world's mortality experience and hence is commonly used for overall comparisons.

Methodologically, double regressions, namely linear and logarithmic equations between e_{10} and ${}_nq_x$ values, were used to derive the life tables. By solving the following equations (3.10) and (3.11) for one initial value of q_0, four different values of ${}_5q_x$ are created (Coale, Demeny et al. 1983:19). The equations are

$$ {}_nq_x = A_x + B_x e_{10} \text{..............}(3.10) $$

and

$$ \log {}_nq_x = A_x^{\,1} + B_x^{\,1} e_{10} \text{..............}(3.11) $$

3.3.3 Ledermann's model life tables

Ledermann (1969) published seven sets of model life tables, each with one and two parameters, by using 154 individual life tables as input. The equations he used for estimation of q_x values were:

$$ \log {}_nq_x = a_{x_0} + a_{x_1} \log {}_nq_j \text{..................}(3.12) $$

$$ \log {}_nq_x = b_{x_0} + b_{x_1} \log {}_nq_i + b_{x_2} \log {}_nq_j \text{..................}(3.13) $$

The seven sets of tables were based on different variables used for of ${}_nq_i$ and ${}_nq_j$ values in the right hand side of the equations (3.12) and (3.13). These seven sets of input values were e_0, q_0, ${}_5q_0$, ${}_{15}q_0$, ${}_{20}q_{30}$, ${}_{20}q_{35}$ and m_{50+}. These tables provide the choice of using the most reliable or easily available piece of information to get the q_x column of the life table for a population.

3.3.4 United Nations model life tables for developing countries

In 1982 the United Nations published a new set of age and sex patterns of mortality for developing countries based on more reliable data. First, clusters of similar life tables, according to age patterns, were formed graphically and statistically. Principal component models were fitted to the deviations of each age pattern of mortality from their own averages (Pathak & Ram, 1992: 101). Clusters for similar sets of q_x values were used to find average logits to reach a common life table. They defined five different patterns: Latin American, South Asian, Far Eastern, Chilean and General.

3.4 Mortality graduation based on relational models

Brass considered similarities of patterns in mortality rates as a reason to search for a simple method to describe the relationship between the mortalities in different countries or at different time periods in the same country (Brass, 1971: 69). He explored mathematical transformations to establish links between the l_x columns of various life tables using two parameters. Brass used the logit system in order to transform the survival curve (l_x) from a finite to an infinite range of values. These life tables serve as baseline models for developing countries where regular data on mortality are not available. His idea was later enhanced by increased parameters; it inspired many demographers and actuaries to explore the logit relationships given by Brass.

Another relational model was a mathematical transformation used by Lee & Carter (1992) in modelling US mortality. This model uses logarithms of the mortality rates in time series as input, and by matrix operations it provides age and time effects of mortality. They termed their model a demographic model. This section discusses the Brass and Lee-Carter models in detail.

3.4.1 Brass logit models

3.4.1.1 The logit system of mathematical transformation

Brass observed, by comparing United Nations average mortality schedules, that a mathematical transformation of **$1-l_x$** produces a linear relationship with x over most of its range (Brass, 1971: 73). This mathematical transformation is called **logit**. He has shown a linear relationship among logits of different life tables from one country at different time periods or among different countries. The logit transformation of **Z** is

$$\operatorname{logit} Z = \frac{1}{2}\log_e \frac{Z}{1-Z} \text{.................}(3.14)$$

Hence:

$$\operatorname{logit}\ [1-l_x] = \frac{1}{2}\log_e \frac{1-l_x}{l_x} \text{.................}(3.15)$$

Brass used the logit transformation to develop standard logit values of $1\text{-}l_x$ using a standard input life table,

$$Y^s{}_x = \operatorname{logit}\,[1-l^s{}_x]\text{...............}(3.16)$$

The linear relation between observed and standard mortality is

$$Y_x = \alpha + \beta . Y^s{}_x \text{.................}(3.17)$$

where $\mathbf{Y_x}$ and $\mathbf{Y^s{}_x}$ are the logit transformations of $\mathbf{1\text{-}l_x}$ values of the study and standard populations. In order to know the level and pattern of mortality one needs estimates of α and β by using equation (3.17). This is the simplest life table system as it needs just two parameter values to derive a whole life table. Brass provided two standard patterns of mortality, the General Standard and the African Standard. The African Standard is based on experience with high infant and child mortality.

The logit transformation provides a representation of mortality that is good for middle ages but is not so good at the beginning and end of the mortality curve compared to the actual pattern. The question of improving the fitting at the ends was a research area studied by different researchers: Zaba (1979), Ewbank, Gomez de Leon & Stoto (1983), Kaneko (1995) and Mitra (1997). In order to overcome the distorted fit, Zaba and Ewbank *et al.* added extra parameters to gain a better fit at young and old ages where the fits seemed distorted. Their methods require the data to be available for the age group 75 and above. In other words a noticeable difference in fitted values will be visible only if these methods are applied on the data with age groups 75 and above. These modified Brass relational systems are discussed in detail in the next sections.

Relational models are useful in conjunction with indirect techniques in situations where mortality data are lacking in developing countries. They provide a means of obtaining complete mortality (i.e. life table) from incomplete estimates. By these techniques of relational-type models, a fairly close but not accurate sense of mortality can be reached.

3.4.1.2 The four-parameter logit life table system

Zaba (1979) defined a new standard $\mathbf{l^N(x)}$ by specifying two constants Ψ and χ such that:

$$l^N(x) = l^S(x) + \psi k(x) + \chi t(x) \text{............}(3.18)$$

and the life table can be derived from the relational equation:

$$\mathbf{\log it\,[1 - l(x)] = \alpha \;+ \beta \,\log it[1 - l_N(x)]} \text{....................}(3.19)$$

So α, β, ψ and χ are the four parameters which are claimed to provide a better fit to the survival values l_x. The parameter α is a scale parameter and β, ψ and χ are shape parameters: β shows the general pattern, ψ allows the curvature in standard patterns corresponding to infant and old ages in any one direction and χ is for similar twists responsible for the deviations in opposite directions, corresponding to infancy and old age. The four-parameter model provided an improved fit compared to Brass's original fitting of mortality; however Zaba warned against using the four parameters as input for future projections, that is, extrapolations.

3.4.1.3 A reducible four-parameter system of model life tables

To make further improvements Ewbank *et al.* (1983) gave a new relational model, as in the following equation:

$$Y_x = \alpha + \beta T(l^S{}_x; \kappa, \lambda) \quad \text{..............(3.20)}$$

where

$$T(p;\kappa,\lambda) = \begin{cases} \dfrac{\left(\dfrac{p}{1-p}\right)^k - 1}{2k} & \text{for } p \geq 0.5 \end{cases}$$

$$T(p;\kappa,\lambda) = \begin{cases} \dfrac{1-\left(\dfrac{1-p}{p}\right)^\lambda}{2\lambda} & \text{for } p < 0.5 \end{cases} \quad \text{................(3.21)}$$

They described this model as a more general form of Brass's transformation. When λ and κ approach zero, equation 3.20 approaches the original logit transformation. This system of transformation is claimed to have more flexibility at the ends where fits were inadequate for Brass and Zaba. As κ affects $T(p;\kappa,\lambda)$ only for $p > 0.5$, it will affect only early ages, and λ accounts for changes at the oldest ages.

So α, β, κ and λ are the parameters to define mortality in terms of level and pattern; α remains the level parameter, β represents the general pattern, κ responds to the changes in young ages only and λ the older ages of the fitted l_x values. This system was claimed to provide a better fit than other relational models.

3.4.2 Lee-Carter Model

Lee & Carter (1992) proposed a model using comprehensive mathematical techniques to decompose a mortality matrix into time and age components. They modelled US mortality between 1900 and 1989. They also modelled the sex differentials in US mortality by analysing data for 1933-1988 (Carter & Lee, 1992). Tuljapurkar, Li & Boe (2000) fitted the Lee-Carter model to G7 countries for five decades time series data for those countries. Booth *et al.* (2000) fitted Lee-Carter to Australian data to compare with the G7 fit by Tuljapurkar *et al.* They found that by selection of a suitable base time period, forecasts could be improved.

The Lee-Carter model is expressed as:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \text{.................}(3.22)$$

or

$$m_{x,t} = e^{a_x + b_x k_t + \varepsilon_{x,t}} \text{.................}(3.23)$$

where

$m_{x,t}$ is the matrix of age-specific death rates for age x at time t;

k_t is the index of mortality over time;

a_x is the average age specific mortality;

b_x is the age specific constant, representing the age component;

$\varepsilon_{x,t}$ is the error term of the equation with mean 0 and variance σ_x^2.

Since there is no direct independent variable (regressor), a matrix decomposition technique was applied, which takes the age-specific death rates matrix and decomposes it into age and time effects. The first vector usually accounts for more than 95 per cent of the overall variation, so by considering only the first vector the estimated values capture much of the variation. However Booth, Maindonald & Smith (2001) show that by consideration of the second and third vectors, small but systematic improvements can be made, which are important methodologically. The method was also used for forecasting in the population forecasts for the USA (Lee & Tuljapurkar, 1994). Lee & Miller (2001) suggest that the Lee-Carter method can provide a useful baseline for planners despite wide forecasting intervals in some cases. A detailed methodology of fitting is discussed in Chapter 4 where actual fitting results are also presented.

3.5 Mortality graduation by mortality laws

Graduating mortality by laws of mortality has been addressed by various researchers (Gompertz, 1977; Makeham, 1977; Heligman & Pollard, 1980; Petrioli, 1981; Mode & Busby, 1982; Anson, 1985; Carriere, 1992; Kostaki, 1992). Mathematical graduation not only serves the purpose of smoothing the irregular fluctuations in data but also provides some parameters for comparison. Mathematical models with fewer parameters (parsimony) are superior so long as they are capable of representing the observed pattern. Table 3.1 shows the functional equations used in the various attempts at the parameterisation of human mortality. The initial attempts were of a nature to represent the mortality for age 30 and above.

It is evident from various studies that human mortality has three distinctive components: it is composed of three different mortality patterns dependent on events related to age or life cycle. The three components are high early childhood mortality, an accident hump in the middle ages and a senescent component at older ages. Most of the earlier models did not take the accident hump into consideration. Also single-component models tend to model a specific part of mortality. The models by Perks, Harper, Weibull and Van der Mean are examples of single-component models.

Thiele was the first to model the whole age pattern of mortality by using three exponential curves with seven parameters, but it was not a continuous curve. Among them, the most frequently used law is that presented by Heligman & Pollard (1980); this 8-parameter model was able to explain the variation in Australian mortality for three different periods. Later Kostaki (1992) introduced one more parameter to present a nine-parameter version of the Heligman-Pollard model. Another model by Carriere (1992) is similar to Heligman-Pollard in that it has the same number of parameters and represents human mortality by three extreme-value distributions. The advantage of the Carriere model is that it assigns three different overall probabilities to the three different age segments of life. Other three-component models have also been given by various researchers. Some of them are polynomials of different orders such as by Krane and Anson. Polynomials are popular for interpolation and graduation as they can be explained by a Taylor power series. Non-polynomials have the advantage that they are derived on the basis of causes of death related to the different stages of the life (Tabeau, Berg Jeths & Heathcote, 2001:6). The models by Heligman & Pollard and Carriere are the advanced forms of non-polynomial mathematical laws of mortality. The components are defined in such a way that each of them vanishes at ages different from those for which they are basically specified (Tabeau *et al.* 2001:6). As in similar attempts by Mode & Busby (1982) and later by Mode (1984; as cited in Anson, 1985: 9), the Heligman-Pollard model provided a very good fit to the empirical data. The qualities of a good mortality model include parameters for sharp comparison and forecasting (Keyfitz, 1982) as well as exact representation of age patterns of mortality. The Heligman-Pollard and Carriere models exhibit these qualities except for some extra number of parameters they tend to possess.

Table 3.1: List of comprehensive mortality laws by the number of parameters and functional equations

Name of researcher	Year	Mortality fn. modelled	Number of parameters	Functions
De Moivre	1725	$\mu(x)$	1	$1/(\omega - x)$
Gompertz	1825	$\mu(x)$	2	Bc^x
Makeham	1867	$\mu(x)$	3	$A + Bc^x$
Opperman	1870	$\mu(x)$	3	$a/\sqrt{x} + b + c\sqrt{x}$
Thiele	1872	$\mu(x)$	7	$a_1 e^{-b_1 x} + a_2 e^{-.5 b_2 (x-c)^2} + a_3 e^{b_3 x}$
Wittestein	1883	$q(x)$	4	$\frac{1}{m} a^{-(mx)^n} + a^{-(M-x)^n}$
Perks	1932	$\mu(x)$	5	$\frac{A + Bc^x}{Kc^{-x} + 1 + Dc^x}$
Harper	1936	$\log_{10} S(x)$	4	$A + 10^{B\sqrt{x} + Cx + D}$
Weibull	1939	$\mu(x)$	2	$\kappa x^a (w-x)^b$
Van der Mean	1943	$\mu(x)$	5	$A + Bx + Cx^2 + I/(N-x)$
Brillinger	1960	$\mu(x)$	4	$\sum_i \left[H_i (x - B_i)^{C_i - 1} + \frac{A_i}{(b_i - x)^{C_i + 1}} + E_i d_i^x \right]$
Beard	1961	$\mu(x)$	3	$\frac{Be^{ux}}{1 + De^{ux}}$
Petrioli	1981	$\{1-l(x)\}/l(x)$	8	$1 \Big/ \left[x^a (\omega - x)^{-b} e^{\frac{c}{2}x^2 + dx} \frac{1}{k} + 1 \right]$
Krane	1963	$\mu(x)$	8	$a + bx + cx^2 + \ldots$

Table 3.1 contd.: List of comprehensive mortality laws by the number of parameters and functional equations

Name of researcher	Year	Mortality fn. modelled	Number of parameters	Functions
Heligman & Pollard	1980	q(x) /p(x)	8	$A^{(x+B)^C} +$ $De^{\{-E(\log(x/F))^2\}} +$ GH^x
Mode & Busby	1982	μ(x)	8	$\mu_0(x) = \alpha_0\beta_0 e^{-\beta_0}$ $\mu_1(x) = \frac{\beta_1\gamma^3_1}{3}$ $-\alpha_1 x \frac{\beta_1}{3}(x-\gamma_1)^3$ $\mu_2(x) = \alpha_2 + \beta_2\gamma_2 e^{\gamma_2 x}$
Mode	1984	μ(x)	11	$e^{a_0} b_0 d_0 (x+c_0)^{d_0} - 1.$ $e^{\{-b_0(x+c_0)\}^{d_0}} + e^{(a_1 - b_1(\log c_1 x)^2} +$ $e^{a_2} b_2 d_0 (x+c_2)^{d_2-1}\ e^{\{b_2(x+c_2)\}^{d_2}}$
Anson	1985	μ(x)/σ	6	$[\sigma(x-\xi)]^4 + \phi[\sigma(x-\xi)]^2 +$ $\tau[\sigma(x-\xi)] + \lambda$
Carriere	1992	S(x)	8	$\sum_{k=1}^{3} \psi_k S_k(x)$

Source: Anson, 1985; Gage & Mode, 1993; Tabeau *et al.*, 2001.

3.6 Choice of models

Model life tables are useful in providing some idea of mortality in the absence of adequate data. Though they are carefully constructed by analysis and evaluation of available data, they suffer from the errors of approximation in computing indices (Mathew, 1997: 22). In modern days, through censuses and demographic and health surveys, better data are more commonly available. Also developing countries today have had different demographic experiences from the developed countries because of the exchange of medical and contraceptive technology in the era of globalisation. Their mortality patterns therefore may differ from those of the now industrially advanced countries when they were developing medical technology. In fact, India has more detailed data available than are required for choosing a model life table so these models are not pursued further.

As the Brass logit life table system relates any mortality experience to the standard one, it is worth using for Indian data to see the goodness of fit. Although the Zaba and Ewbank *et al.* models are improvements on Brass, these models are not suitable for forecasting and the data limitations do not allow them to be applied to Indian mortality. In the present situation, when SRS data for India are generally published only for the ages 70+, these methods are difficult to apply.

The Lee-Carter model, for which the first vector usually explains more than 95 per cent of the overall variation, allows forecasting from a time series of mortality. Since data for Indian mortality are available according to time series, this method can also be tried with Indian data. Notably the method has not been tested for any developing-country data so far. This model serves as an efficient tool for graduating mortality. It has more utility for forecasting as it proceeds without any subjective assumptions. It is worth trying with Indian data to see the nature and pattern of fit and viability of results. Though this model represents the time index of mortality, it assumes that the age component remains constant over time.

Among the laws of mortality, the H-P model was successfully tested on the various data sets. Its logical capacity to model the three different components of mortality gives the model a theoretical base for the modelling. Unlike many other attempts it covers the entire age range. Among the others, which cover the entire age ranges, the H-P model has been fitted extensively to various mortality experiences across countries. It has been observed that despite some irregularities in obtaining weighted least square estimates of parameters, it fits reasonably well to the data (McNown & Rogers, 1989a, b; Rogers & Gard, 1991; Congdon, 1993). Also it has more meaningful explanations as it represents the probability of death. It has all the qualities a mortality model should have; so it is justifiable to test with Indian data to find its suitability and usefulness. The Carriere model is similar with a more statistically acclaimed form and should represent mortality as well as the Heligman-Pollard model. Fitting of the Carriere model will provide the evidence to choose between these two models.

Chapter 4: Selected Mortality Models-Fitting to Indian data

All differences in this world are of degree, and not of kind, because oneness is the secret of everything.

Swami Vivekananda

4.1 Introduction

This chapter shows the fitting to Indian data of the four mortality models selected in Chapter 3: the Brass logit model, the Heligman-Pollard model, the Carriere model and the Lee-Carter model. The emphasis is on the usefulness of mortality models in terms of whether they adequately characterise mortality in India.

The Brass logit model is fitted to five yearly life tables for males and females for India for 1971-75 (India ORG, 1984b), 1976-80 (India ORG, 1985c), 1981-85 (India ORG, 1989b), 1986-90 (India ORG, 1994b) and 1991-95 (India ORG, 1998c). These life tables have 0-1 as a first age group, while in time series data on age-specific death rates the first age group is 0-4. For the Heligman-Pollard model, intercensal life tables of India for males and females for 1951-60 and 1961-70 have been used. For the Carriere model intercensal life tables for 1951-60 and 1961-70 were at first used, but because of non-convergence of the model the SRS life tables for 1970-75, 1981-85 and 1991-94 were also used. For fitting the Lee-Carter model, the age specific death rates for males and females for the years 1971-1995 from the sample registration system were used, and 1971, 1979, 1987 and 1995 were selected to show the fit.

4.2 Fitting the Brass logit system

The Brass system (Brass, 1971) requires a minimum of two parameters to provide a relation between a standard and a study life table. As concluded by many others (Zaba, 1979; Ewbank *et al.*, 1983; Mitra, 1995, 1997), the method tends to provide a systematically distorted fit. The Brass two-parameter system was fitted to five life tables for males, females and persons, using the General Standard. Normal equations were minimised for least square estimates of the coefficients α and β. The equation can be written as:

$$\sum Y_x = n\alpha + \beta.\sum\ Y^s{}_x \text{.................}(4.1)$$

where Y_x is the logit of $1\text{-}l_x$ of Indian life tables and Y^s_x is the logit of Brass General Standard. Estimated values of α and β are shown in Table 4.1. The parameters α and β are highly significant statistically, but from Figure 4.1 it may also be noted that for 1991-95 errors are greater at older ages. The details of the fit to each life table are given in Tables 4.2 to 4.6. Similar error patterns for all the five life tables were observed. The results show that the fits are not only poor at very young and very old ages but also at the middle ages. Decline in parameters α over time has been observed for males, females and persons; meaning an increase in the life expectancy for the population. The slope β was found to be increasing in all cases; an increasing slope implies a limited decline in life expectancy. Overall the model predicts the mortality patterns responsible for increases in life expectancy in all cases.

Table 4.1: Estimated values of Brass α and β for Indian life tables, 1970-95

Males					
Time	**Parameter**	**Estimate**	**SE**	**Student's t**	**Prob(>|t|)**
1970-75	α	-0.2381619	0.016027	-14.86	4.33E-09
	β	0.8367094	0.038464	21.75	5.22E-11
1976-80	α	-0.2924675	0.017205	-16.99	9.22E-10
	β	0.8352229	0.041292	20.22	1.22E-10
1981-85	α	-0.3871119	0.015870	-24.39	1.36E-11
	β	0.8337590	0.038088	21.89	4.85E-11
1986-90	α	-0.4702991	0.015823	-29.72	1.31E-12
	β	0.8682515	0.037975	22.86	2.91E-11
1991-95	α	-0.5360511	0.017474	-30.67	9.03E-13
	β	0.8860797	0.041936	21.12	7.34E-11
Females					
1970-75	α	-0.1827838	0.011596	-15.76	2.20E-09
	β	0.7754369	0.027830	27.86	2.82E-12
1976-80	α	0.2611814	0.011220	23.28	2.40E-11
	β	-0.7334969	0.026920	-27.24	3.70E-12
1981-85	α	-0.3673256	0.010672	-34.42	2.30E-13
	β	0.7459865	0.025612	29.12	1.67E-12
1986-90	α	-0.4618558	0.010114	-45.66	7.94E-15
	β	0.7554709	0.024274	31.12	7.61E-13
1991-95	α	-0.5444899	0.006459	-84.29	5.18E-18
	β	0.7694658	0.015502	49.63	2.93E-15

Figure 4.1: Brass logit fit to SRS life tables and errors, India, 1991-95

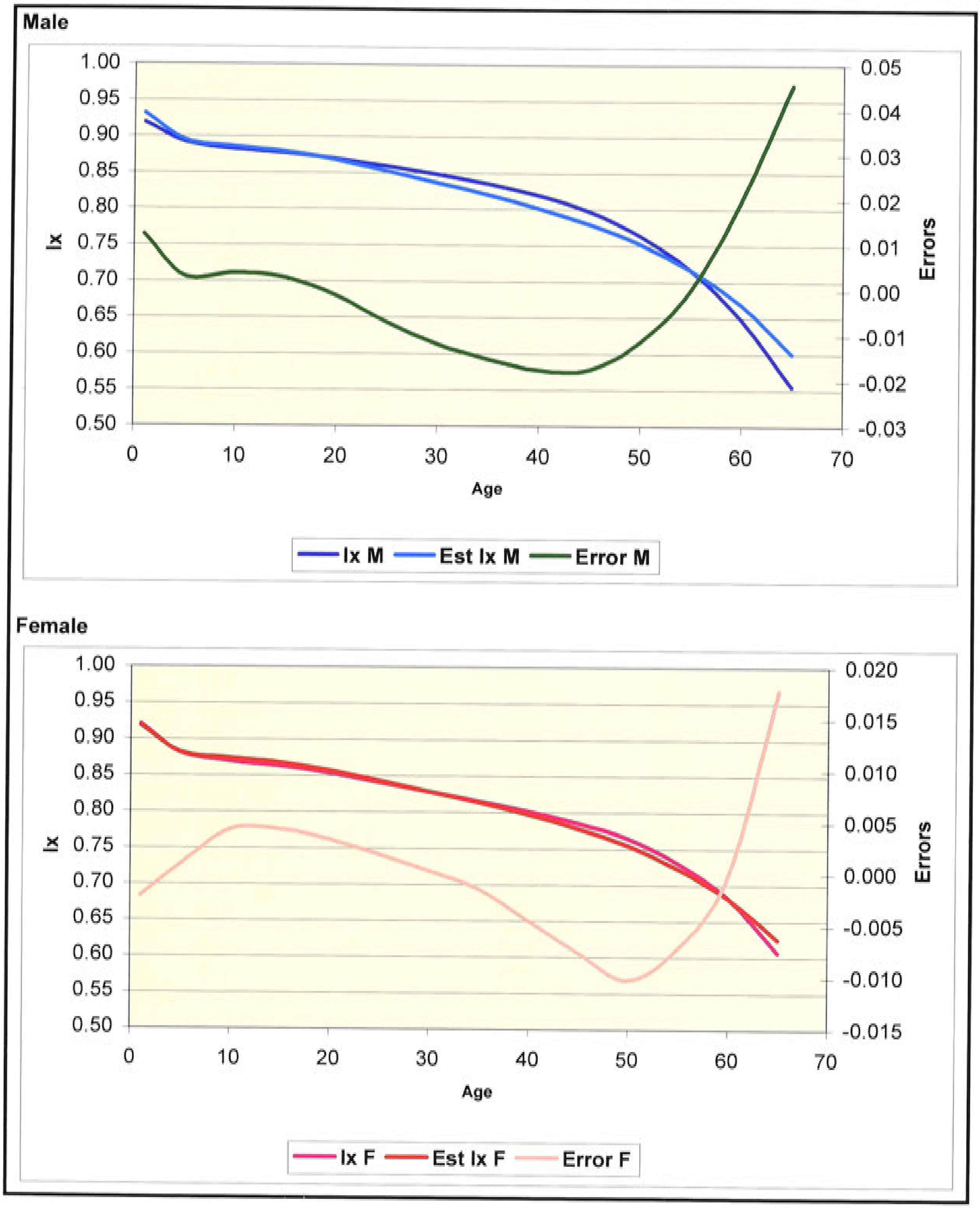


Table 4.2. Brass logit fit to Indian data, 1970-75

Age	lx M	lx F	Est lx M	Est lx F	Error M	Error F
1	870	865	873	847	3	-18
5	804	776	815	786	11	10
10	786	756	802	772	16	16
15	778	747	792	762	14	15
20	769	734	775	745	6	11
25	759	718	753	723	-5	5
30	747	701	732	701	-15	0
35	732	683	710	680	-22	-3
40	710	664	686	656	-25	-7
45	683	642	658	630	-25	-12
50	643	612	625	598	-18	-14
55	589	570	583	559	-6	-11
60	515	514	531	510	16	-4
65	420	431	462	446	43	15

Table 4.3. Brass logit fit to Indian data, 1976-80

Age	lx M	lx F	Est lx M	Est lx F	Error M	Error F
1	879	873	884	857	5	-15
5	819	794	831	803	11	9
10	805	776	818	791	13	14
15	797	769	809	782	12	13
20	789	758	793	767	5	9
25	778	743	773	747	-5	5
30	768	727	752	728	-15	1
35	753	712	731	709	-22	-3
40	735	695	709	688	-26	-7
45	707	675	682	664	-24	-11
50	670	650	650	635	-19	-15
55	615	612	610	599	-5	-13
60	541	557	558	553	17	-3
65	442	475	490	493	48	18

Table 4.4. Brass logit fit to Indian data, 1981-86

Age	lx M	lx F	Est lx M	Est lx F	Error M	Error F
1	896	896	902	884	6	-12
5	848	831	855	836	7	6
10	834	814	844	826	10	12
15	827	806	836	818	9	11
20	819	795	822	804	3	9
25	809	783	804	787	-5	4
30	798	769	786	769	-13	1
35	785	755	767	752	-18	-3
40	768	739	746	732	-22	-7
45	743	720	722	710	-21	-10
50	709	696	692	683	-17	-13
55	657	659	654	648	-3	-10
60	589	608	604	604	16	-4
65	492	527	537	544	45	17

Table 4.5. Brass logit fit to Indian data, 1986-90

Age	lx M	lx F	Est lx M	Est lx F	Error M	Error F
1	913	913	920	903	7	-10
5	874	858	879	862	6	4
10	862	843	869	853	7	9
15	856	837	862	845	6	9
20	848	826	850	834	2	8
25	838	813	833	818	-5	5
30	828	800	816	802	-12	2
35	815	788	798	786	-17	-2
40	796	774	778	768	-18	-6
45	775	757	755	748	-19	-10
50	740	735	727	722	-14	-12
55	691	700	689	690	-2	-10
60	623	650	640	647	17	-3
65	528	572	572	588	44	16

Table 4.6. Brass logit fit to Indian data, 1991-95

Age	lx M	lx F	Est lx M	Est lx F	Error M	Error F
1	919	921	931	919	12	-2
5	892	881	895	882	3	1
10	882	870	886	874	4	4
15	876	863	879	867	3	4
20	869	854	867	857	-1	3
25	859	841	852	843	-7	2
30	848	828	836	828	-12	0
35	835	815	820	814	-15	-1
40	819	802	801	797	-18	-4
45	797	786	779	778	-18	-8
50	763	764	752	754	-11	-10
55	716	730	716	723	0	-7
60	648	683	668	683	20	0
65	555	608	601	625	46	18

4.3 Fitting the Heligman-Pollard model

Chaurasia (1993) fitted the Heligman-Pollard (H-P) model to Indian abridged life table data. He used the software provided by the United Nations: MortPak Lite. This software uses only pre-fixed routines of estimation, curtailing and limiting the flexibility of estimation. Some preliminary analysis of mortality, such as trends in parameter, was presented. The study came up with general findings like declining trends of mortality. In this chapter to fit the H-P model, SAS's NLIN procedure was used. Also, complete life tables derived from census data were used in order to monitor the fits for the individual ages for the periods 1951-60 and 1961-70. These life tables do not show short-term trends but they are available for single years of age giving more points to test the fitting strengths and weaknesses of the model. The H-P model is

$$f(x) = A^{(x+B)^C} + De^{\{-E(\log(x/F))^2\}} + GH^x \text{.............}(4.2)$$

where

$$q_x / p_x = f(x)\text{.............}(4.3)$$

or

$$q_x = f(x)/[1+f(x)]\text{..............}(4.4)$$

where q_x is probability of death between ages x and x+1
p_x is probability of survival between ages x and x+1
A is the parameter representing level of mortality at age below one year
B is age displacement to infant mortality
C represents mortality decline in childhood
D is the severity of the accident hump
E represents the spread of the accident hump
F represents the location of the accident hump
G represents the level of senescent (older age) mortality
H represents the geometric rise in mortality at older ages

Weighted regression estimates of the parameters A to H were obtained by non-linear techniques of curve fitting (Seber & Wild, 1989: 27). The weights were $(1/q_x)^2$ as used by Heligman & Pollard (1980).

Figure 4.2 presents the observed and fitted probabilities of death. As can be seen in the case of females for 1951-60, the fit is not very close at the middle and old ages. This is because of the presence of two modes in the data, a phenomenon not characteristic of model mortality patterns. The H-P model makes no provision for the second mode (which is in fact the accident hump) in its schedule.

Table 4.7 presents the estimated parameters for the four life tables. Instability (variability) in the values of parameters D and F is evident across the four life tables. Such variability has also been reported previously in parameter estimates (Dellaportas, Smith & Stavropoulos, 2001: 276). Congdon (1993: 244) noted that A, B and C have large influences on the whole estimation process. This was the case in the present estimation as well, where A, B and C were responsible for good overall estimation. In other words, if A, B and C assumed values of the expected order, all eight parameters were estimated within their stable ranges.

Figure 4.2 Observed and fitted q(x) by H-P model, India, 1951-70

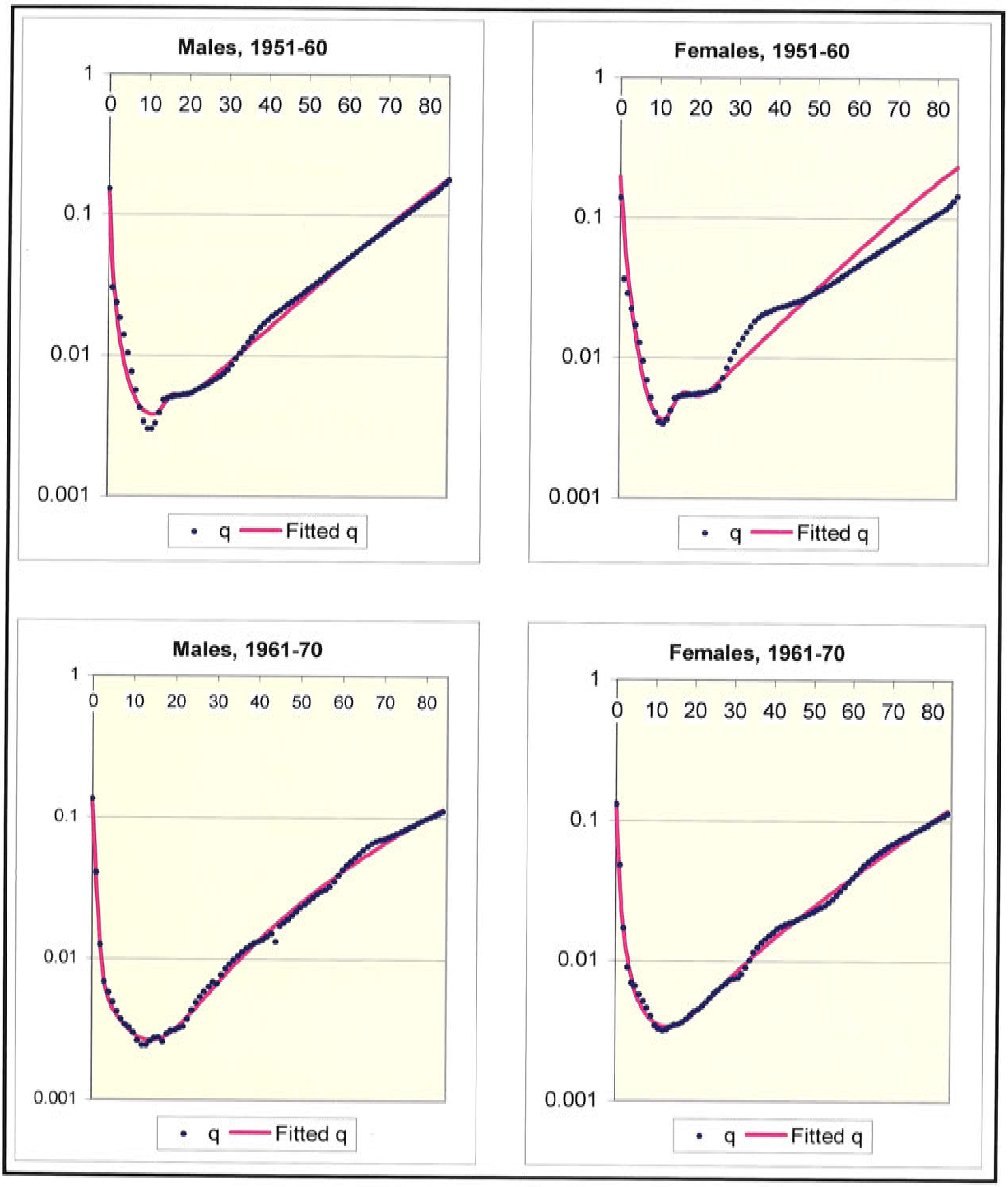


Table 4.7: Estimated parameters for the Heligman-Pollard Model, India.

Parameters	1961-70		1951-60	
	Males	Females	Males	Females
A	0.15020	0.04587	0.05067	0.15323
B	0.98799	0.26321	0.20000	0.60000
C	0.85829	0.37085	0.34774	0.54284
D	-0.02560	-0.00178	0.00106	0.00167
E	0.110	1.600	36.713	26.444
F	328.630	22.396	16.116	15.901
G	0.00665	0.00221	0.00147	0.00138
H	1.03790	1.05024	1.06099	1.06551

Unstable trends were observed by Forfar & Smith (1987) when fitting the H-P model to various English life tables. Estimates of parameters by Forfar & Smith for England for 1841-1971 have a mix of two distinct trends (Benjamin & Soliman, 1993: 53).

4.4 Fitting the Carriere model

The Carriere (1992) model is represented by three extreme value distributions, given by the following equations for the survival ratio S(x):

$$S(x) = \sum_{k=1}^{3} \psi_k S_k(x) \text{...........}(4.5)$$

$$S_1(x) = \exp\left\{-\left(\frac{x}{m_1}\right)^{m_1/\sigma_1}\right\} \text{.......}(4.6)$$

$$S_2(x) = 1 - \exp\left\{-\left(\frac{x}{m_2}\right)^{m_2/\sigma_2}\right\} \text{.......}(4.7)$$

$$S_3(x) = \exp\left[e^{(m_3/\sigma_3)} - e^{(x-m_3)/\sigma_3}\right] \text{..........}(4.8)$$

where m_1, m_2 and m_3 represent the means of three extreme value distributions; σ_1, σ_2, and σ_3 are the standard deviations of the extreme value distributions; and $S_1(x)$, $S_2(x)$ and $S_3(x)$ are the extreme value distributions. $S_1(x)$ is the Weibull distribution and its functional shape exhibits a monotonic declining trend. $S_2(x)$ is the inverse Weibull distribution and is uni-modal. This captures the accident hump in mortality distributions. $S_3(x)$ is the Gompertz distribution with a geometric rise appropriate for the older age mortalities. S(x) is the equivalent to the life table probability of survival, l_x, when the radix is unity. Then q_x is estimated as:

$$q_x = 1 - S(x+1)/S(x).......(4.9)$$

The Carriere model was applied to the Indian life tables for 1951-60 and 1961-70 for males and females derived from the census. However the model did not converge for these data sets. To further test the model, it was applied to the five-yearly SRS-based life tables for 1971-75, 1981-85 and 1990-94 for males and females. The fits are shown in Figures 4.3 and 4.4 for males and females respectively. It is clear that in five out of six cases the fitted values of $\mathbf{q_x}$ do not agree with the observed ones, while $\mathbf{S_x}$ seems to fit closely. The Carriere model has converged but with large variability in the values of the parameters. Table 4.8 presents the values of the parameters for the six cases. The estimated ranges of parameters m_2, σ_1, σ_2 and Ψ_2 are very large, rendering them inappropriate for interpretation and use for forecasting. The model neither captures the age patterns properly nor provides a stable set of parameters for Indian data.

Figure 4.3 Observed and fitted S_x and q_x (on log scale) by Carriere model, males, India, selected periods

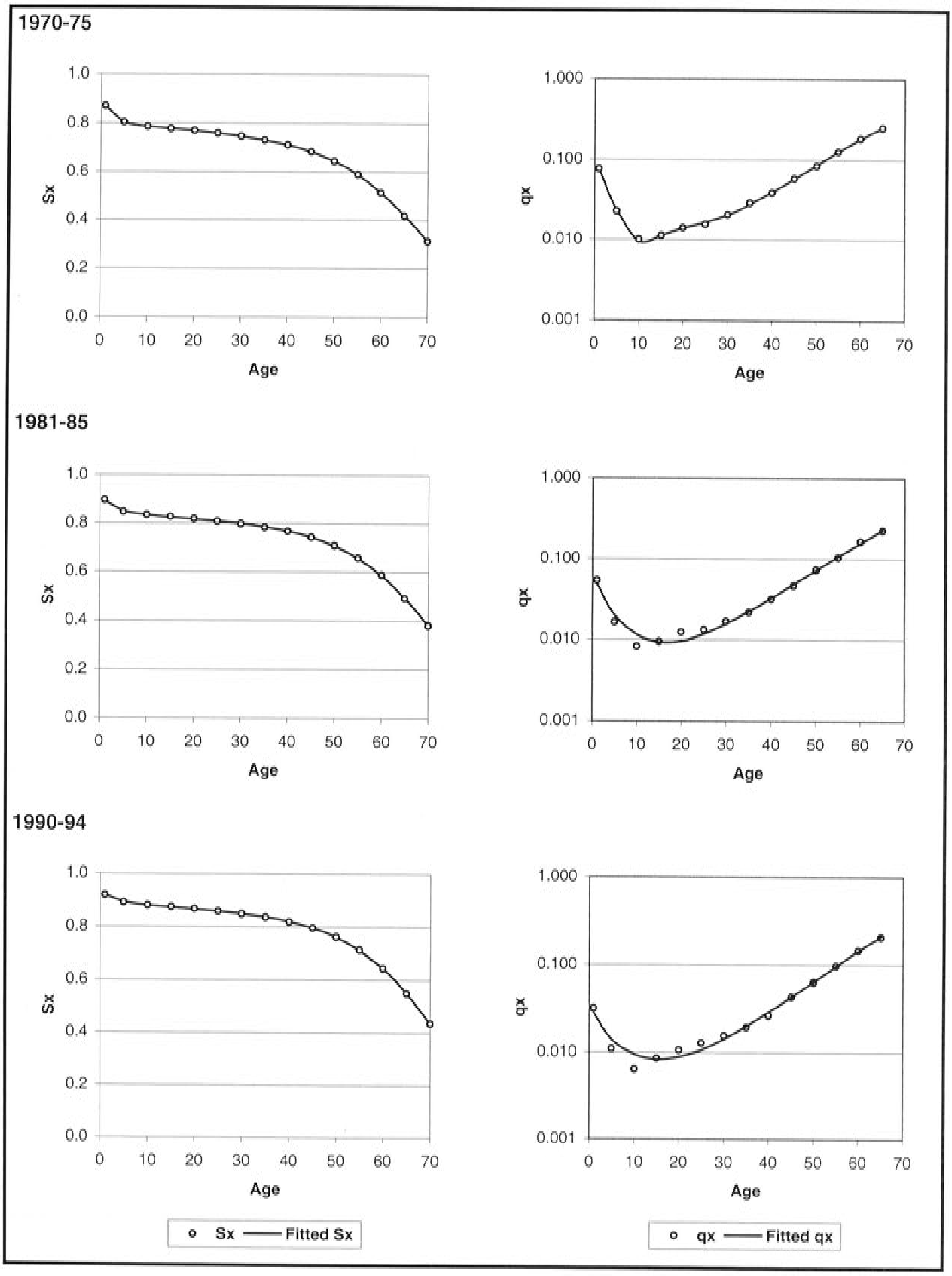


Figure 4.4 Observed and fitted S_x and q_x (on log scale) by Carriere model, females, India, selected periods

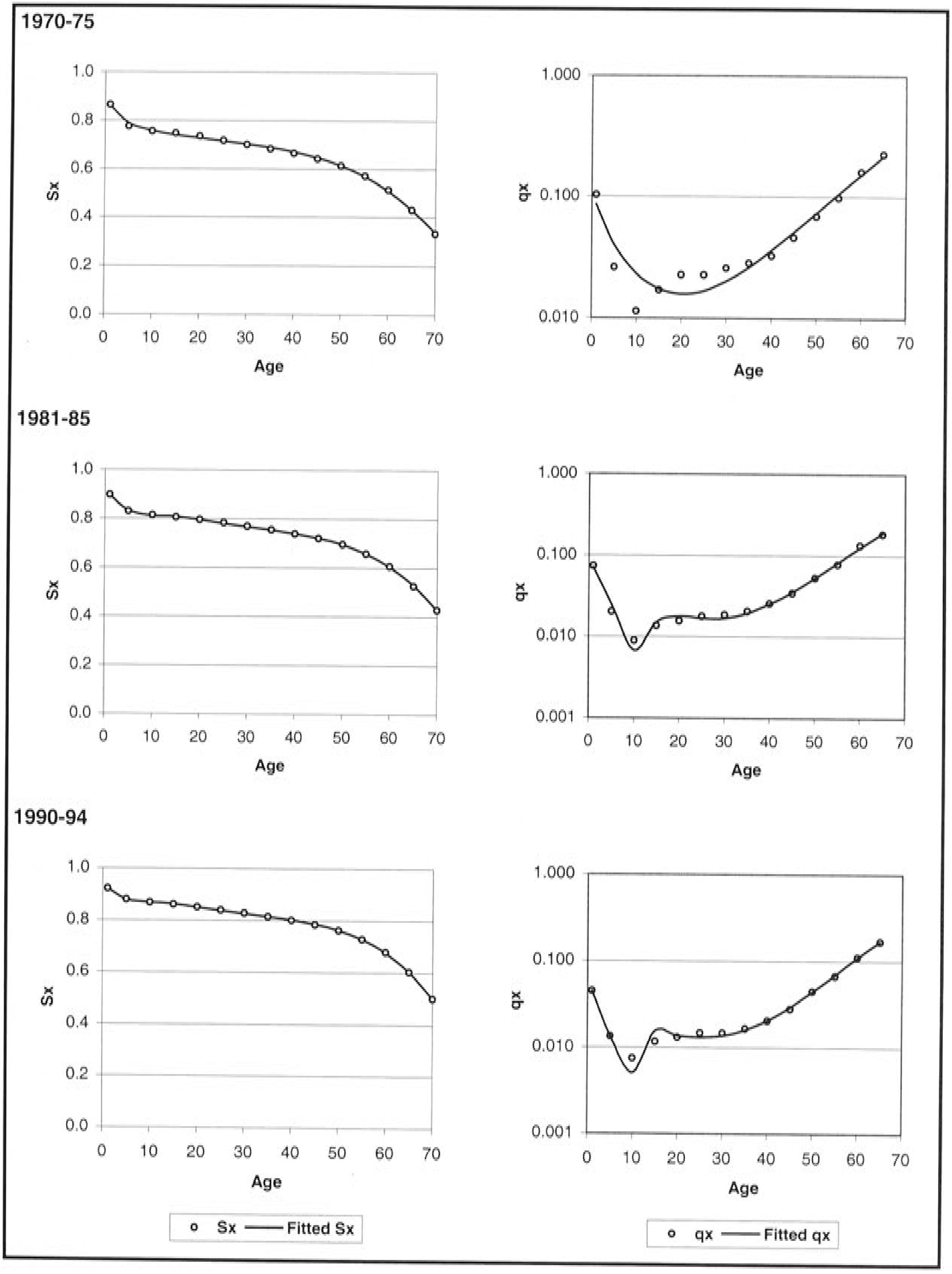


Table 4.8: Parameters of Carriere model, India, males and females, selected years

Parameter	Males			Females		
	1971-75	1981-85	1990-94	1971-75	1981-85	1990-94
m_1	6.22	1.71	7.58	3.14	12.14	93.74
m_2	10.32	100.00	100.00	0.08	11.67	10.12
m_3	69.51	73.25	73.94	73.56	74.86	74.31
σ_1	8.88	4.34	30.41	7.54	19.23	230.26
σ_2	6.06	0.05	1E-08	0.03	4.27	0.37
σ_3	11.63	12.03	11.57	12.66	10.31	9.28
Ψ_1	0.245	0.189	0.178	0.301	0.271	0.359
Ψ_2	0.072	1E-08	1E-08	1E-08	0.054	0.026
Ψ_3	0.683	0.811	0.822	0.699	0.675	0.615

Given the performance of the Carriere model it was also fitted to USA data (Center on the Economics and Demography of Ageing, 2001c) to assess its performance with good-quality data. The results, shown in Appendix 4A, confirm Indian findings that this model proved difficult to converge. Though it seemingly provides close fits to empirical survival ratios, when converted to probabilities of death they do not exhibit the right patterns of mortality. Figures provide the observed S_x and fitted S_x by the Carriere model. The S_x values were transformed to q_x. From the figures presenting the observed and fitted q_x values, it is clear that there is a massive disagreement in observed and fitted patterns of mortality at ages 0-30.

4.5 Fitting the Lee-Carter model

The Lee-Carter model is defined as

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \;\;\ldots\ldots\ldots\ldots\ldots\ldots(4.10)$$

where

$m_{x,t}$ is the matrix of age specific death rates

k_t are the indices of level of mortality over time

a_x defines the general pattern of mortality by age in the form of age specific constants

b_x are the age specific constants; indicating the rate of change in specific age groups

$\varepsilon_{x,t}$ is the error term of the equation with mean 0 and variance σ_x^2

Since equation (4.10) does not have any regressor, Lee & Carter (1992) used a singular value decomposition (SVD) on the logarithms of age specific death rates to segregate time (k_t) and age (b_x) effects. They call the time effect the 'mortality index'. This model does not identify systematic age effects because the simple assumption is made that age effects remain constant throughout the period of analysis.

A singular value decomposition provides the eigen values and eigen vectors of a matrix. The mathematical expression is written as

$$X_{m,n} = U_{m,n} D_{n,n} V'_{n,n} \dots\dots\dots\dots(4.11)$$

where **X** is an **m x n** matrix, **U** is an **m x n** matrix and **V** are the matrices of the orders **n x n**. **V′** is the transpose of **V**, and **D** is a diagonal matrix **n x n** with the singular values **D(i,i)**.

The L-C model was fitted to the time series data for 1970 to 1997 for males and females in India. The SRS data used was grouped into 5-year age groups.

The steps involved in the decomposition and estimation of k_t and b_x are as follows:
The matrix of age specific death rates $\mathbf{m}_{x,t}$ is arranged in the form so that **m<n** (15<28), a precondition for the SVD. Thus the ASDRs were arranged by years as the columns (n=28) and age groups as rows (m=15). The $\mathbf{m}_{x,t}$ then were transformed to logarithms. The mean adjusted age specific death rates were then decomposed by SVD. The estimation of k_t was done by the following relation:

$$k_t = U_{1,t}\ D_{t,t}\ \sum_t V'_{t,1} \dots\dots\dots\dots(4.12)$$

And b_x values were estimated by the V component of the SVD as:

$$b_x = V'_{x,1} / \sum_x V'_{x,1} \dots\dots\dots\dots(4.13)$$

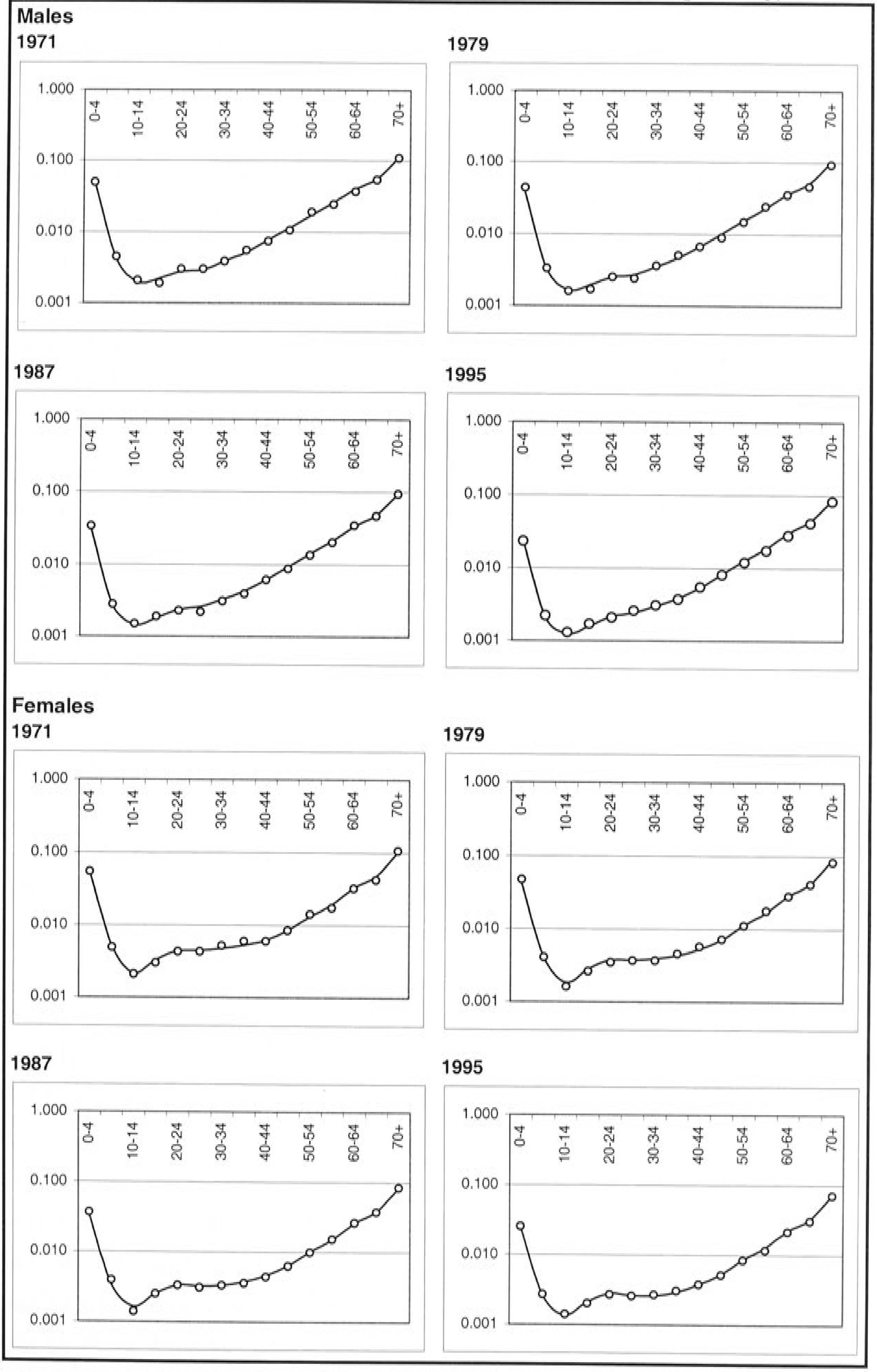
For x = 1 to 15

The estimates of the ASDRs are obtained by using the estimates of k_t and b_x in the following

$$m_{x,t} = e^{(a_x + k_t\ b_x)} \dots\dots\dots\dots(4.14)$$

The results are presented in Figure 4.5 for males and females for selected years; fits are good for all selected years. An overall observation can be drawn from these figures that the fitted rates are slightly on the low side for almost all the age range. Lee & Carter (1992) adjusted k_t to reproduce the actual number of deaths as a means of compensating for the weighting implicit in fitting to the logarithms of rates rather than to actual rates. In the case of Indian mortality, the number of deaths is not available since only ASDRs were published. Thus the adjustment of k_t cannot be made by adjusting to the number of deaths. Lee & Miller (2000) also mentioned that adjustment can be made by reproducing the expectation of life at birth; however, such adjustment involves substantial smoothing because of the nature of life table calculations. The gains by adjusting are likely to be of small magnitude.

Figure 4.5: Observed and fitted ASDRs by Lee-Carter method, India, 1971-95 (Observed and fitted rates are represented by circles and line respectively)



4.6 Conclusion

The review of mortality models in Chapter 3 identified four models for application to Indian data. In this chapter these models have been applied to Indian data to assess their usefulness for further consideration in forecasting Indian mortality.

The Brass relational model has been shown to be of limited value for modelling Indian mortality since the fits were not good at both ends of the age scale. The Lee-Carter model provides a useful way of measuring the overall improvement in the level of mortality and fits are good. However, it is said that the model is limited in that it does not allow for changes in age pattern over time. In the case of Indian mortality it tends to fit well, so the model will be fitted to the time series data and will be used for forecasting.

The Carriere model has a convenient statistical form but either it failed to converge with Indian data sets or its estimated parameters were out of the range. As the Carriere and H-P models are comparable to each other in their approach to representing mortality, the H-P model is chosen for further analysis because it provided better results. Yuen (1997) also compared the H-P and Carriere models and found that the H-P model obtained stable and better fits to data for Hong Kong assured-lives life tables for 1993 and Hong Kong female life tables for 1991. Though the Carriere model has superior logical statistical expression, the H-P model provides a better fit in many cases. The Carriere model is not considered further in this thesis.

The Heligman-Pollard model fits age patterns of Indian mortality well, but problems arose concerning parameter variability; for example the parameters D took on negative values. The values of the parameters E and F were also highly unstable. This instability seems to be resolved when the parameters A, B and C assume values of the right order (as suggested by Heligman and Pollard for Australian data). There seems to be a need to investigate this instability of estimation and possibly improve the model to increase the stability. When the H-P model is fitted to Indian data, it has been observed that the model provides a good fit in many cases but may also provide some negative parameters owing to the non-linear-weighted regression estimations. While weighted regression estimates depend on the values from the three different parts of the curve, it may be possible that stability in estimation may be achieved by a more parsimonious option. It was suggested by various researchers that by fixing the values of one or two parameters to a feasible constant, stability was achieved in estimation (Rogers, 1986; Congdon, 1993). Congdon (1993) also noticed that overparameterisation is a concern with the H-P model. Thus, reducing the number of parameters will help to achieve a more parsimonious model and hence resolve the problem of negative parameter estimation. It is the first step to achieving the parsimony by having the minimum possible number of parameters without reducing the accuracy in fitting.

Chapter 5: Reducing Parameters by Developing a New Pre-teen Mortality Model

Perfection is achieved, not when there is nothing more to add, but when there is nothing left to take away.

Antoine de Saint Exupery

5.1 Introduction

As discussed in Chapter 4, the Heligman-Pollard (H-P) model requires improvement to reduce the variation in parameter estimates when fitting to actual data. It is clear that the H-P model is correct in defining three phases of age-related mortality change. For the last phase, older-age mortality, as applied by the H-P model, two parameters are required: one showing the base mortality and the other the geometric rise in mortality. Again as specified by the H-P model, the middle phase, where the accident hump occurs, needs three parameters to show the location, severity and spread of the hump. This chapter shows that the early, declining-mortality phase can be modelled with fewer parameters than three.

Empirical evidence shows that the minimum of the mortality curve is within the age range 9-15 years (Chiang, 1984) in almost all populations. Mortality declines typically just after birth until it reaches its minimum level. During infancy the decline is much faster and this decline has been captured by mathematical formulae (Choe, 1981; Krishnamoorthy, 1982; Pathak, Pandey & Mishra, 1991; Chauhan, 1997; Mathew, 1997). Anson (1985) estimated the minimum value by parameterisation of mortality. Sankar (1996) has discussed the application of reliability models through analysing the 'risk of mortality' and undertaking the theoretical estimation of hazard function . In this chapter a new statistical model for the first phase of mortality is proposed and tested. This model has the capacity to capture the declining phase of mortality until its minimum. The added advantages are that the new model effectively requires estimation of only one parameter when fitted as the first part in isolation and two parameters when fitting for the entire age range. The model itself is a probability density function. Data for Australian, Swedish and Japanese mortality have been used for testing.

5.2 Overview for improvement in the first part of the curve

The new model of early-age mortality used in this study is similar to the H-P model; however, the new model has certain advantages over the H-P because of its robust fitting procedure. The new model remains simple, as only a single parameter estimation is required, contrary to past models (Heligman & Pollard, 1980; Mode & Busby, 1982; Carriere, 1992), which require the estimation of up to three parameters to achieve the same goal. When used with the entire age range, the new model also needs a scalar parameter included in the model to place the model in the right place on the vertical axis.

The new model, a finite range model, was initially introduced for reliability analysis by Mukherjee & Islam (1983); it was used successfully to model the distribution of deaths in infancy (Chauhan, 1997) using data from Sweden, the USA and India. Later Krishnamoorthy & Rajna (1999) affirmed that the model fits well for deaths in infancy and it also fits for under-five mortality. In this chapter several statistical characteristics of the new model are derived and presented. Other advantages of using the new model are: (1) the model itself is a specific statistical function, a probability density function (pdf); in statistical analysis, pdf plays a wider and more significant role than a simple mathematical formula; (2) with the use of pdf as a formula, continuous representation of mortality becomes possible.

5.3 Introduction of a new mortality function for the pre-teen ages

The model is

$$f(x) = \frac{p}{x}\left(\frac{x}{\theta}\right)^{p} \quad \text{..............(5.1)}$$

$$0<x<\theta;\ p, \theta >0;\ 0<p<1$$

Equation 5.1 presents the mathematical form of the new model, which has one variable x and two parameters p and θ. Function f(x) is the relative mortality level where variable 'x' represents age in years. Parameter θ is defined in such a way that variable 'x' can take its maximum possible value (upper bound) when equal to θ. Parameter p is the shape parameter, defining the flatness of the model. Plotted graphically, this curve assumes a mirror J-shape. Higher values for parameter p correspond to a lower distribution of deaths during the early years in the range. On the other hand if p carries a lower value, this results in a larger share of deaths during the early ages in the range 0 to θ. Mathematical forms of derivations relating to the new model are given below.

The cumulative density function (cdf) of (5.1) is represented as:

$$F(X < x) = \left(\frac{x}{\theta}\right)^{p} \quad \text{................(5.2)}$$

$0<x<\theta$; p, θ >0; $0<p<1$

Note $F(X<\theta)=1$ indicating the relative nature of f(x).

Other statistical characteristics are given by the following equations.

The Moment Generating Function is:

$$M_x(t) = \frac{p}{\theta^p} \sum_{s=0}^{\infty} \frac{\theta^{s+p}}{s+p} \frac{t^s}{s!} \quad \text{..................(5.3)}$$

The r^{th} order moment about zero can be calculated by the following identity:

$$\mu_r = \left[\frac{d^r}{dt^r} M_x(t)\right]_{t=0} \quad \text{.............(5.4)}$$

By solving equations (5.3) and (5.4) the expression for the mean can be derived and the value of the mean is:

$$\text{Mean} = \left[\frac{p}{p+1}\right]\theta \text{.............(5.5)}$$

Similarly the variance is given as:

$$V(x) = \left[\frac{p}{(p+1)(p+2)}\right]\theta^2 \text{..................(5.6)}$$

and finally the median is expressed as:

$$\text{Median} = (1/2)^{1/p}\,\theta \text{............(5.7)}$$

Other statistical characteristics can similarly be derived.

5.4 Development of the methodology for fitting

The regularity of distribution of the probability of death in the first phase of life follows a declining pattern; the pace of decline has been taken as a key factor to derive the methodology for fitting, which assumes that the ratios of the probabilities will remain constant. The declining but maintained (in a specific way) pace takes a non-linear path. This analogy has been used mathematically in the equations given below. To hold the equations true the three following assumptions need to be fulfilled.

5.4.1 Assumptions

1. Child deaths (under age 5) are not misreported disproportionately with respect to other deaths.
2. Deaths in the first phase do not suffer from omission.
3. No seasonality exists for various age-specific deaths.

The nature of the probability suggests that it can ever assume the maximum of all possible odds in favour of one event under examination. In the present case the probability of death q(x) is taken as the variable of analysis, so occurrence of death in the early phase becomes the 'event' in this situation. The probabilities of death will add to unity, when added up to the age corresponding to the minimum value of mortality. These specifications will be represented in the form of the value of the parameter.

If an assumption is made (based on observation of the data) for the value of θ, estimation of the parameter 'p' is required in order to fit the model. The identified minimum for the q(x) values is taken at θ and 'p' is estimated by the following procedure.

If m is the observed minima of the q(x) curve, then the cdf of x which signifies the probability that a person of age less than x will die before reaching X, as defined by equation 5.2 becomes:

$$F(X < x) = \left[\frac{x}{m}\right]^{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.8)$$

Proportionate probabilities have been observed to follow a unique path and probability under five has been taken as a guiding rule for the fitting. Let the proportion of the probability of infant or child death (under age five) to the total probability of death till the minimum be ψ. Hence

$$\psi = \frac{\Pr.\{\text{a person will not survive to fifth birthday}\}}{\Pr.\{\text{a person will not survive to age m}\}} \quad \text{.......(5.9)}$$

In equation (5.8) by having x as one and m as θ, the cumulative probability of dying before the fifth birthday given the minimum of q(x), will be given as:

$$F(X < 5) = [5/m]^{p} \quad \text{.................(5.10)}$$

By equations (5.9) and (5.10) the estimate of p will be

$$p = \left(\log(\psi) \big/ \log(5/m) \right) \quad \text{...........(5.11)}$$

5.5 Testing the function for the pre-teen ages

The new age pattern of mortality from birth to pre-teen ages is tested with data sets. First of all it requires the selection of data sets for testing the model. The details of the testing of the new model with empirical data are provided below.

5.5.1 Data for testing

It is highly desirable to use data of good quality when fitting a new model to ascertain that, in the case of a poor fit, the poor performance of the model cannot be attributed to data quality. Data from Australia, Japan and Sweden are used for the testing. Australian mortality data are available for a century and free from irregularities. Sweden has had good-quality data on vital events for more then a century. Data for Sweden (Center on the Economics and Demography of Ageing, 2001a) and Japan (Center on the Economics and Demography of Ageing, 2001b) have been downloaded from the Berkeley Mortality Database. All data used in testing are for single years of age.

5.5.2 Testing with Australian data

Data on Australian mortality for selected time periods, extracted from the diskette provided with the 1995/97 life tables (Australia. Office of the Australian Government Actuary, 1997), have been used to test the model. The selected time periods are 1932-34, 1953-55, 1970-72 and 1990-92. The testing has been done for males and females separately.

Figure 5.1 provides the observed and fitted probabilities of deaths for Australian males for the four selected time periods. The graphs are on logarithmic scales to highlight the smaller variations at low-mortality ages. Values fitted by the new model are very close to the empirical values of q_x for Australian males as the values of the error sum of squares are very small. Table 5.1 shows the error sum of squares for Australian males and females for the selected time periods: the values are of very small order. It can be concluded that the proposed model provides a satisfactory fit to Australian data.

Figure 5.2 provides the observed and fitted probabilities of death for Australian female life tables for the four selected time periods. The fitted values are very close to those observed for all the periods, but the fit is better for the recent periods. For 1932-34, the estimated q_x corresponding to ages 1-2 has been underestimated to a minor extent.

Table 5.1: Error sum of squares for fit of the new model for Australian males and females, 1932-1992

	Error sum of squares	
Year	Males	Females
1932-34	2.83E-06	1.99E-06
1953-55	2.34E-08	1.36E-07
1970-72	7.11E-08	8.12E-09
1990-92	1.92E-08	1.68E-09

5.5.3 Testing with Japanese data

Japan has experienced a sharp decline in overall mortality since 1950 (Tuljapurkar *et al.*, 2000). Time series data for single years are available for 1950-1996 for males, females and persons. Five equidistant years for both males and females have been selected: 1960, 1969, 1978, 1987 and 1996.

Figure 5.3 provides the observed and fitted probabilities of death for Japanese males for the five selected time periods. It is evident from the figures that the fit is good for Japanese males. Table 5.2 provides the error sum of squares for males and females. For males the lowest error sum of squares has been observed for 1987, followed by 1996. The model also performs satisfactorily in the case of Japanese females. Figure 5.4 provides the observed and fitted probabilities of death for females at pre-teen ages. The lowest values of error sum of squares are observed for 1996 and the highest were for 1960. The fact that errors are of the order of 10^{-7} (or smaller) suggest that the errors are more random than systematic.

Table 5.2: Error sum of squares for fit of the new model for Japanese males and females, 1960-1996

	Error sum of squares	
Years	**Males**	**Females**
1960	3.15E-07	3.66E-07
1969	1.16E-07	1.30E-08
1978	2.41E-08	2.41E-08
1987	1.24E-08	2.17E-08
1996	1.69E-08	6.02E-09

5.5.4 Testing with Swedish data

Vital statistics and other demographic data from Sweden are mainly considered to be of good quality. Data are available for a long time period, 1861-1999. Data for four years for males and females have been selected for fitting: 1891, 1921, 1951 and 1981.

Figure 5.5 provides a comparison of the observed and fitted probabilities of death for Swedish males: the fit is good for all the years except 1891. The amount of error is very small, suggesting that the proposed model fits well with Swedish data also. Figure 5.6 provides observed and fitted probabilities for Swedish females; except for the year 1891 the fit is very good. For 1891, the fit has some bias in the older ages but the amount of difference is again negligible; for other years the errors are very small. Table 5.3 shows that the fit is best for 1981 among the four female data sets.

Table 5.3: Error sum of squares for fit of the new model for Swedish males and females, 1891-1981

	Error sum of squares	
Years	**Males**	**Females**
1891	1.53E-04	1.10E-04
1921	9.75E-06	5.77E-06
1951	4.72E-07	1.26E-07
1981	1.66E-07	3.21E-08

5.6 Fitting the entire age range

As seen in the last section, the new model fits well for the pre-teen age ranges for the data from Australia, Japan and Sweden. However, the application of the new model to this first phase of human mortality must be also tested with the whole age range. Entire age ranges are fitted by modifying the H-P model. Fitting has been done for the same data sets as in the previous section, but for the entire age range.

The proposed pre-teen model gives probabilities of death by their cumulative density function. The proposed new model of mortality for the entire age range becomes:

$$q_x = f_1(x) + f_2(x) + f_3(x) \qquad \text{............(5.12)}$$

where

$$f_1(x) = \Delta F(X < x)$$
$$f_2(x) = De^{-E\{(\log(x/F))^2}$$
$$f_3(x) = GH^x$$

and $\Delta F(X < x)$ is the first difference of successive values of cdf, the $F(X<x)$ is the cdf as given in Equation 5.2, and $f_2(x)$ and $f_3(x)$ are taken from the Heligman-Pollard model. The fitting is done by the 'NLIN procedure' of SAS 8.0 (Luginbuhl & Schlotzhauer, 1987). The weighted least-square estimate of the non-linear equation (e.g. 5.13) has been obtained by minimising the error sum of squares (Seber & Wild, 1989: 27). For the estimation of parameters in cases of extreme values and values with larger disagreements, weighted regressions are used. The method to obtain convergence, the Gauss-Newton algorithm, is applied in iterations, each of which minimises the error sum of squares. Revision in the values of estimated parameters takes place until convergence is reached.

For a model

$$y = \alpha_0 + \beta_1 x_1 + \beta_2 x_2 + \text{..............} + e \qquad \text{.............(5.13)}$$

where y is the dependent variable, also called the response variable. Other variables x_1, x_2 and so on are independent variables, also called explanatory or regression variables. β_1, β_2 and so on are the regression coefficients, α_0 is the intercept and **e** is the random error for the regression. Using ordinary least squares (OLS), the parameters for a linear regression model (5.13) can be obtained by solving:

$$\beta = (X^T X)^{-1} X^T Y \text{..................}(5.14)$$

where **X** is a column vector consisting of (x_1, x_2,.......) and $\mathbf{X}^T$ represents the transpose of **X**. **Y** is a column vector of (y_1, y_2,......). $\boldsymbol{\beta}$ is a column vector of the (β_1, β_2....).

The estimation of $\boldsymbol{\beta}$ for the weighted regression may be solved by using a weight scheme as follows:

$$\beta = (X^T W X)^{-1} X^T W Y \text{..................}(5.15)$$

where **W** represents the column vectors with (w_1, w_2,.......) that provide weights to the estimation. For estimation of the new model 5.12, the weights W used are:

$$W' = [(1/q_1)^n, (1/q_2)^n, \text{...........}] \quad \text{......}(5.16)$$

In most of the cases, n is taken as 2.

Where parsimony is lacking, unstable parameter estimation is expected. The problem of over-parameterisation in the H-P model has been acknowledged by researchers (Dellaportas *et al.*, 2001). It has also been suggested (Congdon, 1993; Dellaportas, *et al.*, 2001) that two parameters can be limited by assigning them feasible constant values. During estimation of the parameters for the new model, the situation arose where convergence was reached by constraining a parameter. In one or two cases, two parameters were also constrained. The new model is more parsimonious and avoids some of the drawbacks in estimating the parameters. The new mortality model tends to stabilise the parameter estimation. The stability of parameters means that the order of parameters does not change across the years, which helps in tracing a trend over time in the parameters. Persistent trends, if observed, help in forecasting parameters over time. Also the more parsimonious model tends to exhibit stabilisation of the parameters by inclusion of a smaller number of parameters in the model.

5.6.1 Inherent assumptions

The model fits human mortality with the following inherent assumptions:

(a) Mortality declines in the first part by reaching its minimum.

(b) The second part has only one peak (known as 'accident hump').

(c) After a certain age, mortality increases monotonically (e.g. 35 and above).

It has been observed that a few mortality data sets show two or more peaks corresponding to young adult ages. Following assumption (b) the model will consider the prominent peak as the peak for fitting the model, in order to minimise the errors. For industrially advanced countries it has been observed that mortality declines after age 90 or above, because of the selectivity of survivorship in any human population (Kannisto *et al.*, 1994; Horiuchi & Wilmoth, 1998). This model does not incorporate any such phenomenon, as the model for the third (last) phase of mortality defined here is the geometrically rising Gompertz function. In the case of India there is little need for declining-mortality concerns and hence this possibility has not been pursued. Though the model does not consider declining mortalities at very old ages, it is shown below that it gives a reasonably close fit to mortality data for Australia, Japan and Sweden.

5.6.2 Testing the entire age range with Australian data

The new mortality model provides a very good fit to Australian mortality data. Figures 5.7 and 5.8 present a comparison of the probabilities of death as observed, fitted with the new model and fitted with the Heligman-Pollard model. The errors are distributed randomly and the new model performs equally well as the H-P model (see Appendix 5A) in fitting the empirical data. The parameters of the new model are more stable and the standard errors of the parameters are very small.

For Australian females it was noticed that the H-P model estimates the parameter B with negative values in most of the cases. While attempts have been made to constrain the value of B to a positive range, the H-P model stops converging or produces a large set of unstable parameters. The new model provides a very close fit to the observed q_x values with stable and consistent estimates of parameters.

Thus, the new model converges with Australian data for females and males. The small values of the error sum of squares in Table 5.4 suggest that when fitted to the entire age range the new model minimises overall error of fitting.

Table 5.4: Error sum of squares for fit of the new model and H-P model, Australia, males and females, selected time periods

	Males		Females	
Time periods	New model	H-P model	New model	H-P model
1932-34	0.005395	0.005925	0.006656	0.008342
1953-55	0.003249	0.002583	0.000728	0.000654
1970-72	0.031128	0.029338	0.002646	0.002817
1990-92	0.010530	0.009587	0.001854	0.001926

5.6.3 Testing the entire age range with Japanese data

Fitted to Japanese data, the model performed equally well as with Australian data. Five data sets for males and females between 1960 and 1996 were used in fitting the new model. Comparison were made with the H-P model.

Figures 5.9 and 5.10 provide the two fitted probability curves along with the observed one for Japanese males and females for the five selected time periods. Stable parameters were estimated for the new model of mortality (see Appendix 5B). The new model fits male Japanese mortality as well as the H-P model does. For females, there were a few irregular disturbances in the data for four out of the five periods. Both models fitted the data on similar lines. The errors due to the two models are comparable, and random in both cases. Table 5.5 provides the error sums of squares for males and females. Except for males in 1978 and 1987, the new model provides a smaller error sum of squares than the H-P model, suggesting that the new model brings stability to the estimation process.

Table 5.5: Error sum of squares for fit of the new model and H-P model, Japan, males and females, selected time periods

	Males		Females	
Time periods	New model	H-P model	New model	H-P model
1960	0.006164	0.007110	0.021229	0.021771
1969	0.071373	0.073679	0.021768	0.023672
1978	0.031202	0.030130	0.027207	0.031709
1987	0.008291	0.008050	0.018359	0.025159
1996	0.030478	0.045121	0.017296	0.034523

5.6.4 Testing the entire age range with Swedish data

When fitted to the Swedish data, both the models treated irregularities in the distribution of probabilities in a similar manner. Figures 5.11 and 5.12 provide the observed and fitted probabilities of death for Swedish males and females for four selected time points. There were more fluctuations in the data for males than for females. Both the models have graduated these errors and obtained smooth age patterns of male mortality. The new model fitted well in all cases (see Appendix 5C). For Swedish females, there were irregular patterns around age 20-29, perhaps due to random fluctuations resulting from small numbers of deaths. Both models graduated these irregularities. Once again the new model has provided good fits for these data sets. Table 5.6 provides an account of the errors. The new model provided the least overall error in all eight cases under study.

Table 5.6: Error sum of squares for fit of the new model and H-P model, Sweden, males and females, selected time periods

	Males		Females	
Time periods	**New model**	**H-P model**	**New model**	**H-P model**
1891	0.058200	0.059654	0.086925	0.087350
1921	0.026803	0.035567	0.010211	0.010253
1951	0.013043	0.015601	0.024331	0.024405
1981	0.004019	0.004029	0.005360	0.005524

5.7 Conclusion

The above analysis and comparison of observed and fitted probabilities of death suggest the following. As far as the pattern of errors is concerned the new model is similar to that of Heligman and Pollard along the no-error line. The concept of a no-error line is somewhat arbitrary however, as the observed data possess certain random errors due to undefined random causes.

When random fluctuations occur, the models fit the data well, with a succession of small positive and negative errors. A fear that the use of a graduating formula might change the observed age pattern is unwarranted as shown especially in the application of the models to Swedish data. Where the purpose is to produce models that can be used to project future mortality, graduation of random fluctuations is desirable. On the basis of the trials in this chapter it can be concluded that model 5.1 can be used to graduate the declining first part of human mortality and the new model (modified Heligman-Pollard) can be used to graduate overall mortality. The next chapter considers the application of the new mortality model to Indian data.

Figure 5.1: Observed and fitted probabilities of death, Australia, males, selected periods

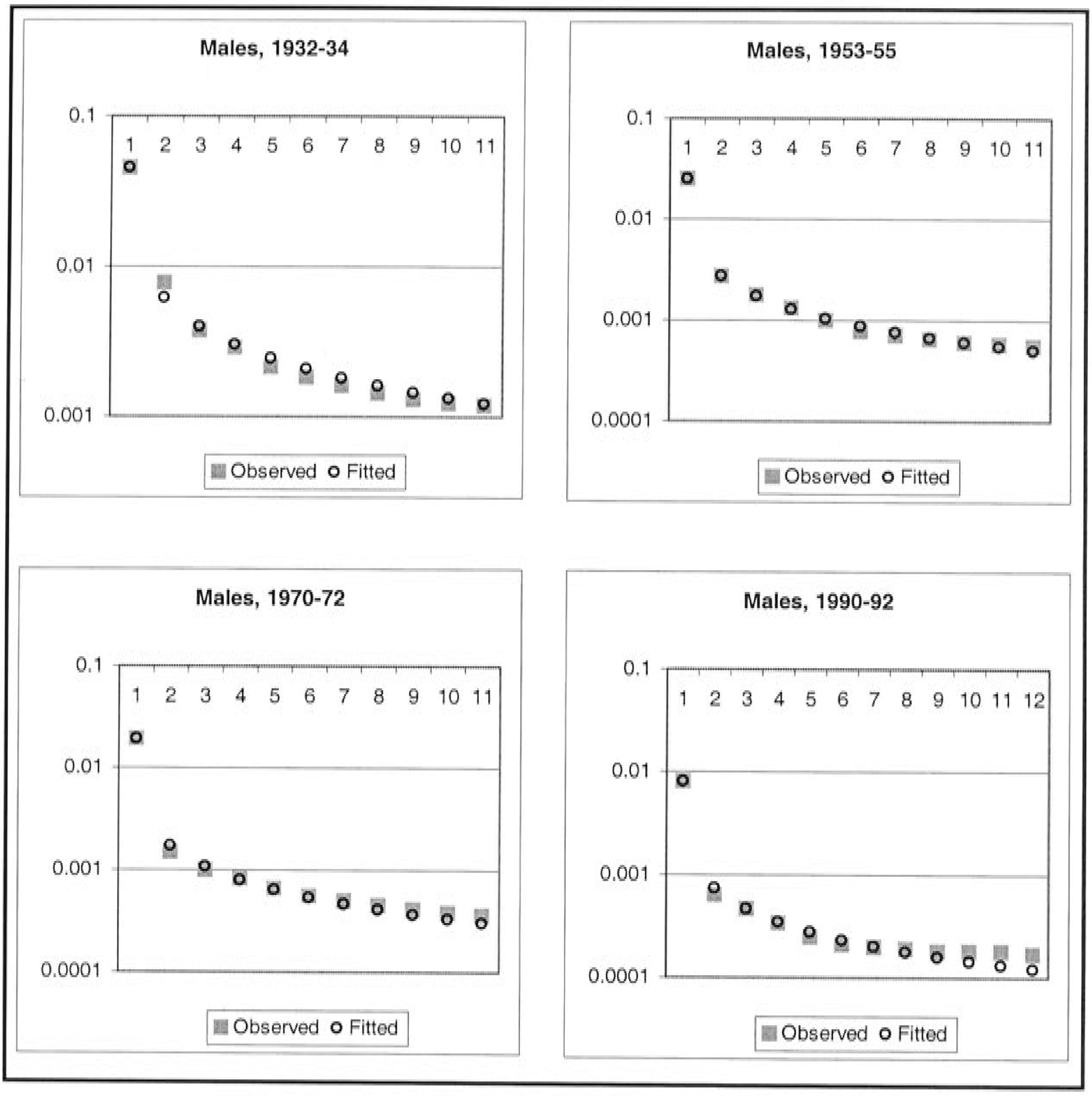


Figure 5.2: Observed and fitted probabilities of death, Australia, females, selected periods

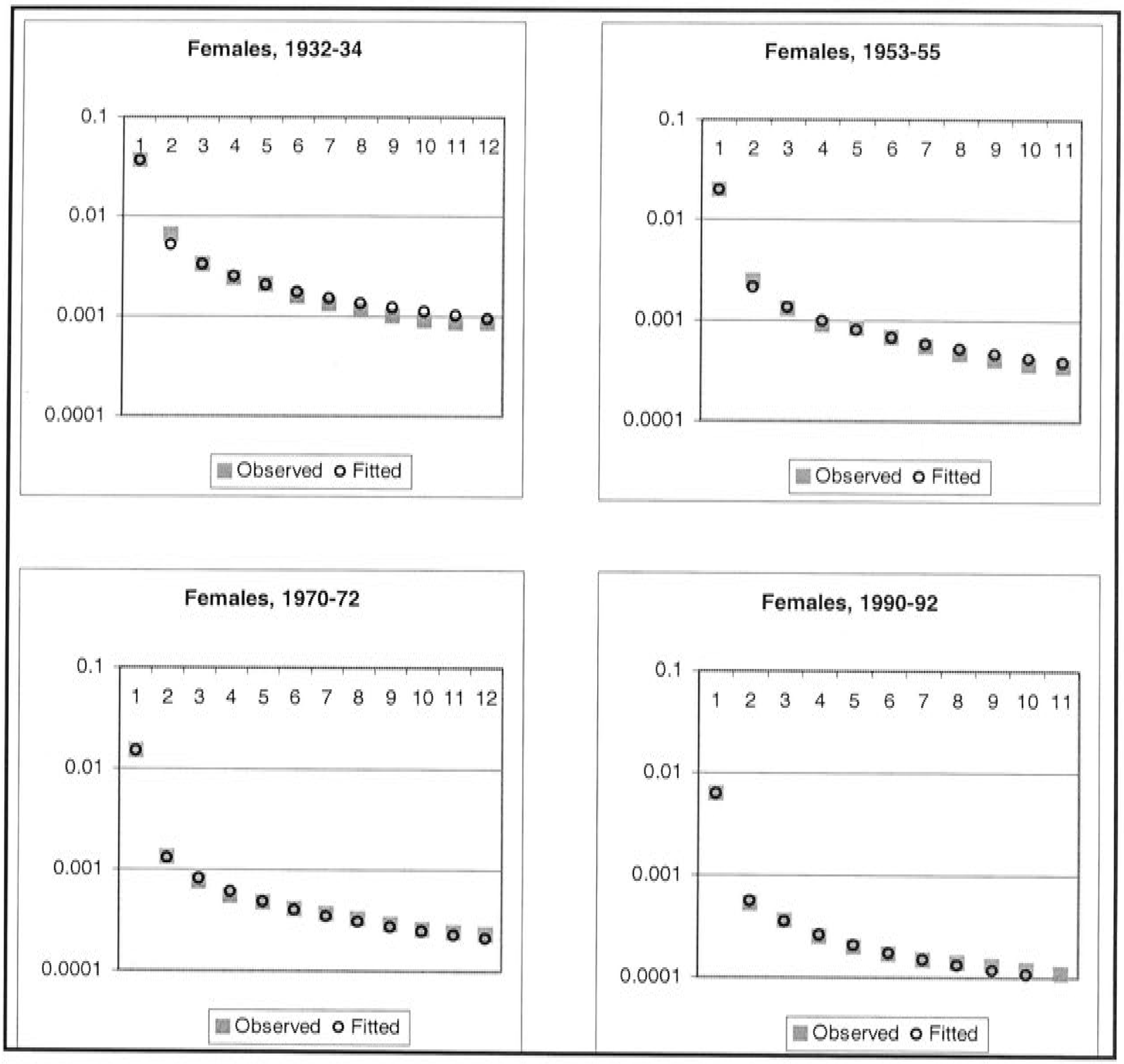


Figure 5.3: Observed and fitted probabilities of death, Japan, males, selected periods

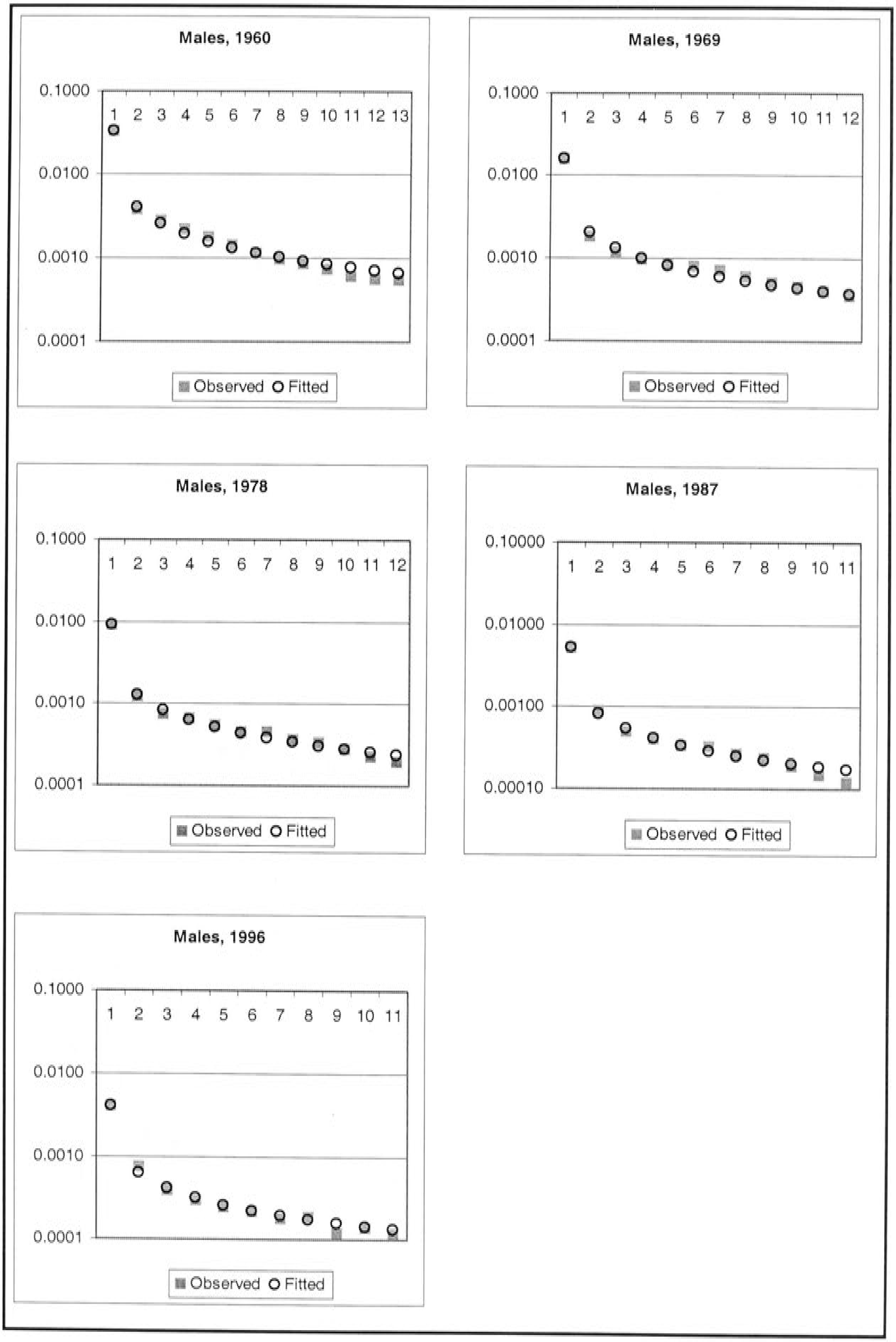


Figure 5.4: Observed and fitted probabilities of death, Japan, females, selected periods

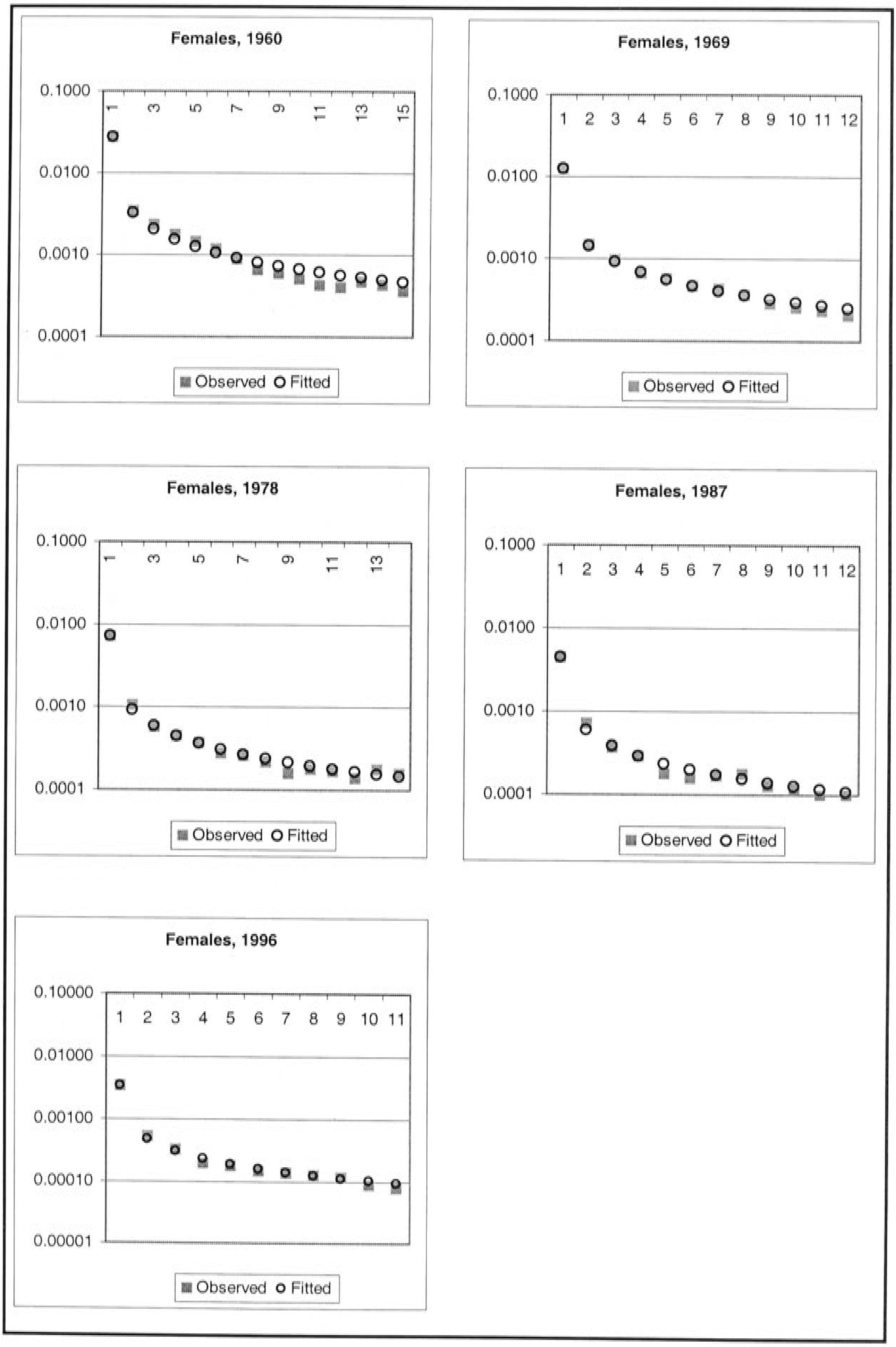


Figure 5.5: Observed and fitted probabilities of death, Sweden, males, selected periods

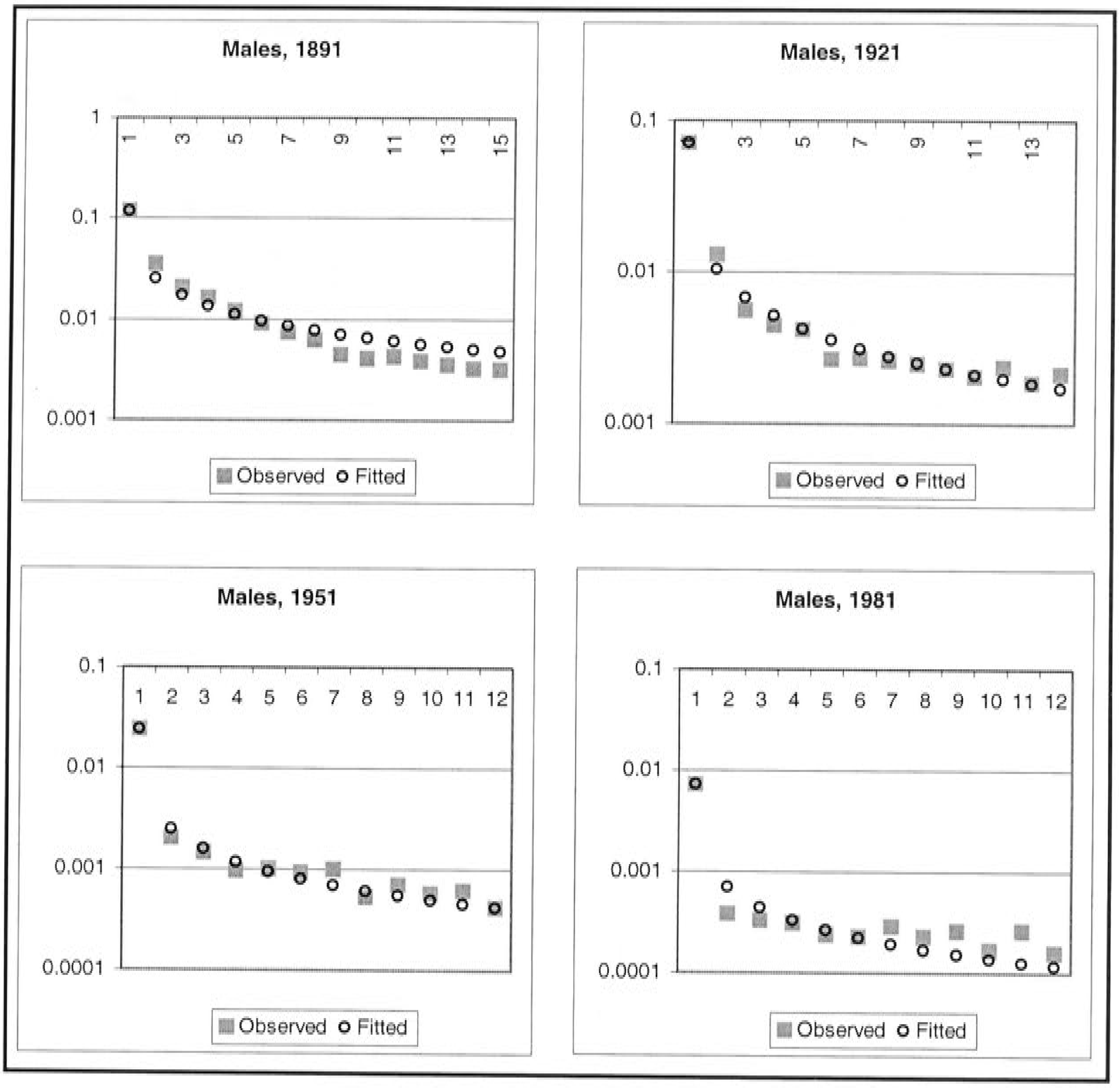


Figure 5.6: Observed and fitted probabilities of death, Sweden, females, selected periods

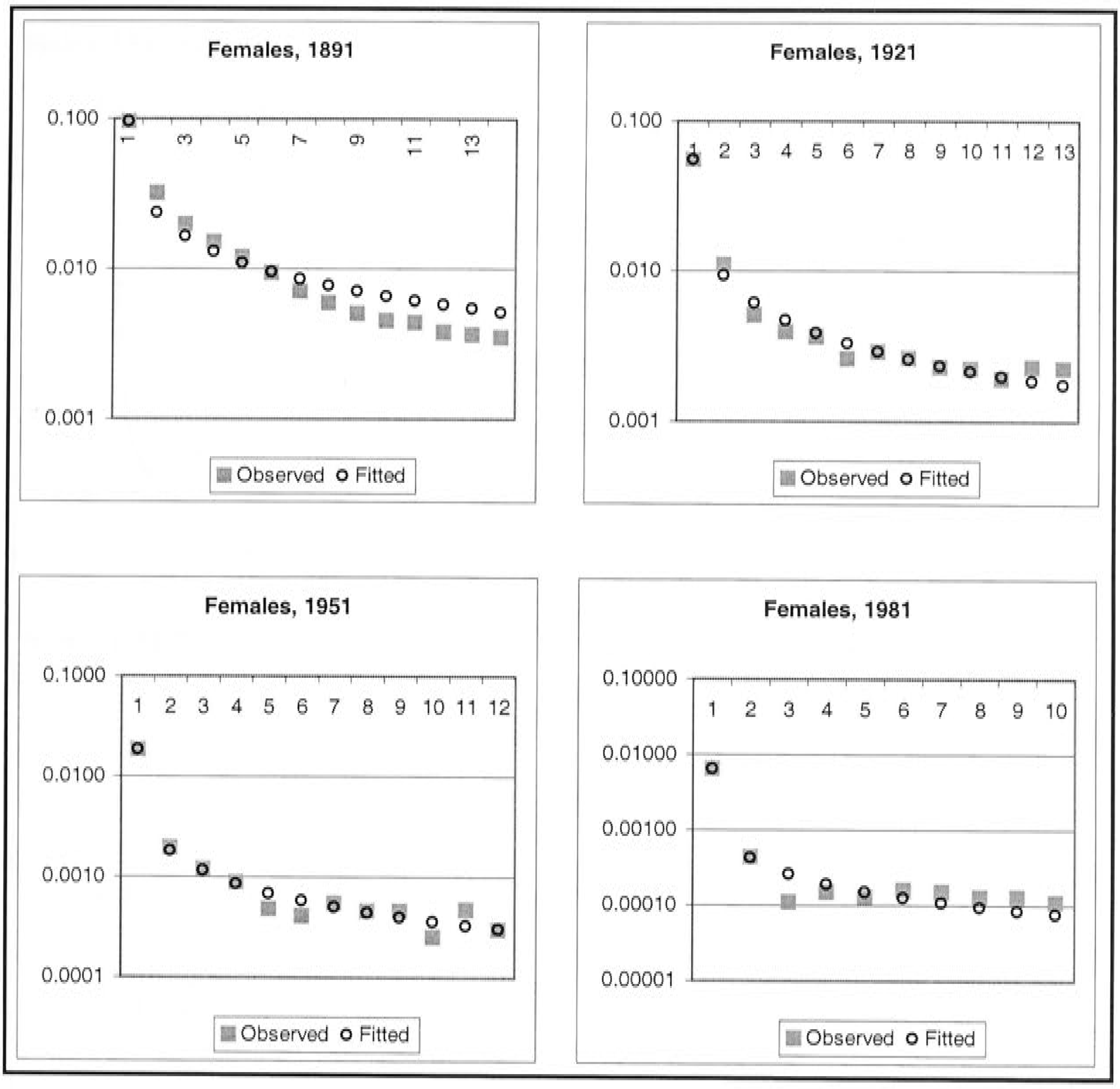


Figure 5.7: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Australia, males, selected periods

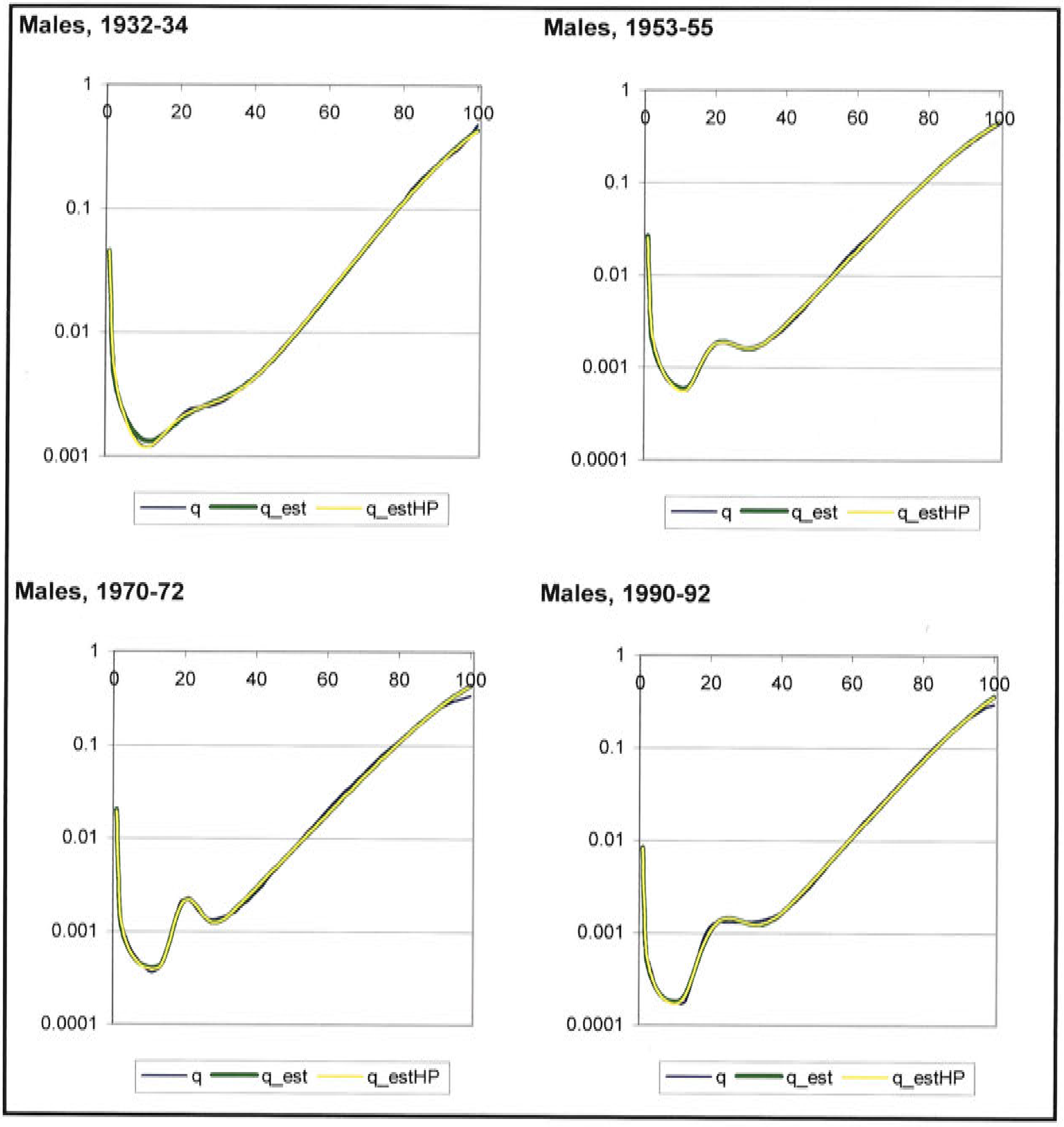


Figure 5.8: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Australia, females, selected periods

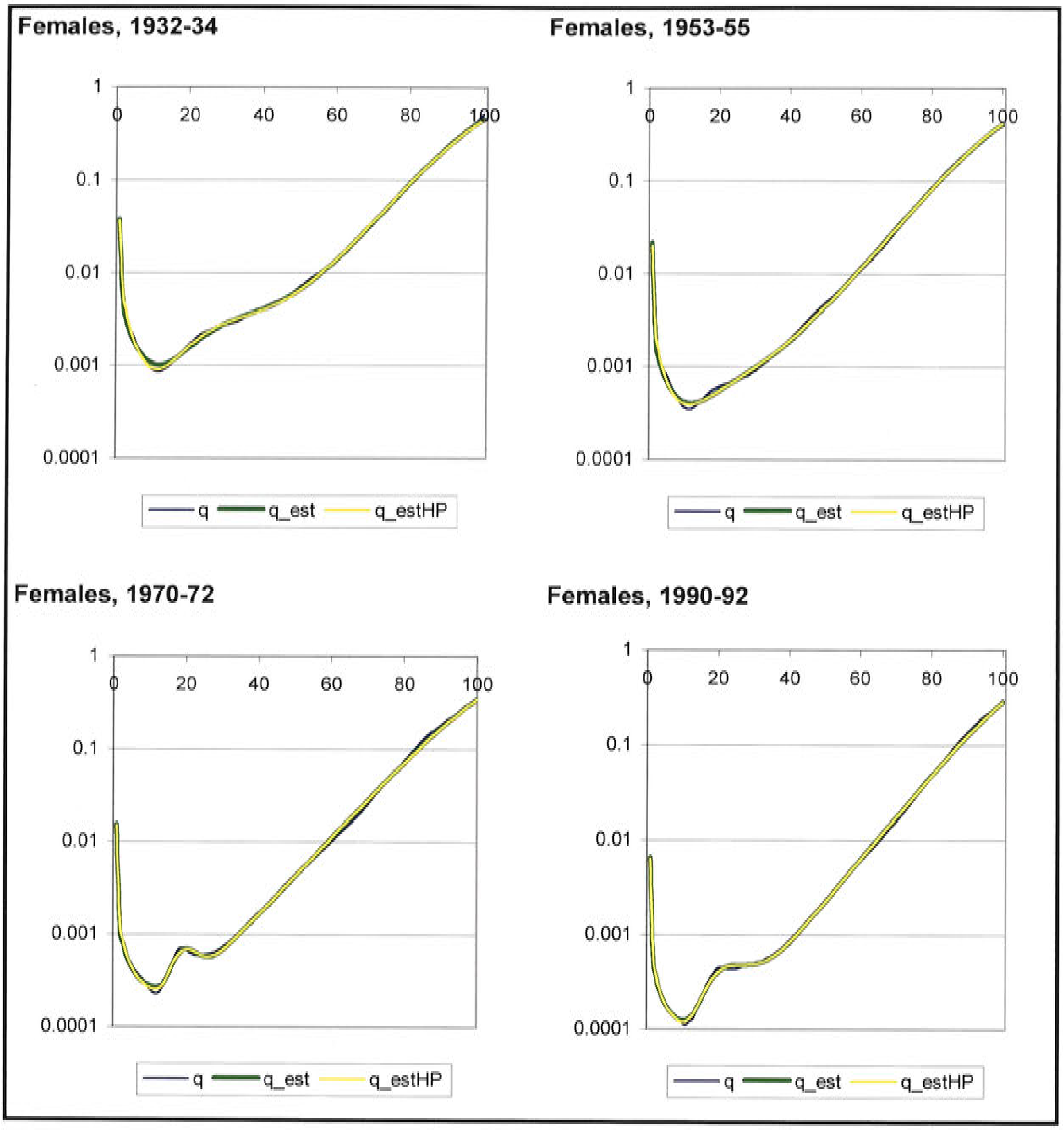


Figure 5.9: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Japan, males, selected periods

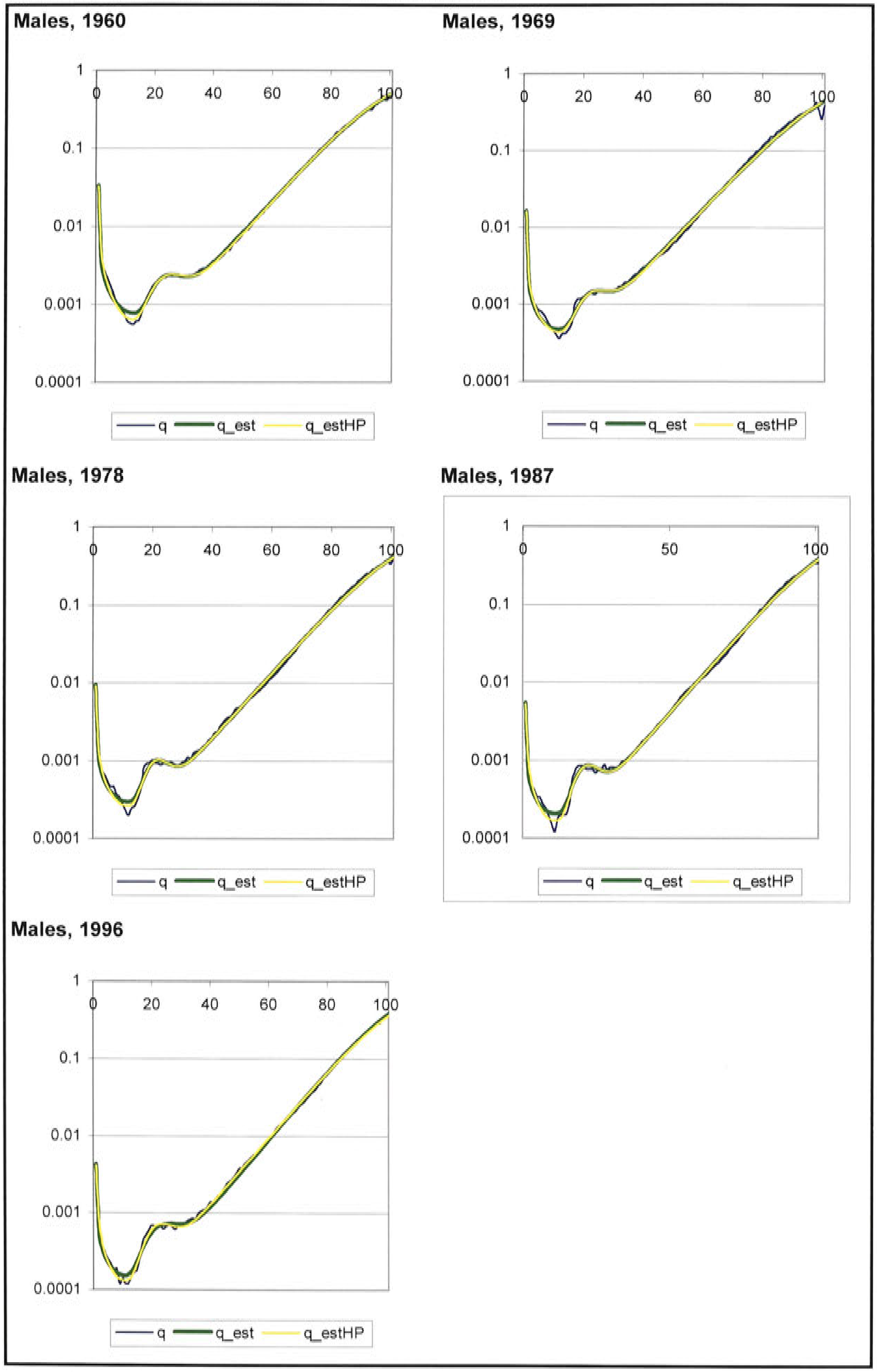


Figure 5.10: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Japan, females, selected periods

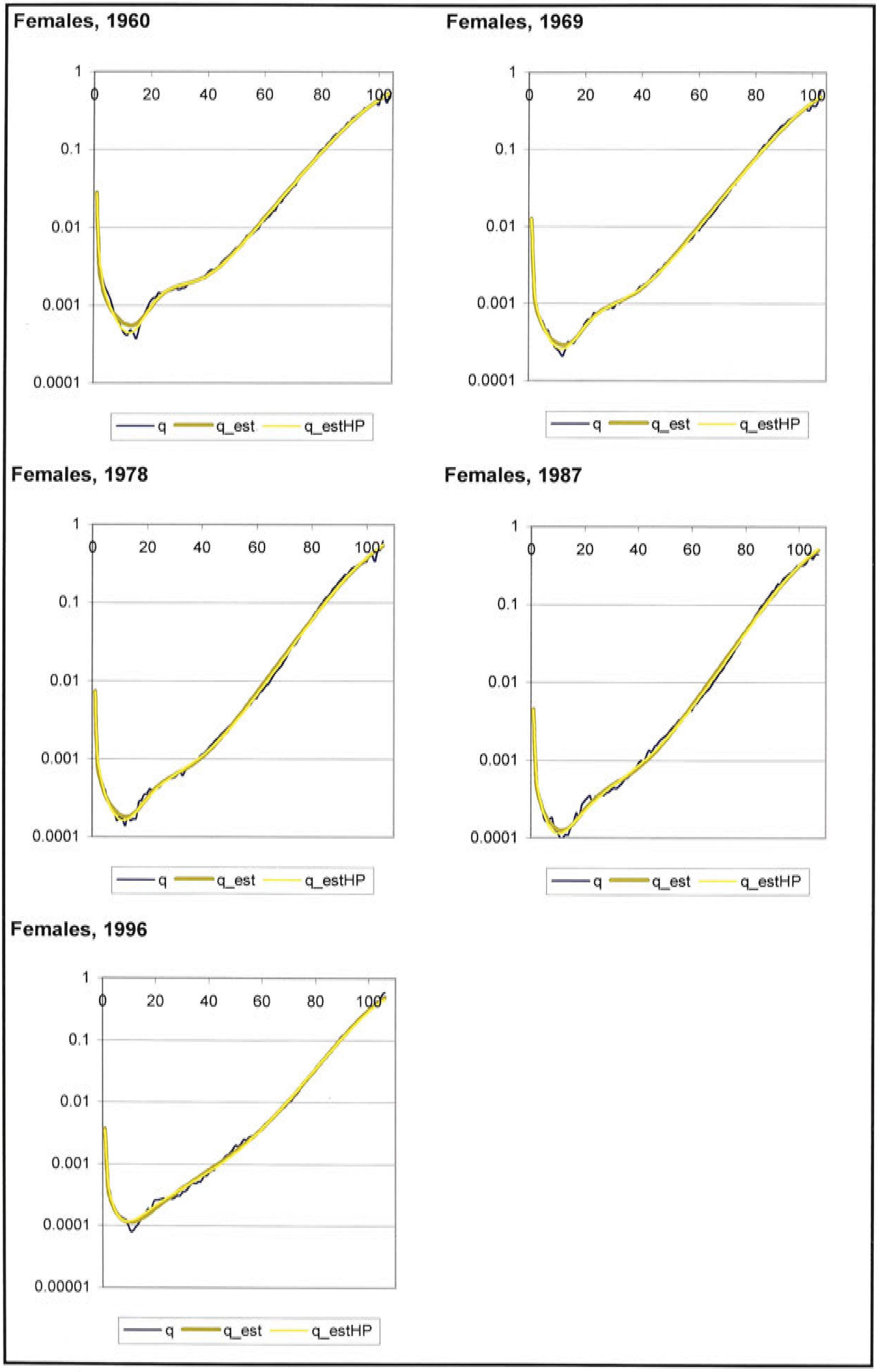


Figure 5.11: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Sweden, males, selected periods

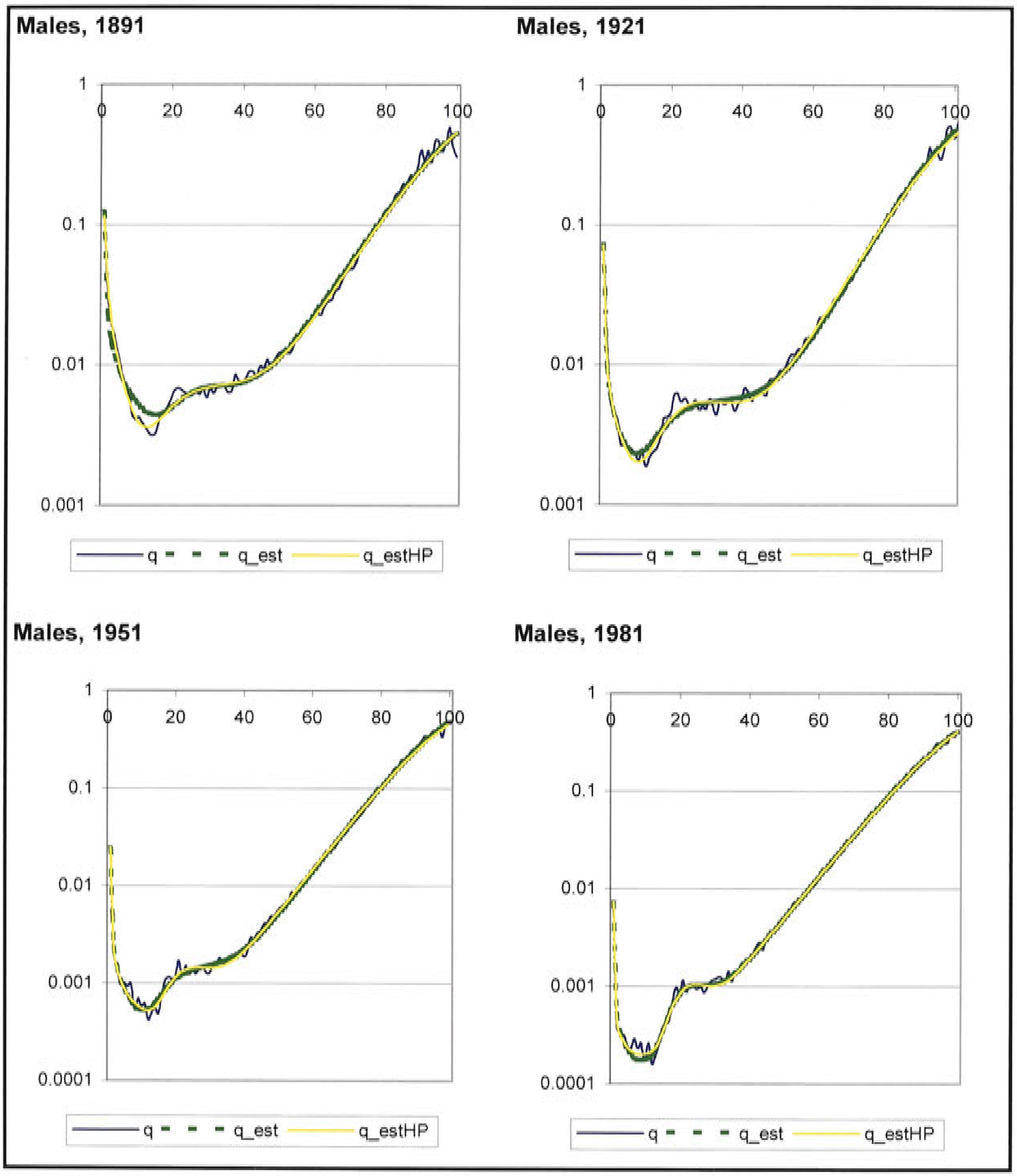
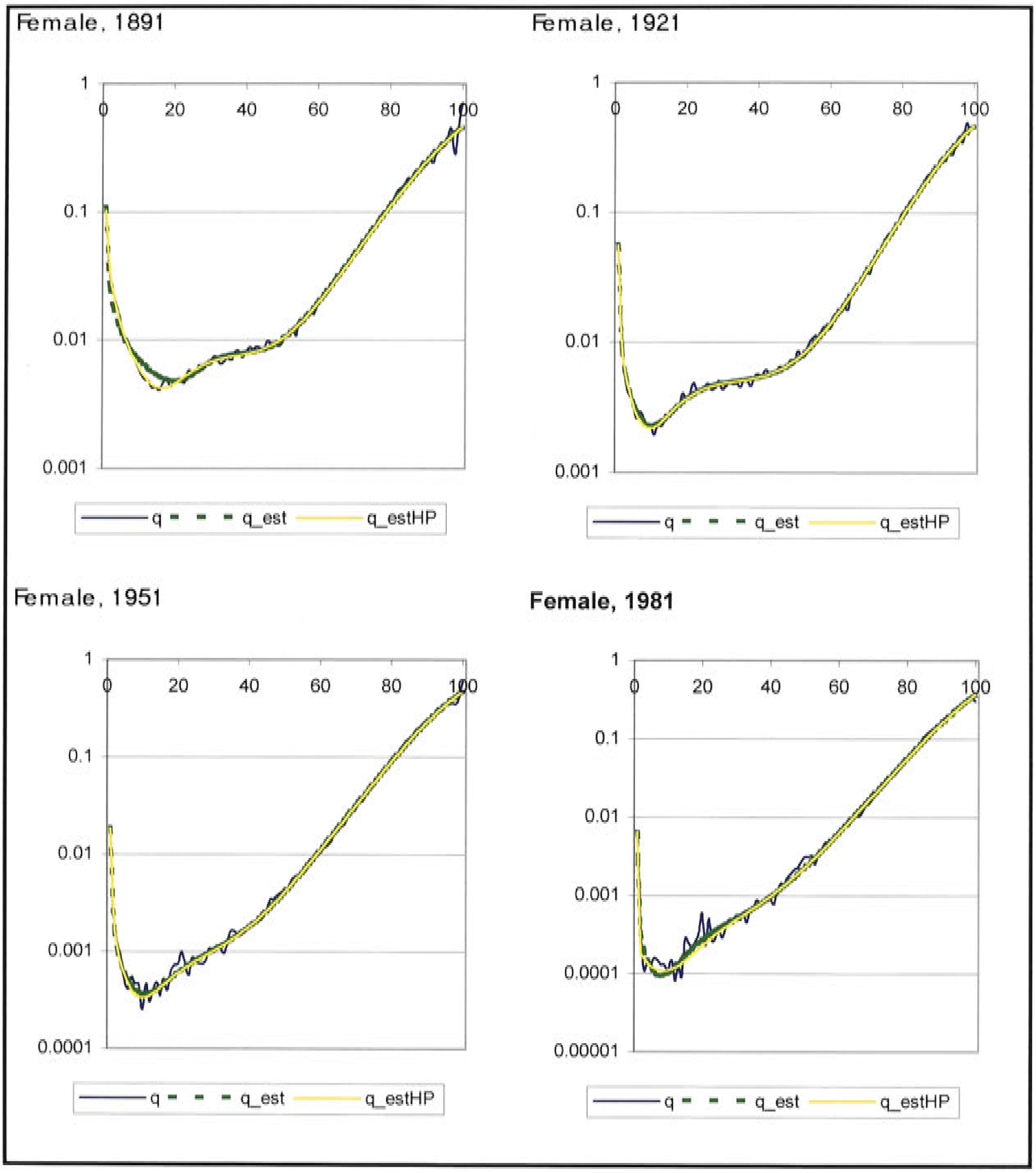


Figure 5.12: Observed and fitted probabilities of death by new and H-P models, whole age ranges, Sweden, females, selected periods



Chapter 6: Fitting the New Mortality Model- Application to Indian Data

In theory, there is no difference between theory and practice. But, in practice, there is.

Jan L.A. van de Snepscheut

6.1 Introduction

This chapter presents the results of fitting the new mortality model to Indian data. Six variants from the available Indian data sets are used in fitting the model; these are sex-specific death rates for India, rural India and urban India for the years 1970-97. As seen in Chapter 5, the new model has been tested with data for Australia, Japan and Sweden; the fittings were done with the single-year probabilities of death. For the application of the model to Indian data, 5-year age group probabilities will be used. This requires slight modification to the fitting methodology.

The new model has converged in all cases without much statistical manoeuvring. The NLIN procedure of SAS requires initial values of parameters to be supplied with precision. Greater flexibility in the initialisation of the parameters for the new mortality model was observed compared to that for the H-P model.

6.2 Methods for abridged rates

In fitting the first part of the curve, one parameter is needed. However, in fitting the new mortality model, a scalar multiplier was also needed to place the first part of the curve in relation to the rest of the mortality curve for the entire age range. The parameter p is now denoted as 'R', as 'p' is used as the default parameter for the statistical packages, especially for SAS. So the functional shape of the new model is:

$$q_x = f_1(x) + f_2(x) + f_3(x) \text{............}(6.1)$$

where

$$f_1(x) = M * \Delta F(X < x)$$
$$f_2(x) = De^{-E\{(\log(x/F))^2}$$
$$f_3(x) = GH^x$$

and

$$F(X < x) = \left(\frac{x}{\theta}\right)^R \text{ ....}(6.2)$$

M is the scalar parameter and the rest are defined as previously. By fitting equation 6.1 to the data sets from India, the parameters M, R, D, E, F, G and H have been estimated.

6.3 Fitting to the age pattern of Indian data

Data for India for the years 1970-1997 have been used to estimate the parameters; they are time-series for 28 years. In total, 168 data sets are fitted to show the ability of the model to represent Indian mortality patterns. The results of the fitting patterns are presented in the following sections of this chapter. As it is not possible to present 168 graphs of the fitted results, data sets at five-year intervals were selected; 1972, 1977, 1982, 1987, 1992 and 1997. These periodically selected graphs of the observed and fitted q_x values are provided to demonstrate the fit; the estimated parameters are discussed in the various sections of this chapter.

6.3.1 Fitting to data for all India

The parameters obtained from fitting the new mortality model to data for India as a whole are shown in Appendix 6A and 6B for males and females respectively; they provide the statistical summary of the parameters and overall estimation. A summary is shown in Tables 6.1 and 6.2. Important variations in parameters when fitted to male and female mortalities in India were noticed.

Figure 6.1 presents graphs for the selected years of the observed and fitted q_x values for males and females in India. As can be noted, the fitted values not only follow the patterns of the empirical data values but also define the movements of the curves smoothly. Each set of fitted q_x values was reached through convergence of the model and the parameters are estimated with statistical significance. The F-values for the fits were observed to be highly statistically significant, F<0.0001 (Appendix 6A and 6B) in most of the cases.

The upper part of Figure 6.2 provides trends in parameters M, R, D, E, F, G and H for Indian males for the time series 1970-97. As seen, the trends in M and R are in generally opposite directions over time. Parameter M is a scalar multiplier that represents the base level of mortality for the initial age group; it is expected it to fall over time. On the other hand, R rises slightly, meaning that deaths are less concentrated at the initial ages. These trends reflect declining infant and early child mortality. M and R counterbalance each other in general but there persist some annual fluctuations in the estimates. These annual fluctuations in M and R may be caused by annual variations in infectious diseases or reporting errors due to age mis-reporting across the 5-year bound. The year-to-year fluctuating trends in parameters M and R represent the steady long-run trends, which are important for determining future mortality and its further analysis.

Parameter D, which represents the intensity of young adult (age groups 15-34) deaths, shows an increasing trend over time, which indicates an increasing relative propensity to die in these ages. Year-to-year fluctuations were also observed for parameter D but a long-run trend is clearly visible. Parameter E shows a declining trend over time, despite the annual fluctuations. The inverse of E represents the spread of the young-adult mortality hump. So decline in E tells us that the hump is decreasingly concentrated around its modal value, F. Parameter F fluctuates between values 20 and 30 with some exceptions in the early 1980s and late 1990s with values around 40. F shows a slightly increasing trend over time, meaning that the accident hump is shifting relatively to the older ages for males in India.

Parameter G shows a declining trend with annual fluctuations. G represents the intercept of the Gompertz curve at the first age group (0-4); H represents the slope of the Gompertz curve. G and H counterbalance each other with G having a declining and H a slightly increasing trend.

The lower part of Figure 6.2 shows the seven estimated parameters for females in India. Parameters M and R exhibit similar patterns to those for males. M has a clear declining trend indicating decline in the base mortality for the first mortality curve. Parameter R shows a slight increase over time, suggesting a small but declining concentration of deaths in the initial age group of the first mortality curve. Both M and R show annual fluctuations for the entire period. M reflects the more profound overall declining trend. R varies between 0.30 and 0.45 with year-to-year fluctuations. Despite fluctuations locally, R exhibits a clear trend.

Parameter D has a declining trend in contrast to that for males. A decline in D shows that the hump in female mortality is getting less severe over time, suggesting decline in mortality in the age range 15-34, which may be due to improvement in the levels of maternal mortality in India in the last three decades (Ch. 2). Parameter E has a rising trend indicating that the mortality hump for females has become more concentrated around the mode over time. A slight decline in F suggests that the hump is shifting to younger ages over time. E and F are counterbalancing, since an earlier mode would be expected to be associated with a smaller spread.

Parameters G and H complement each other by showing opposite trends over time; however, in general their trend is flatter over this time period than is the case for males. On the other hand, the year-to-year fluctuations are more prominent for females than for males. G has a slight decreasing trend while H has a slightly increasing trend; this means that the level of older-age (senescent) mortality is falling slightly while the relative share at the oldest ages is increasing.

6.3.2 Fitting to data for rural India

Figure 6.3 presents the selected graphs for observed and fitted q_x values for rural males and females in India. The fitted q_x values represent observed mortality well and the fits are highly statistically significant ($F<.0001$; see Appendix 6C and 6D). A summary of estimated parameters is presented in Tables 6.3 and 6.4. Figure 6.4 presents the graphs of the estimates of the parameters of the new model. Parameters M and R show similar patterns for males and females. M shows a declining trend while R shows a slightly increasing overall trend for both sexes. Decline in M indicates a decline in the level of mortality at younger ages. Increase in R shows that there is less concentration of deaths in the early ages.

Parameter D has an increasing trend for males and a decreasing trend for females, as is the case of India as a whole. As for India as a whole, the increasing trend for males indicates that the accidental component of male mortality is getting more severe, but control in maternal mortality may be the cause for D declining for females. The values for females are more stable and the decline is sharp.

Values of parameter E are relatively low for females compared to males, which suggests that the female hump is more spread than the male hump, but the trends are opposite, suggesting convergence for the two sexes. F has been increasing for males and decreasing for females, showing that the modal ages have been shifting in reverse directions.

The trends in G and H counterbalance each other over time. Female trends in these parameters were more steady than those for males; however, fluctuations were evident in both cases and they exhibit similar trends.

6.3.3 Fitting to data for urban India

The selected graphs of the observed and fitted $\mathbf{q}_x$ values are presented in Figure 6.5 for urban males and females in India: the fitted $\mathbf{q}_x$ values exhibit good fit for the new model. Estimations converged with high statistical significance in most of the cases (see Appendices 6E and 6F). The summary of estimated parameters is given in Tables 6.5 and 6.6; Figure 6.6 provides the graphs of parameters. Parameter M shows a declining trend for males and females. For males more systematic year-to-year fluctuations were noted, but still the long-term trend is evident; for females the trend was steady except for two outlier values. Decline in M represents a decline in the level of mortality in the initial age group. Parameter R shows a slight declining trend for males but a very slight increasing trend for females. Year-to-year fluctuations in R for males and females are visible, but trends can also observed over the long run.

Unlike the all-Indian and rural Indian experiences, D has an increasing trend for males as well as for females. The minuscule increasing trends for males and females are less evident than the fluctuations. E shows a slight declining trend for males but stays more or less steady for females over time. Year-to-year fluctuations in E were more prominent for females than for males. F has shown a slow declining trend in both cases and shows more fluctuations than E. Fluctuation in F occurs because of the low levels of mortality and the difficulty of identifying a modal value in this situation. A slightly decreasing trend for males and an increasing trend for females are observed.

G and H show parallel trends to those of the total Indian and rural trends. A more prominent decline in G is noticed for females than males. H declines for females but remains steady for males. For females, the fluctuations are least when compared to the G and H for all the six cases. Urban males also exhibit less fluctuation in G and H than all-Indian and rural males, perhaps urban areas have enjoyed better of health care status as evidenced by the longevity figures (Ch. 2) during the last three decades.

6.4 Observations

The new model of mortality provides a good representation of mortality in India. Appendices 6A to 6F show that in all cases the F-statistics are significant at the 1 per cent level and usually at .01 per cent level (F<.0001).

Fitting has been done by the weighted non-linear estimation of the parameters with the weights varying from $(1/q_x)^{1.1}$ to $(1/q_x)^2$. Most of the parameters were estimated without constraints; in some cases, constraining was required and yielded stable sets of parameters. In using the parameters D, E, F, G and H of the H-P model for the second and third phases of Indian mortality, overparameterisation was tackled by fixing values of one or two parameters to a constant value. This approach has been used by other researchers in similar situations (Rogers, 1986; Congdon, 1993; Dellaportas *et al.*, 2001).

In some cases, constraining of either of the parameters E or F was required for assuring the convergence of the models. Interestingly, constraining E yielded a set of parameters which were highly significant in most of the cases. Parameter D was significant in most of the cases at a level of significance between 5 and 10 per cent. The modular shape of G and H has more statistical stability as G and H tend to be significant at levels of 1 to 5 per cent in most of the cases. The parameters M and R were also estimated with very high statistical significance: they were mostly significant at the one per cent level.

Of the 1176 cases of parameter estimation from the 168 data sets, 1029 were significant at the 5 per cent level or higher. If 5-10 per cent significance levels are also included, then the number of significant parameters becomes 1137, and in only 39 cases were parameters not statistically significant. Table 6.7 and Figure 6.7 summarise the number of significant parameters (at the 5 per cent level) for each estimation for every data set analysed. The parameter E was not significant for 30 of the 168 fits, G and R each 3 times, D twice and

M only once. It was observed while fitting that constraining parameter E has always yielded other parameters that are stable and highly significant. In other words the fitted q_x values agree with empirical q_x values with very high statistical significance. As expected in any statistical procedure, some parameters do not yield statistically significant results, but they need to be included in the set of parameters as they complete the mortality representations.

6.5 Conclusion

Mortality in India has been adequately represented by the new mortality model. The parameters estimated are stable (of similar order) except in a few cases when fluctuating values have been observed. In every case of fitting, convergence has been achieved in the fitted values in respect of the empirical values.

Estimates for India and rural areas are more stable than the urban series of parameters. The reasons for instability lie with the numerical instability of weighted least squares estimation, particularly in the cases of inconsistent changes, which have been noticed for some of the urban q_x values. As is the aim of any statistical process, apart from the few fluctuations, there is a steady series of parameter values that pave the way for the forecasting of the parameters for the future.

The graphs of parameters provide considerable information on mortality representation in India. Parameter M shows a declining trend in all the cases for males and females in India. Parameter R shows a slight increasing trend in most of the cases while constant in other cases. Increase in R means less concentration of early age deaths among all pre-teen age deaths, which is likely in a declining mortality regime. Parameter D has an increasing trend for males but the opposite for females. This shows that for males, the hump has become more intense but the reverse applies for females. Parameter E shows variable patterns but may be considered more or less constant. So the severity of the hump varies little. Parameter F shows increasing trends for males but declining trends for females. F represents the mode of the accident hump: for females the hump is shifting to younger ages but for males it is moving to higher ages. Parameters G and H counterbalance each other in the six cases of male and female mortality.

Estimated sets of parameters may be used as the base for the forecasting. As advanced time series techniques like autoregressive integrated moving average (ARIMA) allow the variations and their structural errors to be represented in a standard manner, this method will be applied to the time series obtained for India.

Figure 6.1: Observed and fitted q_x, India, males and females, 1970-97 (observed and fitted q_x are represented by solid squares and circles respectively)

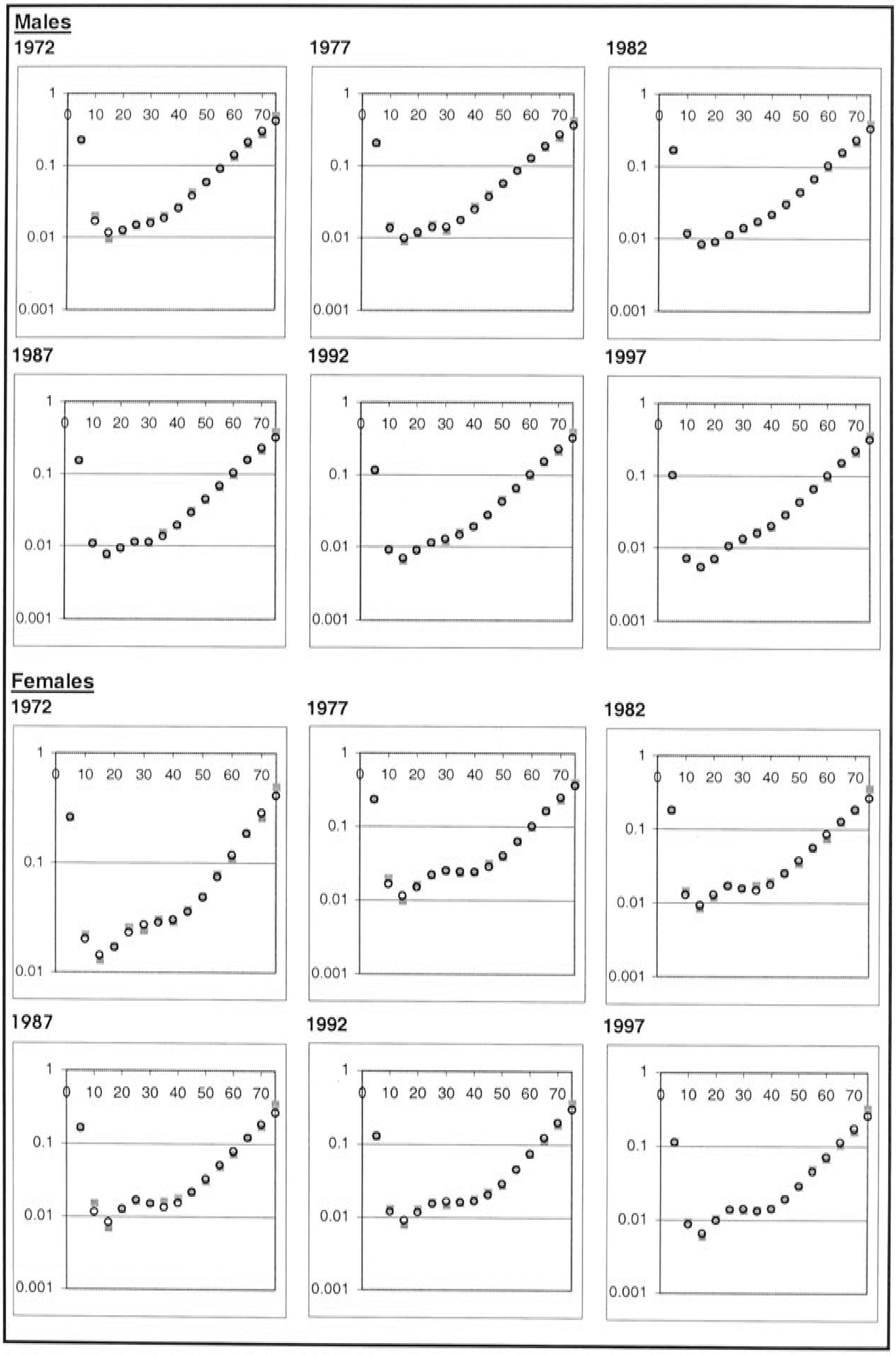


Figure 6.2: Trends in parameters M, R, D, E, F, G and H, India, males & females, 1970-97

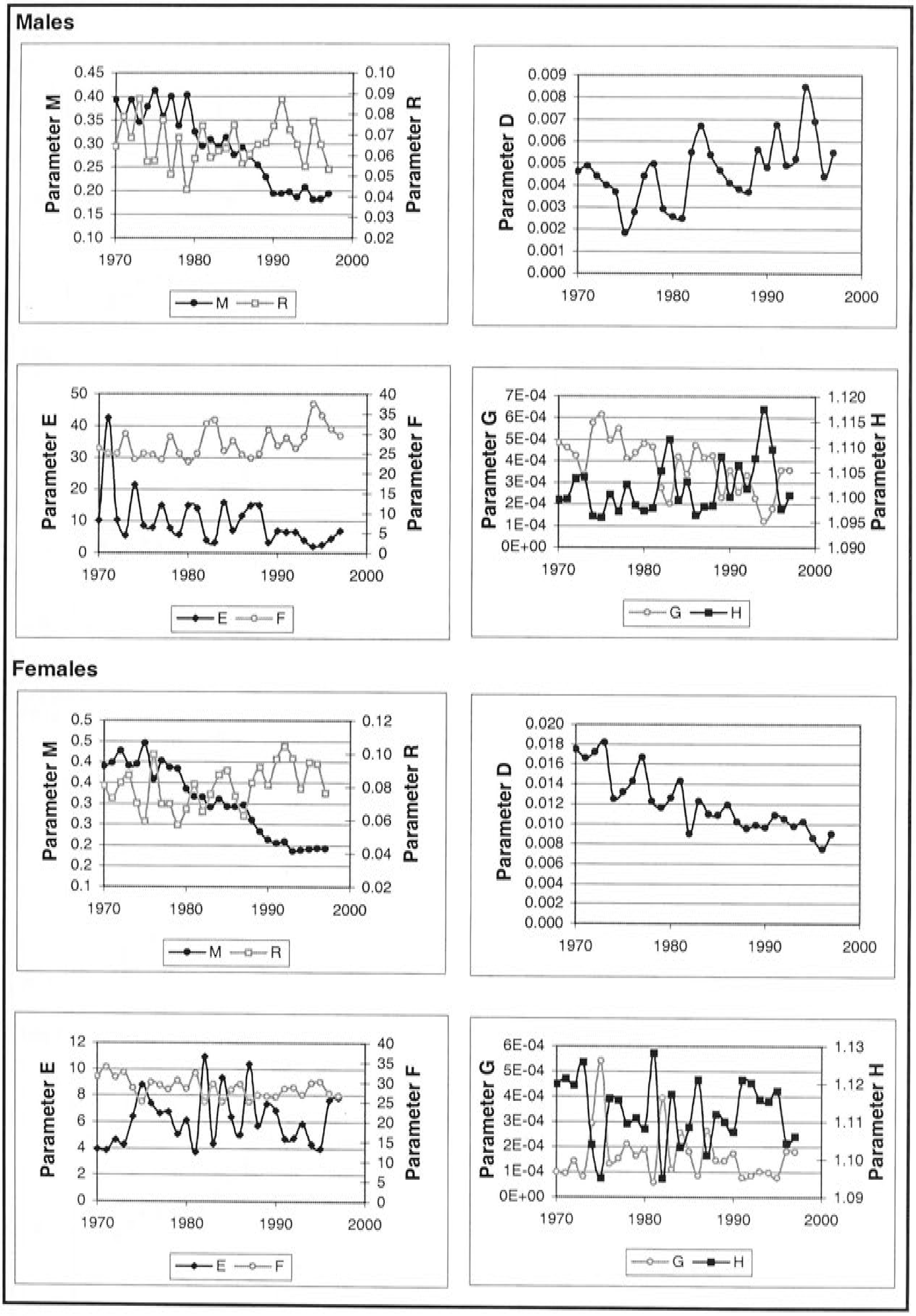


Figure 6.3: Observed and fitted q_x, India, rural, males and females, 1970-97 (observed and fitted q_x are represented by solid squares and circles respectively)

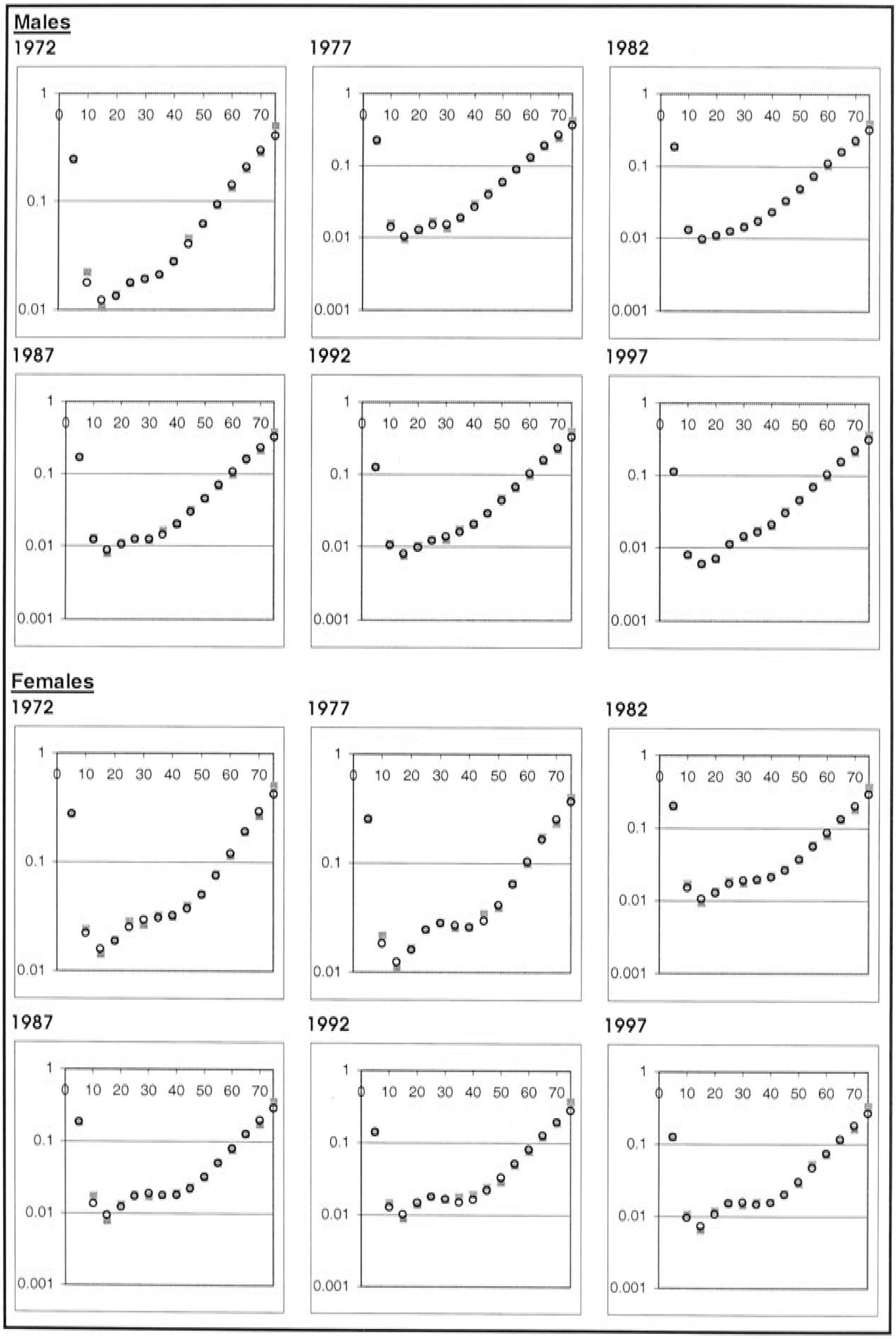


Figure 6.4: Trends in parameters M, R, D, E, F, G and H, India, rural, males & females, 1970-97

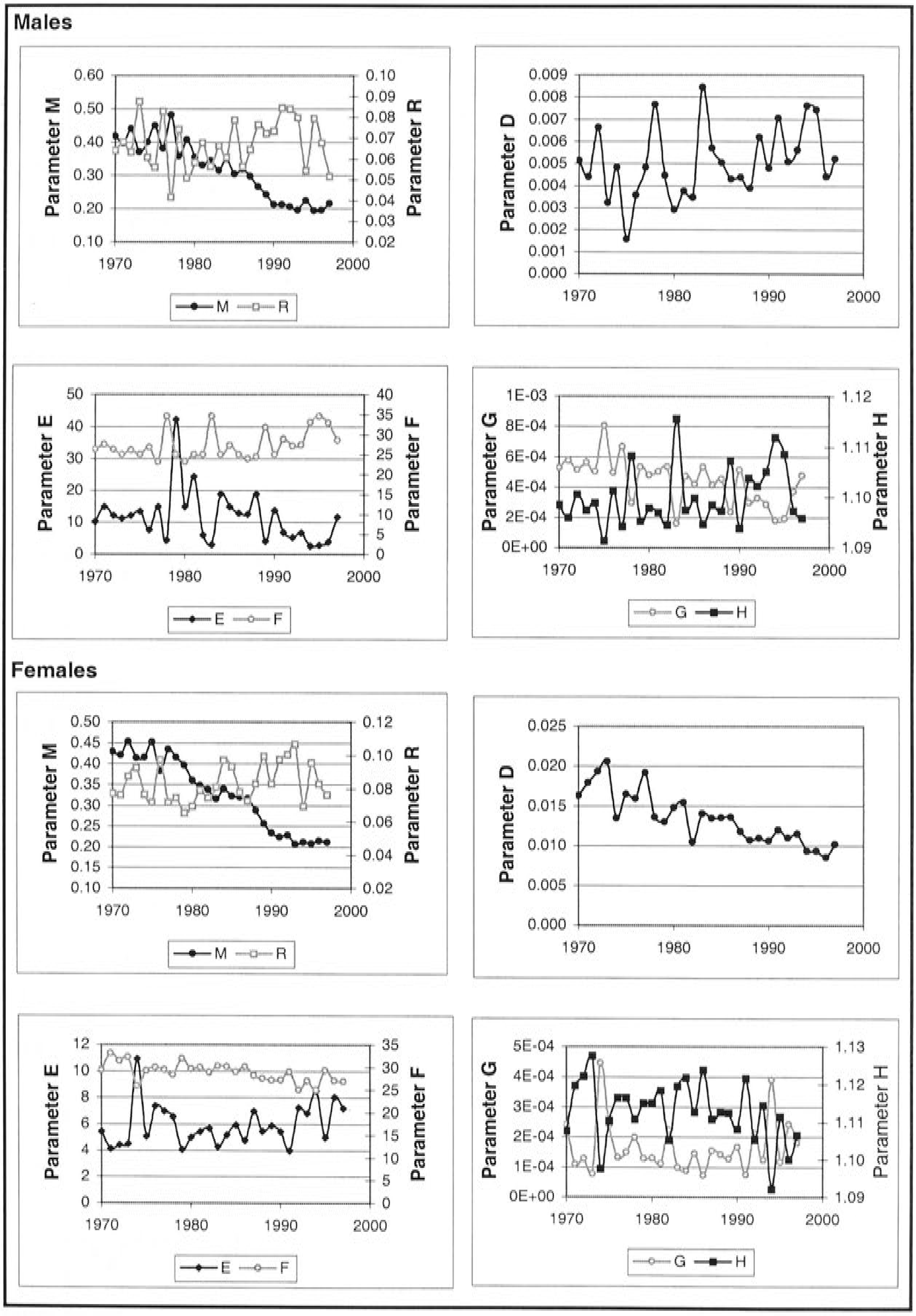


Figure 6.5: Observed and fitted q_x, India, urban, males and females, 1970-97 (observed and fitted q_x are represented by solid squares and circles respectively)

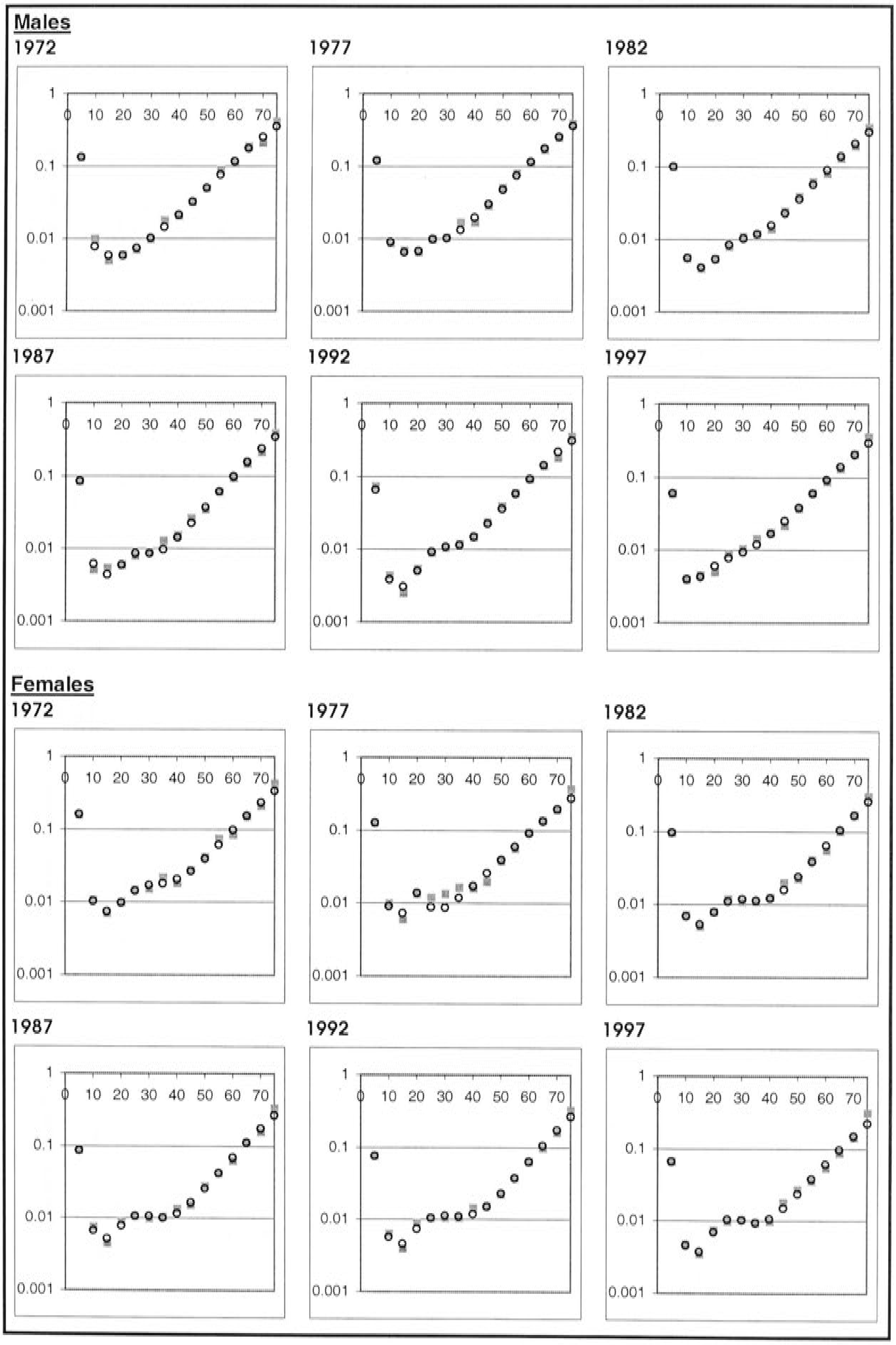


Figure 6.6: Trends in parameters M, R, D, E, F, G and H, India, urban, males & females, 1970-97

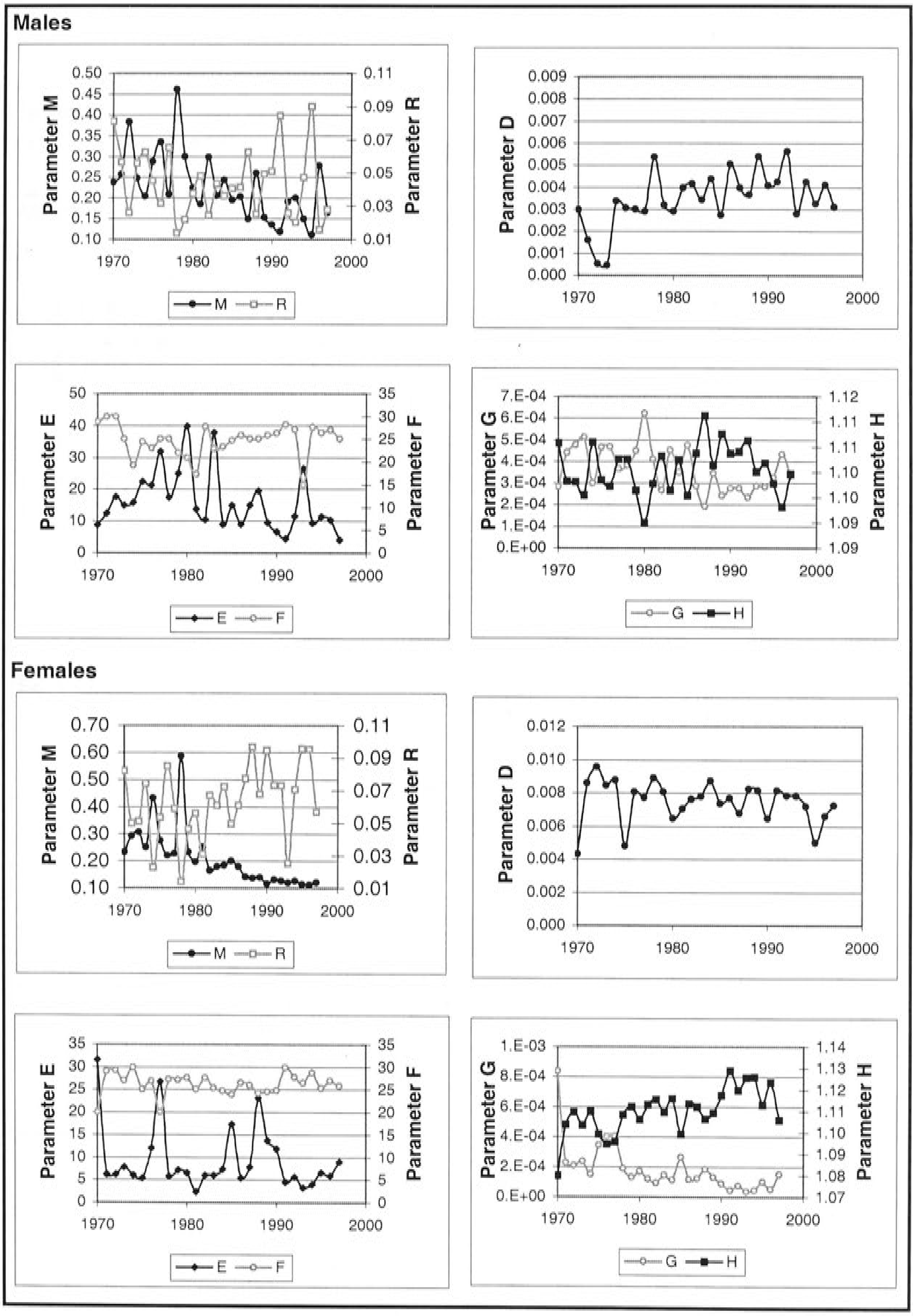


Figure 6.7: Number of statistically significant parameters per estimation; India, rural and urban, males and females, 1970-97 (for $\alpha<.05$)

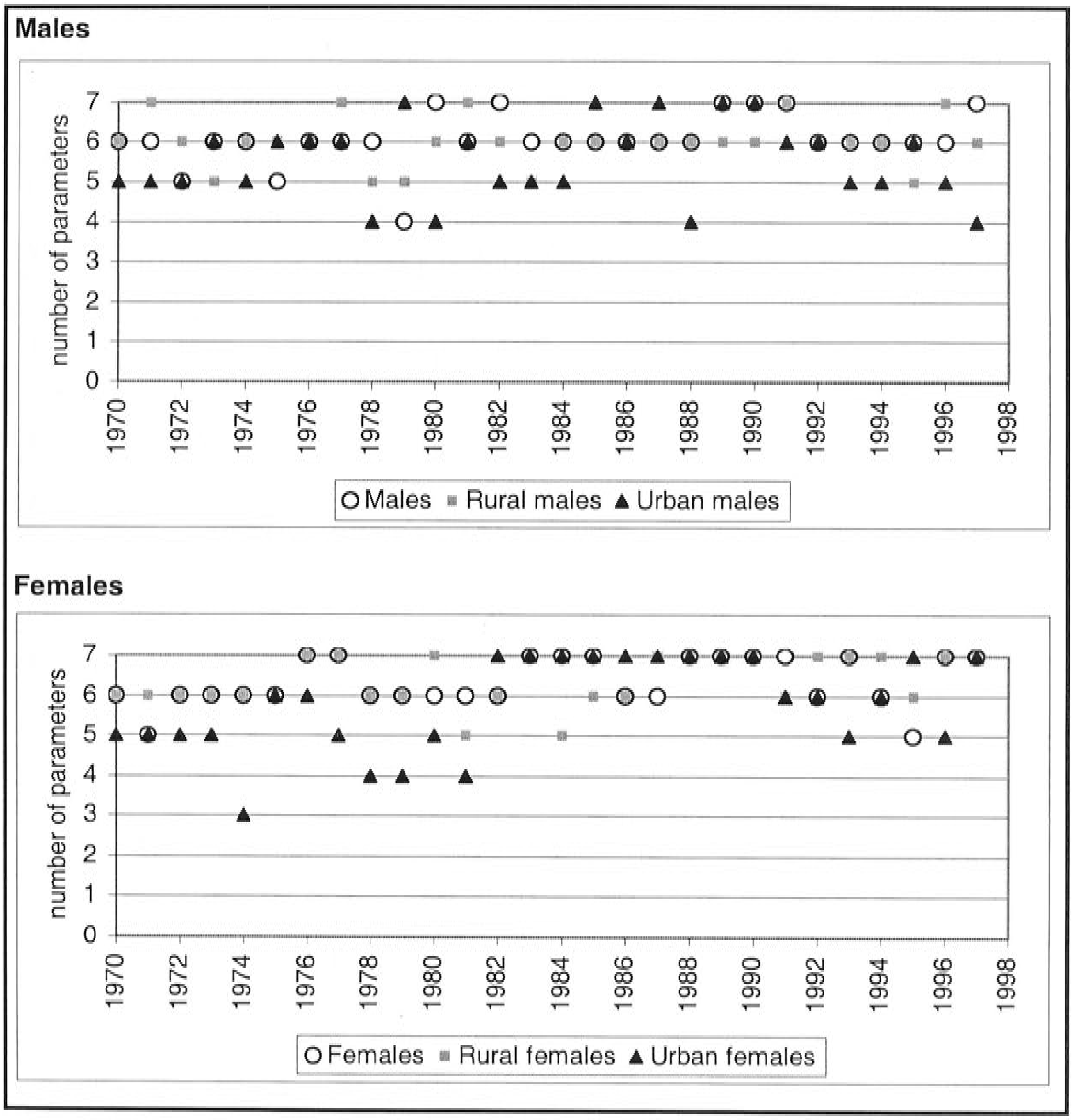


Table 6.1: Estimated parameters of the new mortality model, India, males, 1970-97

Years	Parameters						
	D	E	F	G	H	M	R
1970	0.00461	10.23	26.35	0.000484	1.0995	0.3930	0.0644
1971	0.00488	42.51	25.00	0.000462	1.0997	0.3535	0.0789
1972	0.00440	10.35	25.00	0.000423	1.1037	0.3929	0.0686
1973	0.00399	5.53	30.00	0.000337	1.1040	0.3454	0.0875
1974	0.00370	21.41	23.59	0.000576	1.0962	0.3780	0.0571
1975	0.00184	8.48	25.00	0.000614	1.0960	0.4120	0.0577
1976	0.00276	7.89	24.64	0.000497	1.1005	0.3579	0.0772
1977	0.00440	15.00	23.45	0.000553	1.0972	0.3997	0.0509
1978	0.00496	7.73	29.23	0.000412	1.1025	0.3378	0.0687
1979	0.00292	5.87	25.00	0.000438	1.0984	0.4025	0.0436
1980	0.00257	15.00	22.85	0.000481	1.0973	0.3248	0.0586
1981	0.00248	14.04	25.00	0.000467	1.0979	0.2949	0.0743
1982	0.00549	4.04	32.49	0.000277	1.1052	0.3086	0.0591
1983	0.00670	3.23	33.47	0.000203	1.1115	0.2936	0.0621
1984	0.00538	15.83	25.67	0.000419	1.0995	0.3128	0.0633
1985	0.00467	7.04	28.10	0.000341	1.1031	0.2766	0.0747
1986	0.00410	11.67	24.68	0.000473	1.0964	0.2911	0.0560
1987	0.00381	14.94	23.81	0.000416	1.0981	0.2737	0.0606
1988	0.00370	14.97	24.94	0.000425	1.0983	0.2541	0.0655
1989	0.00562	3.25	30.90	0.000229	1.1081	0.2290	0.0660
1990	0.00482	6.92	26.99	0.000356	1.1000	0.1941	0.0744
1991	0.00673	6.66	28.92	0.000256	1.1063	0.1940	0.0871
1992	0.00490	6.62	26.30	0.000333	1.1017	0.1968	0.0725
1993	0.00521	4.06	29.24	0.000228	1.1077	0.1863	0.0654
1994	0.00846	2.05	37.52	0.000122	1.1174	0.2069	0.0548
1995	0.00689	2.57	34.52	0.000178	1.1095	0.1817	0.0766
1996	0.00440	4.52	31.17	0.000358	1.0977	0.1819	0.0655
1997	0.00548	6.84	29.48	0.000357	1.1004	0.1937	0.0533

Table 6.2: Estimated parameters of the new mortality model, India, females, 1970-97

Year	Parameters						
	D	E	F	G	H	M	R
1970	0.01750	3.95	31.40	0.000099	1.1200	0.3901	0.0808
1971	0.01660	3.83	33.81	0.000095	1.1214	0.3978	0.0732
1972	0.01720	4.62	31.28	0.000142	1.1197	0.4269	0.0827
1973	0.01820	4.29	32.42	0.000082	1.1258	0.3907	0.0870
1974	0.01250	6.39	28.46	0.000291	1.1039	0.3946	0.0702
1975	0.01320	8.79	25.00	0.000541	1.0950	0.4447	0.0593
1976	0.01430	7.41	29.95	0.000133	1.1161	0.3594	0.0997
1977	0.01670	6.64	29.19	0.000153	1.1157	0.4023	0.0698
1978	0.01230	6.75	28.09	0.000210	1.1094	0.3862	0.0699
1979	0.01160	5.06	30.37	0.000164	1.1110	0.3836	0.0572
1980	0.01260	6.12	28.43	0.000190	1.1080	0.3355	0.0668
1981	0.01430	3.73	32.25	0.000059	1.1282	0.3163	0.0817
1982	0.00900	10.93	25.00	0.000393	1.0949	0.3160	0.0653
1983	0.01230	4.36	29.50	0.000110	1.1173	0.2912	0.0757
1984	0.01100	9.35	25.00	0.000255	1.1032	0.3104	0.0873
1985	0.01090	6.38	28.00	0.000180	1.1086	0.2930	0.0898
1986	0.01190	5.02	29.48	0.000085	1.1210	0.2928	0.0744
1987	0.01020	10.37	25.00	0.000264	1.1011	0.2961	0.0626
1988	0.00955	5.72	26.73	0.000146	1.1119	0.2606	0.0824
1989	0.00989	7.37	26.41	0.000144	1.1099	0.2328	0.0920
1990	0.00965	6.88	26.38	0.000173	1.1073	0.2133	0.0812
1991	0.01090	4.72	28.39	0.000078	1.1210	0.2052	0.0967
1992	0.01050	4.79	28.66	0.000086	1.1203	0.2094	0.1046
1993	0.00980	5.87	26.84	0.000102	1.1159	0.1859	0.0970
1994	0.01020	4.30	29.85	0.000096	1.1154	0.1881	0.0791
1995	0.00859	3.99	30.01	0.000077	1.1182	0.1903	0.0949
1996	0.00749	7.66	27.16	0.000182	1.1043	0.1924	0.0939
1997	0.00904	7.82	26.64	0.000180	1.1060	0.1918	0.0765

Table 6.3: Estimated parameters of the new mortality model, India, rural males, 1970-97

Year	Parameters						
	D	E	F	G	H	M	R
1970	0.00513	10.15	26.40	0.000530	1.0985	0.4179	0.0639
1971	0.00440	15.00	27.51	0.000576	1.0960	0.3961	0.0681
1972	0.00663	12.11	26.31	0.000516	1.1006	0.4400	0.0634
1973	0.00323	11.25	25.00	0.000566	1.0975	0.3705	0.0874
1974	0.00484	12.09	26.13	0.000505	1.0989	0.3990	0.0607
1975	0.00158	13.54	25.00	0.000809	1.0913	0.4499	0.0559
1976	0.00359	7.62	26.84	0.000500	1.1014	0.3802	0.0829
1977	0.00484	15.00	23.26	0.000668	1.0942	0.4816	0.0417
1978	0.00766	4.29	34.56	0.000296	1.1082	0.3580	0.0740
1979	0.00446	42.19	25.00	0.000533	1.0953	0.4068	0.0509
1980	0.00291	15.00	23.24	0.000480	1.0979	0.3547	0.0583
1981	0.00375	24.34	25.00	0.000506	1.0970	0.3300	0.0677
1982	0.00347	6.02	25.00	0.000537	1.0945	0.3448	0.0564
1983	0.00843	3.06	34.67	0.000161	1.1156	0.3148	0.0663
1984	0.00569	18.90	25.00	0.000472	1.0975	0.3509	0.0607
1985	0.00505	14.95	27.27	0.000420	1.0998	0.3046	0.0786
1986	0.00431	12.84	25.04	0.000533	1.0947	0.3194	0.0564
1987	0.00439	12.64	23.93	0.000416	1.0985	0.2976	0.0645
1988	0.00389	18.87	24.43	0.000454	1.0973	0.2659	0.0764
1989	0.00619	4.01	31.78	0.000237	1.1073	0.2427	0.0723
1990	0.00479	13.71	25.00	0.000515	1.0938	0.2126	0.0734
1991	0.00707	6.87	28.90	0.000300	1.1038	0.2125	0.0843
1992	0.00509	5.19	27.15	0.000329	1.1023	0.2064	0.0839
1993	0.00560	6.72	27.38	0.000287	1.1051	0.1954	0.0798
1994	0.00759	2.59	33.01	0.000179	1.1118	0.2242	0.0544
1995	0.00742	2.79	34.56	0.000192	1.1086	0.1946	0.0794
1996	0.00442	3.83	32.91	0.000373	1.0973	0.1958	0.0678
1997	0.00522	11.64	28.64	0.000475	1.0959	0.2164	0.0516

Table 6.4: Estimated parameters of the new mortality model, India, rural females, 1970-97

Year	Parameters						
	D	E	F	G	H	M	R
1970	0.016322	5.41	29.36	0.000241	1.1076	0.4290	0.0773
1971	0.017900	4.14	33.09	0.000110	1.1196	0.4210	0.0764
1972	0.019384	4.40	31.47	0.000127	1.1222	0.4534	0.0876
1973	0.020551	4.50	32.27	0.000077	1.1276	0.4145	0.0929
1974	0.013465	10.91	25.83	0.000447	1.0976	0.4154	0.0768
1975	0.016472	5.08	29.24	0.000239	1.1103	0.4520	0.0719
1976	0.015930	7.35	29.99	0.000132	1.1166	0.3836	0.0975
1977	0.019204	6.96	29.47	0.000150	1.1165	0.4347	0.0720
1978	0.013603	6.56	28.42	0.000199	1.1108	0.4151	0.0748
1979	0.012983	4.07	32.01	0.000129	1.1151	0.3973	0.0654
1980	0.014805	5.01	29.66	0.000131	1.1151	0.3605	0.0696
1981	0.015466	5.43	29.93	0.000112	1.1184	0.3481	0.0791
1982	0.010508	5.69	28.92	0.000224	1.1053	0.3385	0.0748
1983	0.014008	4.28	30.31	0.000100	1.1196	0.3155	0.0810
1984	0.013505	5.19	30.16	0.000089	1.1218	0.3398	0.0970
1985	0.013522	5.92	28.99	0.000146	1.1127	0.3227	0.0932
1986	0.013596	4.79	30.17	0.000073	1.1239	0.3201	0.0782
1987	0.011826	6.98	28.22	0.000155	1.1108	0.3167	0.0728
1988	0.010722	5.46	27.58	0.000141	1.1127	0.2884	0.0830
1989	0.010970	5.90	27.26	0.000127	1.1125	0.2561	0.0996
1990	0.010639	5.43	27.24	0.000168	1.1082	0.2338	0.0831
1991	0.011966	3.99	29.01	0.000076	1.1217	0.2233	0.0978
1992	0.011020	7.25	25.00	0.000212	1.1054	0.2288	0.1006
1993	0.011486	6.84	27.07	0.000125	1.1145	0.2065	0.1069
1994	0.009379	8.50	25.00	0.000392	1.0922	0.2109	0.0696
1995	0.009365	5.00	29.38	0.000116	1.1114	0.2076	0.0958
1996	0.008573	8.05	27.10	0.000243	1.1001	0.2151	0.0831
1997	0.010163	7.17	26.90	0.000181	1.1066	0.2115	0.0764

Table 6.5: Estimated parameters of the new mortality model, India, urban males, 1970-97

Year	Parameter						
	D	E	F	G	H	M	R
1970	0.00298367	8.88	28.84	0.0002816	1.10579	0.237098	0.0814173
1971	0.00158701	12.35	30.00	0.0004414	1.09829	0.257459	0.0565682
1972	0.00051684	17.60	30.00	0.0004775	1.09815	0.383097	0.0262963
1973	0.00045146	15.00	25.00	0.0005121	1.09547	0.247501	0.0561742
1974	0.00335902	15.84	19.26	0.0002996	1.10603	0.204722	0.0627329
1975	0.00306729	22.41	24.37	0.0004669	1.09855	0.287408	0.0455199
1976	0.00299408	21.19	23.07	0.0004681	1.09728	0.334773	0.0316403
1977	0.00290691	31.96	25.00	0.0003671	1.10261	0.209073	0.0655924
1978	0.00536496	17.58	25.00	0.0003830	1.10258	0.460491	0.0139946
1979	0.00320323	25.00	22.03	0.0004513	1.09641	0.300000	0.0219853
1980	0.00291010	39.80	20.93	0.0006235	1.08999	0.224315	0.0377160
1981	0.00397193	13.84	17.31	0.0004105	1.09776	0.184527	0.0484409
1982	0.00415947	10.43	27.78	0.0002672	1.10331	0.298204	0.0245919
1983	0.00343585	37.89	22.92	0.0004516	1.09647	0.205344	0.0437258
1984	0.00437364	9.01	23.33	0.0003519	1.10247	0.242828	0.0362564
1985	0.00273603	15.00	24.72	0.0004744	1.09541	0.194828	0.0409357
1986	0.00506102	9.00	25.85	0.0002863	1.10386	0.203689	0.0414648
1987	0.00396781	15.00	25.00	0.0001926	1.11122	0.149648	0.0628520
1988	0.00366709	19.40	25.00	0.0003452	1.10142	0.260460	0.0255078
1989	0.00539264	9.51	25.93	0.0002430	1.10760	0.152406	0.0495842
1990	0.00408624	6.64	26.33	0.0002761	1.10370	0.135153	0.0511492
1991	0.00424174	4.47	28.20	0.0002755	1.10414	0.117785	0.0844790
1992	0.00562780	11.49	27.15	0.0002349	1.10630	0.190000	0.0259271
1993	0.00280457	26.43	15.00	0.0002872	1.10013	0.200000	0.0202152
1994	0.00422457	9.48	27.62	0.0002828	1.10186	0.149845	0.0475090
1995	0.00326122	11.36	26.45	0.0003270	1.09776	0.111801	0.0903355
1996	0.00409300	10.37	27.05	0.0004331	1.09313	0.277542	0.0158911
1997	0.00310379	3.98	25.00	0.0003365	1.09969	0.163931	0.0280024

Table 6.6: Estimated parameters of the new mortality model, India, urban females, 1970-97

Year	Parameters						
	D	E	F	G	H	M	R
1970	0.004314	31.50	20.00	0.000840	1.0799	0.2339	0.0823
1971	0.008618	6.22	29.03	0.000226	1.1038	0.2925	0.0502
1972	0.009581	6.22	29.24	0.000207	1.1097	0.3072	0.0511
1973	0.008449	7.71	26.91	0.000238	1.1035	0.2509	0.0746
1974	0.008778	6.01	29.92	0.000151	1.1100	0.4335	0.0230
1975	0.004799	5.38	25.00	0.000347	1.0993	0.2750	0.0536
1976	0.008047	12.04	26.80	0.000405	1.0946	0.2211	0.0852
1977	0.007705	26.77	20.00	0.000406	1.0958	0.2277	0.0591
1978	0.008886	5.73	27.25	0.000188	1.1085	0.5876	0.0140
1979	0.008057	7.15	27.12	0.000134	1.1122	0.2337	0.0467
1980	0.006463	6.49	27.56	0.000169	1.1063	0.1964	0.0563
1981	0.007060	2.30	25.00	0.000121	1.1130	0.2508	0.0310
1982	0.007629	6.01	27.53	0.000095	1.1156	0.1654	0.0670
1983	0.007794	5.96	25.38	0.000145	1.1098	0.1798	0.0613
1984	0.008730	7.34	24.77	0.000113	1.1159	0.1850	0.0726
1985	0.007380	17.27	23.87	0.000265	1.0994	0.2008	0.0497
1986	0.007695	5.49	26.56	0.000116	1.1135	0.1787	0.0611
1987	0.006803	7.78	26.01	0.000124	1.1121	0.1420	0.0775
1988	0.008243	23.16	24.32	0.000181	1.1064	0.1377	0.0967
1989	0.008151	13.70	24.52	0.000132	1.1093	0.1395	0.0680
1990	0.006477	11.87	24.86	0.000089	1.1174	0.1155	0.0949
1991	0.008151	4.51	29.91	0.000044	1.1289	0.1317	0.0736
1992	0.007819	5.66	27.77	0.000074	1.1200	0.1270	0.0732
1993	0.007817	3.29	26.56	0.000036	1.1256	0.1200	0.0254
1994	0.007200	4.02	28.76	0.000044	1.1259	0.1256	0.0707
1995	0.005009	6.57	25.34	0.000099	1.1131	0.1144	0.0955
1996	0.006584	5.90	26.94	0.000050	1.1235	0.1118	0.0954
1997	0.007245	9.03	25.80	0.000152	1.1060	0.1206	0.0572

Table 6.7: Number of significant ($\alpha < 0.05$) parameters out of estimated seven parameters of the new mortality model, India, males and females, 1970-97

Year	Males			Females		
	India	Rural	Urban	India	Rural	Urban
1970	6	6	5	6	6	5
1971	6	7	5	5	6	5
1972	5	6	5	6	6	5
1973	6	5	6	6	6	5
1974	6	6	5	6	6	3
1975	5	6	6	6	6	6
1976	6	6	6	7	7	6
1977	6	7	6	7	7	5
1978	6	5	4	6	6	4
1979	4	5	7	6	6	4
1980	7	6	4	6	7	5
1981	6	7	6	6	5	4
1982	7	6	5	6	6	7
1983	6	5	5	7	7	7
1984	6	6	5	7	5	7
1985	6	6	7	7	6	7
1986	6	6	6	6	6	7
1987	6	6	7	6	7	7
1988	6	6	4	7	7	7
1989	7	6	7	7	7	7
1990	7	6	7	7	7	7
1991	7	7	6	7	6	6
1992	6	6	6	6	7	6
1993	6	6	5	7	7	5
1994	6	6	5	6	7	6
1995	6	5	6	5	6	7
1996	6	7	5	7	7	5
1997	7	6	4	7	7	7

Chapter 7: Forecasting the Parameters of Death for India

My specific goal is to revolutionize the future of the species. Mathematics is just another way of predicting the future.

Ralph Abraham

7.1 Introduction

The expectation of life at birth in India in 1993-97 stood at 61.8 years for females and 60.4 years for males (India ORG, 2000b). According to the UN's medium variant projections, these estimates are likely to increase to 72.1 years for females and 68.8 years for males by the period 2020-25 (United Nations, 1999: 228). Despite rapid expansion in infant and child immunisation coverage, the infant mortality rate remained at 71 per thousand live births in India in 1997 (India ORG, 1998a). Past improvements in life expectancy among women may be linked to the remarkable decline in maternal deaths that in itself has been associated with the decline in fertility and the successful reduction in the frequency of pregnancy and childbirth (McNay, 2000). Indirect estimates show that the maternal mortality ratio (MMR) declined from 892 during the period 1972-76 to 568 in 1982-86 (Bhat, Navaneetham & Rajan, 1995). Recent estimates for the country put MMR at 540 in 1991-92 (IIPS, 1995) and 424 in 1997-98 (IIPS, 2000).

As in most other developing countries, mortality forecasts in India are generally limited to extrapolation of expectation of life at birth. However, for purposes of health planning and monitoring the achievement in human development, it may be necessary to go beyond such measures and investigate changes in trends and patterns in age-specific mortality. From a methodological point of view, mortality forecasting can be done in two ways, either by forecasting the designated mortality measure directly or by forecasting the estimated parameters representing the mortality pattern in the population in question. This chapter presents the application of the latter approach to Indian mortality. This approach is applied using two models, the new mortality model and the Lee-Carter model. Specifically the estimated parameters of the two models are forecasted to 2020.

In the case of the new model, the approach followed was that of forecasting each of the parameters of the model separately using the univariate ARIMA procedure, and then combining the forecast parameters to produce probabilities of deaths and the expectation of life at birth. Forfar & Smith (1987) and McNown & Rogers (1989a, b, 1992) used a similar approach to forecast the parameters of the Heligman-Pollard model for England and the USA, respectively. To some minute degree, the parameters exhibit dependence but by choice of the right models, this dependence is subdued for all practical reasons. As is demonstrated by Forfar & Smith (1987) and McNown & Rogers (1989a, b, 1992), and the results presented in the sections to follow in this chapter, ensuring the internal consistency of each of the univariate models at the selection stage has the tendency to overcome this problem. Of course, the other possibility is to undertake simultaneous forecasting of the model parameters using multivariate ARIMA technique, but in a situation such as the present one, where the exact relationship between the parameters is not well understood, the use of multivariate ARIMA procedure may introduce even more statistical problems than the one its application tries to solve. Also the independence of parameters is demonstrated by their movements in different directions with time. For the Lee-Carter model, the parameter $\mathbf{k_t}$ is forecasted and applied in the model using the $\mathbf{a_x}$ and $\mathbf{b_x}$ values obtained for the fitting period. Saboia's (1974) paper represents one of the earliest works on the application of ARIMA models in demographic forecasting.

7.2 Time series analysis of parameters: a description of the method

The appropriate procedure for forecasting the parameters of the two models is an auto-regressive integrated moving average (ARIMA) time series model (Box, Jenkins & Reinsel, 1994; Hamilton, 1994; Enders, 1995). In univariate ARIMA models, a time series is modelled only in terms of its own past values and a disturbance (Johnston & DiNardo, 1997). For a variable, say X, such a model can be expressed as:

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-3}, \ldots, U_t) \ldots\ldots\ldots\ldots\ldots\ldots (7.1)$$

where X_t is the value of any time series process at time t and U_t is a component responsible for unwarranted changes in the process. U_t is termed error and is also called 'white noise'.

To make equation 7.1 operational, three specifications are needed: the reduced form of the equation, the desired number of lags and some knowledge about the structure of the error term, U_t. For instance, if X is assumed to be an autoregressive time series process of order p, equation 7.1 can be written as:

$$X_t = \alpha_0 + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots\dots\dots + \alpha_p X_{t-p} + U_t \dots\dots\dots\dots\dots\dots (7.2)$$

where the αs are coefficients of autoregressive components. This process is called an AR process and is expressed as AR(p). If the structure of the error term U_t, is defined in terms of moving averages then

$$U_t = \beta_0 + e_t + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots\dots\dots + \beta_q e_{t-q} \dots\dots\dots\dots (7.3)$$

where e_ts are the error components at time point t. Equation 7.3 is called a model of moving average (MA) of order q and written as MA(q). The βs are the coefficients of the moving average components. By combining equations 7.2 and 7.3, the following is obtained:

$$X_t = \alpha_0 + \alpha_1 X_{t-1} + \dots\dots + \alpha_p X_{t-p} + e_t + \beta_1 e_{t-1} + \dots\dots + \beta_q e_{t-q} \dots\dots\dots\dots\dots (7.4)$$

Equation 7.4 may be rewritten as

$$X_t - \alpha_1 X_{t-1} - \dots\dots - \alpha_p X_{t-p} = \alpha_0 + e_t + \beta_1 e_{t-1} + \dots\dots + \beta_q e_{t-q} \dots\dots\dots\dots\dots (7.5)$$

Equation 7.4 is called an ARMA process with parameters p and q, where p specifies the autoregressive component and q the moving average component and the model (process) is denoted as ARMA (p, q).

To fit an ARMA model, it is necessary to achieve stationarity in the series. The stationarity assumption ensures that the process reverts to its long-term mean value: the values of the series need to be distributed around its long-term mean. If the mean and autocovariance of a series do not depend on time, the series is considered to be stationary (Hamilton, 1994: 45). However, if a process is non-stationary then it might attain infinite values in some cases or may explode when forecasted. Stationarity in a series can be achieved by differencing the original data. In practice, first or second-order differences of the given data are usually sufficient to achieve stationarity (Farnum & Stanton, 1989: 469; Box *et al.*, 1994: 185). The order of differencing used to achieve stationarity is denoted by the value, d, for a time series model. When the value of d is zero it means the original data is stationary and used as it is the analysis. When the stationarity condition holds true for equation 7.4, the process is called an Auto Regressive Integrated Moving Average, ARIMA (p, d, q), where p and q are defined as before and d is the order of difference needed to make the series stationary.

If the operator , L, is defined as the lag such that:

$$L X_t = X_{t-1}$$

$$L^2 X_t = X_{t-2}$$

in general, $$L^p X_t = X_{t-p}$$

and another operator used is the backward difference operator, ∇, which is defined as:

$$\nabla X_t = X_t - X_{t-1}$$

in general, $$\nabla^d X_t = \nabla \{\nabla^{d-1} X_t\}$$

then equation 7.5 can be converted into an ARIMA model of the following form:

$$\nabla^d X_t - \alpha_1 L \nabla^d X_t \ ...... - \alpha_p L^p \nabla^d X_t = \alpha_0 + e_t + \beta_1 e_{t-1} + ... + \beta_q e_{t-q} \qquad (7.6)$$

or

$$(1 - \alpha_1 L ..... - \alpha_p L^p)\ \nabla^d X_t = \alpha_0 + e_t + \beta_1 e_{t-1} + ... + \beta_q e_{t-q} \qquad (7.7)$$

The following steps were taken in the process of fitting the ARIMA models:

1. Test the data for stationarity using random walk with drift (r-w) tests. In cases where the original data were non-stationary, stationarity was achieved in most cases after using the first order difference.

2. Identify p (autoregressive order) and q (moving average order). For this purpose, a test called 'extended sample autocorrelation function' (ESACF) was employed (Tsay & Tiao, 1984; Wei, 1990: 126), which suggests possible combinations of p and q. Each of the determined models was then estimated from the data.

3. Finally, the model with the lowest 'Akaike information criteria' (AIC) statistics (Box *et al.*, 1994: 200) along with its forecasting capacity (minimum standard error) was chosen from all the fitted models.

The following is a brief example of an ARIMA model, taking X to be for one of the estimated parameters in Chapter 6 (parameter F for rural females). Using the steps set out above, the estimates of the coefficients of the autoregressive and moving average orders provide the movement of the various parameters over time. L represents the order of the lag operator for the autoregressive process. Here e_t are the measures of white noise in the models. Figures provided in parentheses below the estimated ARIMA coefficients are their respective standard errors. The ARIMA model chosen for the series was of order (2,1,2), which together with its estimated coefficients is written as follows:

$$\underset{(0.60)}{(1+0.56L} - \underset{(0.29)}{0.26L^2)}\ \nabla F(t) = e_t - \underset{(0.60)}{0.29e_{t-1}} - \underset{(0.49)}{0.62e_{t-2}} \qquad \text{.......(7.8)}$$

Model 7.8 has two AR coefficients in the left-hand side and two MA coefficients in the right hand side. In this model, none of the coefficients is statistically significant. The value of d in the model is one as the order of ∇ is unity.

The standard errors of parameters obtained from the ARIMA fit are used to calculate confidence intervals for the forecasted parameters; in this analysis 95 per cent confidence intervals are used. The ARIMA method of forecasting was applied to the parameters of both the new model and the Lee-Carter model.

Once an ARIMA (p, d, q) model that satisfied the above conditions was chosen for each series (parameters R, M, D, E, F, G & H), forecasts (Enders, 1995) were generated for 1998 to 2020.

7.3 Fitted ARIMA models for the parameters of the new mortality model

Table 7.1 presents the ARIMA models estimated for each of the seven parameters from the time-series data covering the period 1970-97 for males in India. For all the seven parameters under investigation, the time series were found to be non-stationary; stationarity was achieved in each case only after taking the first difference of the series. Each model, except that for parameter M, provides one or more statistically significant AR coefficient (at the 5 per cent level). For four of the seven parameters, the models converged when the moving average of the model was of order zero (i.e. q=0). MA coefficients of non-zero order were estimated for three models, those for E, M and R.. For the models for parameters E and M, the coefficients of the error terms were not statistically significant, while for R they were found to be significant, which suggests that R incorporates some random fluctuations. For every parameter, three to five suggested models were tested and the final model was selected based on the AIC test and its ability to forecast the future. In the case of parameter M, random fluctuations necessitated that the parameter be transformed to achieve stabilisation and the logistic transformation was found suitable.

This transformation provided an adequate model as presented in Table 7.1. The AR and MA coefficients are significant at the 10 per cent level but its capacity to forecast was the main basis for its selection.

Table 7.1 ARIMA models for parameters of the new mortality model, India, males, 1970-1997

Parameters	ARIMA Models
D	$\left(1+0.36L+0.63L^2+0.42L^3+0.38L^4+0.16L^5\right)\nabla D(t)=e_t$ (0.22) (0.22) (0.25) (0.25) (0.26)
E	$(1-0.96L)\,\nabla E(t)=e_t+0.05e_{t-1}$ (0.12) (0.23)
F	$\left(1+0.51L+0.58L^2+0.47L^3+0.35L^4+\right)\nabla F(t)=e_t$ (0.20) (0.22) (0.22) (0.22)
G	$\left(1+0.41L+0.33L^2+0.38L^3\right)\nabla G(t)=e_t$ (0.20) (0.23) (0.22)
H	$\left(1+0.35L+0.22L^2\right)\nabla H(t)=e_t$ (0.20) (0.22)
M (Logistic)	$(1+0.54L)\,\nabla M(t)=e_t-0.08e_{t-1}$ (0.29) (0.35)
R	$(1-0.06L)\,\nabla R(t)=e_t-1.00e_{t-1}$ (0.14) (0.23)

Note: Figures in parentheses are the standard errors of the coefficients.

In most cases, the coefficients of the higher-order AR process were not statistically significant; nevertheless they were retained in the models as they improved the forecasts. Parameter E provides the best fit as it clearly exhibited a first-order AR component responsible for the variation in the long run. In all cases, non-negative forecasts were obtained. It was noted that models with a higher-order AR process generally provided minimum standard errors for the forecasts (see Appendix 7A). Stable forecasts for males were obtained.

Figure 7.1 provides the ARIMA forecast for each of the seven parameters for the male population of India. Parameters D, G and H have relatively wide confidence intervals owing to more pronounced fluctuations in the estimated values of parameters for the period 1970-97. The parameters D and G represent the base mortality for the middle and last phases of the curve suggesting that a fluctuating decline in mortality for the period of observation was prevalent; this may be responsible for the annual fluctuations in forecasts.

Table 7.2 provides results of the ARIMA models for females for the period 1970-1997. For each parameter the first difference of the time series was sufficient to achieve stationarity. For Parameter F, all of the AR and MA coefficients were statistically non-significant but smaller standard errors of forecast, the basis for better forecasts, prompted their inclusion. For parameters D, E and M, all the coefficients were statistically significant. Parameters G, H and R exhibit some higher-order AR coefficients that are statistically non-significant, but these coefficients were helpful in improving the efficiency of forecasts (see Appendix 7B). The forecasts presented non-negative values of the parameters for the future.

Figure 7.2 provides the ARIMA forecast for the seven parameters for India's female population. Parameters E, G and H show wide confidence intervals. The estimated E model has more random fluctuations, while G and H have a few outlying points that contribute to the larger standard errors. Parameters D and M have very efficient estimation and forecasting, as their standard errors are remarkably low.

Table 7.2 ARIMA models for parameters of the new mortality model, India, females, 1970-1997

Parameters	ARIMA Models
D	$\nabla D(t) = e_t - \underset{(0.13)}{0.83} e_{t-1}$
E	$(1 + \underset{(0.13)}{0.41} L)\ \nabla E(t) = e_t - \underset{(0.25)}{0.50} e_{t-1}$
F	$(1 - \underset{(3.78)}{0.20} L - \underset{(3.78)}{0.39} L^2 - \underset{(1.59)}{0.11} L^3 + \underset{(1.21)}{0.06} L^4 - \underset{(1.44)}{0.026} L^5 - \underset{(0.50)}{0.09} L^6)\ \nabla F(t) = e_t - \underset{(3.85)}{1.00} e_{t-1}$
G	$(1 + \underset{(0.21)}{0.78} L + \underset{(0.26)}{0.58} L^2 + \underset{(0.26)}{0.30} L^3 + \underset{(0.21)}{0.24} L^4)\ \nabla G(t) = e_t$
H	$(1 + \underset{(0.23)}{0.88} L + \underset{(0.32)}{0.70} L^2 + \underset{(0.35)}{0.47} L^3 + \underset{(0.35)}{0.39} L^4 + \underset{(0.32)}{0.10} L^5 + \underset{(0.24)}{0.17} L^6) \nabla H(t) = e_t$
M	$\nabla M(t) = e_t - \underset{(0.18)}{0.46} e_{t-1}$
R	$\left(1 + \underset{(0.22)}{0.68} L + \underset{(0.25)}{0.69} L^2 + \underset{(0.26)}{0.33} L^3 + \underset{(0.23)}{0.01} L^4 + \right) \nabla R(t) = e_t$

Note: Figures in parentheses are the standard errors of the coefficients.

Table 7.3 provides the ARIMA models for rural Indian males. Stationarity was achieved in each case for the first difference. Almost all AR coefficients for the parameters F and H were statistically significant despite the fact that models with higher orders of p were fitted, suggesting that the parameters were related to their lags of as high as the fourth or fifth order. Parameter G converged for the MA process of second order, with a statistically significant coefficient for the first MA process. This means that random fluctuations were responsible for the changes in the values. For parameter M, a logistic transformation facilitated convergence of ARIMA (1,1,1) with the AR coefficient being highly statistically significant. For the selected models, each of the seven parameters had at least one statistically significant coefficient. The forecasts obtained were stable (see Appendix 7C).

Figure 7.3 shows the estimated and forecasted parameters for the rural male population of India. Parameters D, F, H and R show slightly increasing trends while E, G and M show declining trends. Parameters D and R show wider confidence intervals because of year-to-year fluctuations. Parameters F, G and M were estimated with narrow confidence intervals.

Table 7.3 ARIMA models for parameters of the new mortality model, rural males, India, 1970-1997

Parameters	ARIMA Models
D	$(1-0.22L)\ \nabla D(t) = e_t - 1.00e_{t-1}$ $\quad(0.25)\qquad\qquad(0.17)$
E	$(1+0.21L)\ \nabla E(t) = e_t - 0.78e_{t-1}$ $\quad(0.24)\qquad\qquad(0.16)$
F	$(1+0.97L+0.77L^2+0.81L^3+0.77L^4+0.31L^5)\ \nabla F(t) = e_t$ $\quad(0.21)\quad(0.26)\quad(0.27)\quad(0.29)\quad(0.23)$
G	$\nabla G(t) = e_t - 1.04e_{t-1} + 0.04e_{t-2}$ $\qquad(0.22)\qquad(0.25)$
H	$(1+0.88L+0.56L^2+0.77L^3+0.56L^4)\ \nabla H(t) = e_t$ $\quad(0.20)\quad(0.27)\quad(0.28)\quad(0.22)$
M (Logistic)	$(1+0.65L)\nabla R(t) = e_t - 0.10e_{t-1}$ $\quad(0.23)\qquad\qquad(0.31)$
R	$(1+0.44L)\nabla R(t) = e_t - 0.65e_{t-1}$ $\quad(0.23)\qquad\qquad(0.21)$

Note: Figures in parentheses are the standard errors of the coefficients.

Table 7.4 provides the ARIMA models fitted to the parameters for rural Indian females. Stationarity was reached for each parameter after the first differences were taken. Parameter G required a logistic transformation after which all the coefficients were statistically significant at the 5 per cent level. Parameter H had the first AR coefficient significant at the 5 per cent level and the second and third at 10 per cent. Although none of the coefficients was statistically significant, the model for parameter F was chosen because it had the narrowest 95 per cent confidence interval compared to all available options. Parameter D was modelled with the first AR coefficient significant at 5 per cent and the second at 10 per cent; suggesting that the prevailing time trends explained the variations. In most of the models except for G, white noise was non-existent or not found to be statistically significant. Parameter R exhibited a sixth-order MA process with first coefficient significant at the 5 per cent level and four out of five of the higher-order coefficients significant at the 10 per cent level. Parameter M had first order AR and MA coefficients estimated to explain the variation. The AR coefficient of M is highly significant. The statistical details of the forecasted values and standard errors are given in the appendix (see Appendix 7D).

Figure 7.4 gives the past and forecasted course of the seven parameters for rural females in India. Parameters D, F and H show declining trends over time. Parameters E and R show slightly increasing trends, while parameters M and G are forecasted to remain constant in the future. Parameters E, G, H and M are forecasted with wide confidence intervals because of prevalent random fluctuations. Parameters D, F and R had narrow confidence intervals.

Table 7.4 ARIMA models for parameters of the new mortality model, rural females, India, 1970-1997

Parameters	ARIMA Models
D	$\underset{(0.20)}{(1+0.52L}+\underset{(0.19)}{0.23L^2})\ \nabla D(t)=e_t$
E	$\underset{(0.20)}{(1+0.82L}+\underset{(0.25)}{0.46L^2}+\underset{(0.21)}{0.33L^3})\ \nabla E(t)=e_t$
F	$\underset{(0.60)}{(1+0.56L}-\underset{(0.29)}{0.26L^2})\ \nabla F(t)=e_t-\underset{(0.60)}{0.29e_{t-1}}-\underset{(0.49)}{0.62e_{t-2}}$
G (Logistic)	$\underset{(0.001)}{(1-1.00L)}\ G(t)=e_t-\underset{(0.248)}{0.99e_{t-1}}$
H	$\underset{(0.20)}{(1+0.80L}+\underset{(0.24)}{0.43L^2}+\underset{(0.21)}{0.37L^3})\ \nabla H(t)=e_t$
M	$\underset{(0.04)}{(1-1.00L)}\ \nabla M(t)=e_t-\underset{(0.23)}{0.21e_{t-1}}$
R	$\underset{(0.22)}{(1+0.67L}+\underset{(0.27)}{0.48L^2}+\underset{(0.29)}{0.40L^3}+\underset{(0.33)}{0.12L^4}+\underset{(0.30)}{0.36L^5}+\underset{(0.27)}{0.38L^6})\nabla R(t)=e_t$

Note: Figures in parentheses are the standard errors of the coefficients.

Table 7.5 provides the ARIMA models for urban Indian males. In all the series, stationarity was achieved after taking the first-order difference. The models for parameters G and H had at least two coefficients statistically significant at the 10 per cent level despite high AR and MA orders. All parameters were modelled with at least one statistically significant AR coefficient. For parameters D and E, the first two AR coefficients were statistically significant. For parameter R, only the second MA coefficient was significant. The first MA coefficient of R was not statistically significant. For the standard errors, see Appendix 7E.

Figure 7.5 provides the trends in parameters for urban males in India. Parameters D, E and G exhibit a slightly increasing trend while F, H, M and R provide slightly decreasing trends. Wider confidence intervals are observed for the urban data set compared to others.

Table 7.5 ARIMA models for parameters of the new mortality models, urban males, India, 1970-1997

Parameters	ARIMA Models
D	$\underset{}{(1+\underset{(0.20)}{0.70}L+\underset{(0.24)}{0.40}L^2+\underset{(0.21)}{0.14}L^3)\,\nabla D(t)=e_t}$
E	$(1+\underset{(0.22)}{0.77}L+\underset{(0.29)}{0.72}L^2+\underset{(0.33)}{0.16}L^3+\underset{(0.30)}{0.13}L^4-\underset{(0.24)}{0.15}L^5)\,\nabla E(t)=e_t$
F	$(1-\underset{(0.28)}{0.04}L)\nabla F(t)=e_t-\underset{(0.19)}{0.76}e_{t-1}$
G (Logistic)	$(1+\underset{(0.35)}{0.12}L+\underset{(0.26)}{0.28}L^2+)\nabla G(t)=e_t-\underset{(0.34)}{0.58}e_{t-1}$
H	$(1+\underset{(0.59)}{0.63}L+\underset{(0.44)}{0.47}L^2+\underset{(0.42)}{0.12}L^3+\underset{(0.31)}{0.07}L^4-\underset{(0.22)}{0.35}L^5)\,\nabla H(t)=e_t+\underset{(0.63)}{0.11}e_{t-1}$
M	$(1+\underset{(0.20)}{0.80}L+\underset{(0.27)}{0.81}L^2+\underset{(0.29)}{0.63}L^3+\underset{(0.27)}{0.25}L^4+\underset{(0.21)}{0.45}L^5)\,\nabla M(t)=e_t$
R	$(1+\underset{(0.29)}{0.26}L+\underset{(0.27)}{0.43}L^2)\,\nabla R(t)=e_t-\underset{(0.28)}{0.53}e_{t-1}$

Note: Figures in parentheses are the standard errors of the coefficients.

Table 7.6 provides the ARIMA models for urban Indian females. In these series, parameter E was already stationary, while the rest of the six parameters attained stationarity after the first difference. Parameter E had an 'average model' ARIMA (0,0,1) providing the best forecast. For parameter G, logistic transformation was needed to provide stable forecasts.

To avoid negative forecasts the outlier value corresponding to 1970 was excluded from the analysis. By excluding the outlier and using logistic transformation, convergence with non-negative forecasts was obtained. Parameter F had no statistically significant coefficients despite high-order AR terms being included in the model. The model for parameter D had only the third AR coefficient significant at the 10 per cent level. For parameter R, all coefficients were statistically significant, suggesting that first, second and third orders of the AR process were important in explaining variation over time. For the parameter M both AR and first MA coefficients were statistically significant. For the details see Appendix 7F.

Table 7.6 ARIMA models for parameters of the new mortality models, urban females, India, 1970-1997

Parameters	ARIMA Models
D	$(1+0.53L+0.44L^2+0.47L^3+0.53L^4-0.09L^5)\ \nabla D(t) =$ (0.92) (0.67) (0.33) (0.47) (0.62) $e_t+0.21e_{t-1}+0.36e_{t-2}$ (0.89) (0.44)
E	$E(t)=e_t+0.10e_{t-1}$ (0.20)
F	$(1+0.43L+0.13L^2+0.29L^3+0.13L^4+0.23L^5+0.36L^6)\ \nabla F(t) =$ (0.64) (0.36) (0.25) (0.27) (0.25) (0.24) $e_t-0.08e_{t-1}$ (0.68)
G (Logistic)	$(1-0.14L)\ \nabla G(t)=e_t+0.06e_{t-1}+0.61e_{t-2}$ (0.41) (0.38) (0.27)
H	$\nabla H(t)=e_t$
M	$(1-0.53L+0.30L^2)\ \nabla M(t)=e_t-1.40e_{t-1}+0.75e_{t-2}$ (0.33) (0.27) (0.28) (0.22)
R	$(1+0.64L+0.62L^2+0.38L^3)\ \nabla R(t)=e_t$ (0.20) (0.21) (0.20)

Figure 7.6 provides the estimated and forecasted trends in parameters for urban females in India. Parameters D, F, H and R show an increasing trend over time. Parameters G and M show slight declines over time. Parameter E is forecasted to remain constant about the value 10. Wide confidence intervals were noted because of large year-to-year fluctuations in

the estimated values of all seven parameters. Compared to all-India and rural data, urban females have wider confidence intervals. Parameter G was highly varying in the original series and logistic transformation brought the stability. Despite the uneven confidence intervals it was included for the analysis as it facilitated the mortality forecasts within the permissible range.

Some parameters tend to fluctuate when forecasted because higher-order ARIMA models are greatly affected by the year-to-year fluctuations, and incorporate them in the forecast. Apart from this, high variation in a parameter also brings the wider fluctuations.

7.4 Forecasts of q_x by the new mortality model

The 1998 to 2020 forecasted parameters have been used to estimate the $\mathbf{q}_x$ values. Figure 7.7 provides estimated and forecasted $\mathbf{q}_x$ for all the six series; obvious declines in mortality in each case have been confirmed by the curves shifting downwards. Figure 7.8 and Table 7.7 provide the forecasted life expectancies for six variants among Indian males and females. Some fluctuations in the expectation of life at birth are evident but their overall trends remain in order. The year-to-year fluctuations are due to fluctuations mainly in the parameters G and H. As G is a parameter for base mortality in older ages but the higher-order ARIMA for the parameter carries the past year-to-year fluctuations to the future. Also the year-to-year fluctuations are caused by the independent estimation of the parameters.

According to the forecasts, the expectation of life at birth in India will reach 78.6 years for males and 79.2 years for females by the year 2020. The average gains in expectation of life at birth were 0.43 and 0.58 years per annum respectively for the period, 1971-1997. The gains were also higher for females than males in rural and urban areas. The gains are forecasted to be 0.66 and 0.69 years per year for males and females for the period 1998-2020. For rural males and females life expectancies are forecast to reach 74.6 and 77.0 years respectively by the year 2020. Urban females are forecast to reach 83.4 years by 2020 compared with 82.3 years for urban males. Rural males have the lowest and urban females the highest of the life expectancies during the period of forecast.

Table 7.7: Forecasted expectation of life at birth by the new mortality model, India, 1998-2020 (in years)

Year	Males			Females		
	All India	Rural India	Urban India	All India	Rural India	Urban India
1998	62.80	60.87	69.71	62.70	60.81	71.23
1999	66.23	57.58	67.74	63.10	62.82	71.78
2000	64.14	58.82	68.29	63.81	68.70	72.33
2001	63.06	63.72	73.59	65.66	65.25	72.89
2002	64.44	65.99	66.82	64.50	69.48	73.44
2003	66.17	63.96	71.43	66.48	66.12	73.99
2004	66.51	61.59	72.10	67.59	69.66	74.54
2005	66.32	62.26	72.49	67.06	67.91	75.09
2006	66.71	65.47	72.52	68.61	70.09	75.64
2007	67.76	67.66	72.67	68.52	68.77	76.19
2008	68.66	67.21	75.01	69.96	70.94	76.74
2009	69.14	65.70	74.80	70.57	70.07	77.29
2010	69.55	66.23	74.44	70.90	71.50	77.84
2011	70.26	68.49	77.48	72.26	71.25	78.39
2012	71.13	70.72	75.58	72.45	72.67	78.94
2013	71.95	71.15	78.12	73.60	72.40	79.49
2014	72.69	70.63	78.31	74.24	73.47	80.04
2015	73.48	71.28	78.63	74.95	73.72	80.59
2016	74.37	72.29	79.80	76.00	74.67	81.14
2017	75.35	72.87	80.27	76.51	74.82	81.69
2018	76.36	73.45	81.41	77.59	75.76	82.24
2019	77.43	74.03	81.93	78.28	76.25	82.79
2020	78.59	74.61	82.27	79.15	77.01	83.35

7.5 Fitted ARIMA models for the mortality index, k_t of the Lee-Carter model

In Chapter 4, the Lee-Carter model was fitted to Indian data by SVD to obtain estimates of the model parameters. This was done for males and females for data for 1970-1997. In order to forecast mortality, it is necessary to forecast only the parameter k_t since a_x and b_x are assumed to remain constant.

In application to data from developed countries, k_t has been shown to be roughly linear and ARIMA (0,1,0) models were used (found suitable) in forecasting (Carter & Lee, 1992; Lee & Carter, 1992; Lee, 2000; Tuljapurkar *et al.*, 2000). The forecasting of k_t for India data was carried out using the ARIMA method described in section 7.2. The orders of the ARIMA models for Indian applications did not, however, follow the (0,1,0) pattern. Without exception, higher order ARIMA models were found to fit best. For purposes of comparison, results obtained from the ARIMA (0,1,0) models are presented along with the optimal models.

Before fitting the ARIMA models, Lee & Carter (1992) adjusted $\mathbf{k}_t$ to remove discrepancies in the total number of deaths created by the log-normal estimation process. Later Lee & Miller (2001) recommended using adjustment of the 'jump off' point for the forecast period. In the present situation, where higher-order ARIMA models have been used for all six variants for India, the fitted k_t are relatively close to the observed values and adjustments are not likely to improve much.

It has been noted that $\mathbf{k}_t.\mathbf{b}_x$ captures about 95 per cent of the variations over time in the Indian case as well as in the case for the developed countries (Lee & Carter, 1992; Lee, 2000: 81; Tuljapurkar *et al.*, 2000: 789; Booth *et al.*, 2001: 4).

7.5.1 Estimated ARIMA models for Indian k_t series

Table 7.8 provides the orders of the ARIMA models of the estimated $\mathbf{k}_t$ for Indian data. Table 7.9 provides details of the ARIMA models for k_t estimated from Indian data. Notably the zero order of 'q' in five cases and the first order in one case was found optimal. This means that structural errors (moving average) were not present in explaining the time series variations in five out of six cases. For rural females, the first order MA coefficient was included in the model but it was not found to be statistically significant. This means that the error component did not explain the variation and all the variation was explained by the autoregressive components.

Table 7.8: Estimated ARIMA model of parameter $\mathbf{k}_t$ for India, males and females, 1970-1997

Series	Models	Series	Models
Males	2,1,0	Females	1,1,0
Rural males	2,1,0	Rural females	3,1,1
Urban males	4,1,0	Urban females	3,1,0

Table 7.9 ARIMA models for k_t by sex and urban-rural area, India, 1970-1997

Parameters	ARIMA Models
Males	$(1+0.37L+0.46L^2)\ \nabla K(t) = e_t$ $\quad (0.19) \quad (0.19)$
Females	$(1+0.34L)\ \nabla K(t) = e_t$ $\quad (0.19)$
Rural males	$(1+0.33L+0.40L^2)\ \nabla K(t) = e_t$ $\quad (0.19) \quad (0.19)$
Rural females	$(1-0.21L-0.73L^2+0.04L^3+)\ \nabla K(t) = e_t - 1.00e_{t-1}$ $\quad (1.00) \quad (0.82) \quad (0.26) \qquad (1.05)$
Urban males	$(1+0.50L+0.82L^2+0.22L^3+0.45L^4)\ \nabla K(t) = e_t$ $\quad (0.20) \quad (0.23) \quad (0.24) \quad (0.27)$
Urban females	$(1+0.61L+0.50L^2+0.12L^3)\ \nabla K(t) = e_t$ $\quad (0.21) \quad (0.22) \quad (0.21)$

In four out of six cases, at least one of the AR coefficients was found to be statistically significant (α=0.05); for total females and rural females, none of the AR coefficients was statistically significant. For total males, both of the AR coefficients were significant at the 5 per cent level. For total females, the AR coefficient was only significant at the higher level, 10 per cent. For urban males, the first two AR coefficients were significant at the 5 per cent and the fourth at the 10 per cent level. For urban females, the first two AR coefficients were significant at the 5 per cent level.

Figures 7.9 and 7.10 show estimated and forecast k_t with confidence intervals for the selected ARIMA process and the ARIMA (0,1,0) for males and females respectively. In choosing the optimal model over the (0,1,1) model, the standard error of the forecast was reduced by one-third to one-half in the three male cases. A remarkable reduction in the standard error was achieved for urban males. This improvement in forecasting k_t will provide better forecast mortality rates for Indian males. For females, the k_t were also forecast with narrower 95 per cent confidence intervals for the optimal model compared to the intervals for ARIMA (0,1,0). The reductions in forecast errors for urban and rural females were remarkable. The improved forecasts will yield age specific death rates for Indian females with narrower confidence intervals.

7.6 Estimation of life expectancy by the Lee-Carter method

By using the $\mathbf{k}_t$, $\mathbf{a}_x$ and $\mathbf{b}_x$ values, the ASDR matrix was constructed for the period 1998 to 2020 for India. Each forecasting period has lower and upper control limits around the central value of each ASDR. The estimating equation can be written as:

$$\mathrm{m}_{x,t} = e^{(a_x + b_x k_t)} \qquad \text{................(7.9)}$$

To provide easily understandable results, life table techniques (Chiang, 1984; Namboodiri & Suchindran, 1987; Pathak & Ram, 1992) have been used to calculate the expectation of life at birth by using estimates of $\mathbf{m}_{x,t}$ for different ages and time periods. Expectation of life can be used to compare mortality levels by means of a synthetic index.

By using the $\mathbf{k}_t$, $\mathbf{a}_x$ and $\mathbf{b}_x$ values estimated and forecasted previously, mortality matrices for six variants for Indian males and females were estimated for the period 1971-2020. Table 7.10 and Figure 7.11 provide the expectation of life at birth for males and females in all India and rural and urban areas. Estimates of the life expectancy for the ARIMA (0,1,0) were about a year lower than those for the optimal model in every case under estimation. The substantial reductions in the 95 per cent confidence intervals were observed due to choosing the appropriate ARIMA models. In ARIMA (0,1,0), only the first differencing of data is used and this more often provides stationarity of the series. The concept of ARIMA differs from the linear regression where independence of events is a prerequisite while, in ARIMA, dependence is ensured by the orders of AR (p) and MV (q). The ARIMA (0,1,0) provides no dependence of values of series in either autoregressive factors (trends) or moving averages (white noise) making it redundant except that it provides standard errors of estimates. While the difference in the estimated and forecasted values between the chosen models and the (0,1,0) models was one year, the remarkable improvement in the standard errors of the chosen models suggests that the rigorous diagnosis and selection of appropriate models is more statistically authentic.

Figure 7.12 presents all the six forecasted series of life expectancies together; they exhibit a linear path in all cases. A clear problem with the forecast is evident from a comparison of the rural-urban differences by sex: for females the original difference of about seven years between rural and urban areas is largely maintained over the years of the forecast. For males, however, the difference reduces substantially and unreasonably, essentially because of relative lack of improvement for urban males.

Table 7.10: Forecast expectation of life at birth by Lee-Carter method, India, 1998-2020 (in years)

Year	Males			Females		
	All India	Rural India	Urban India	All India	Rural India	Urban India
1998	62.41	60.54	66.64	63.69	62.62	69.54
1999	62.55	61.46	66.74	64.04	62.47	70.12
2000	62.81	61.62	65.55	64.54	63.37	70.26
2001	63.34	61.90	66.77	64.98	63.45	70.66
2002	63.72	62.44	67.17	65.44	64.16	71.16
2003	64.02	62.84	67.00	65.89	64.39	71.52
2004	64.42	63.18	67.79	66.34	64.99	71.88
2005	64.80	63.58	67.69	66.78	65.30	72.29
2006	65.15	63.99	67.72	67.23	65.84	72.69
2007	65.51	64.36	68.44	67.67	66.20	73.06
2008	65.88	64.74	68.48	68.11	66.70	73.45
2009	66.23	65.13	68.66	68.55	67.09	73.84
2010	66.58	65.50	69.11	68.99	67.57	74.23
2011	66.93	65.88	69.14	69.43	67.98	74.61
2012	67.28	66.25	69.43	69.86	68.44	75.00
2013	67.63	66.62	69.80	70.29	68.86	75.38
2014	67.97	66.99	69.89	70.73	69.31	75.77
2015	68.32	67.36	70.18	71.16	69.74	76.15
2016	68.66	67.72	70.45	71.59	70.19	76.54
2017	69.00	68.09	70.60	72.02	70.62	76.92
2018	69.34	68.45	70.90	72.45	71.06	77.31
2019	69.67	68.81	71.13	72.87	71.49	77.69
2020	69.67	69.17	71.31	73.30	71.94	78.07

7.7 Comparisons of other forecasted results for India

This section provides comparison of forecasted mortality from the new mortality model, Lee-Carter model, and with the previous forecasts by the 'technical group on population projections' and the United Nations.

7.7.1 Comparison of the new mortality model with the Lee-Carter model

Figure 7.13 provides the difference between the life expectancies for the six cases of Indian mortality by the two methods. The gap is positive on the side of the new mortality model with few exceptions, and the gap increases with time; the difference reaches above five years in each case by 2020. The Lee-Carter model forecasts exhibit linear trends as they depend on only one parameter; on the other hand, the new mortality model moves with the inherent fluctuations from year-to-year. The new mortality model considers all possible variations in future mortality through its parameters because it incorporates the age pattern of mortality more effectively. The Lee-Carter method, in contrast, defines all the improvements in mortality as due to movements in the mortality index only, which to a degree incorporates relative age-wise improvements indirectly.

7.7.2 Comparison of the mortality forecasts by previous sources to the new mortality model and Lee-Carter model

Previous mortality projections for India have been made in the context of population projections, which are based mainly on assumptions that mortality will not rise beyond a specific upper limit. The technical group on population projections (India ORG, 1996) made the assumption that the upper limit of the expectation of life at birth was 75 years for males and 80 years for females. The model used to project life expectancy was for males

$$\ln(e_0 - 75) = a + b(\text{time}) \quad \text{.....................(7.10)}$$

and for females

$$\ln(e_0 - 80) = a + b(\text{time}) \quad \text{.....................(7.11)}$$

The projected expectations of life achieved by equations (7.10) and (7.11) are presented in Table 7.11. The pooled category has been created by pooling the data available from major states in India.

Table 7.11: Projected values of e_0, India, 1996-2016 (in years)

Year	Males		Females	
	India	Pooled	India	Pooled
1991 (Base yr.)	59.00	58.72	59.70	59.08
1996	61.18	58.85	62.04	63.70
2001	63.10	62.74	64.25	65.86
2006	64.76	64.79	66.19	66.30
2011	66.18	66.18	67.89	68.19
2016	67.41	67.24	69.38	69.08

Source: India. ORG, 1996:6.

The United Nations (1999) also provides projected expectations of life at birth for India. The medium variant projections are presented in Table 7.12. The values tend to be on the low side when compared to the projections by the two methods presented in this chapter.

Table 7.12: Projected values of e_0, India, 1995-2025 (in years)

Year/Period	Males	Females
1995-2000	62.3	62.9
2000-2005	63.5	64.9
2005-2010	64.9	66.9
2010-2015	66.2	68.6
2015-2020	67.3	70.3
2020-2025	68.8	72.1

Source: UN, 1999: 228.

As seen from Tables 7.11 and 7.12, the projections of expectation of life at birth from both the sources provide much lower values than those projected by the new model. The upper bound fixed by the technical group on population projections constrains the increase in expectation of life in their projections. For United Nations projections, similar results may be expected as the base data are supplied from government, sources in this case the Registrar General of India, who also officiates on the technical group on population projections. Pollard (1987) also suggested the upper-limit approach to linear forecasting of age specific death rates, but it is not easy to predict such an upper bound for countries with wide regional variations like India.

A comparison of the gain in expectation of life at birth by the various methods is presented in Table 7.13. Females are projected to be the higher gainers compared to males in all the forecasts.

Table 7.13: Comparison of estimated and forecasted gains in life expectancy, India (average annual increase in years)

Method/	Males			Females		
Source	All India	Rural India	Urban India	All India	Rural India	Urban India
By new mortality model						
1971-1997	0.43	0.47	0.45	0.58	0.48	0.35
1997-2020	0.66	0.57	0.52	0.69	0.23	0.50
1971-2020	0.58	0.53	0.51	0.61	0.57	0.52
By Lee-Carter method with selected ARIMA models						
1971-1997	0.42	0.49	0.30	0.54	0.59	0.48
1997-2020	0.35	0.34	0.27	0.40	0.39	0.34
1971-2020	0.38	0.41	0.28	0.47	0.49	0.41
By Lee-Carter method with ARIMA (0,1,0)						
1971-1997	0.48	0.50	0.33	0.54	0.58	0.46
1997-2020	0.30	0.30	0.20	0.39	0.42	0.32
1971-2020	0.38	0.40	0.26	0.46	0.49	0.39
Registrar General of India						
1991-2016	0.34			0.39		
United Nations						
1997-2022	0.26			0.37		

For India, the estimates provided by the Lee-Carter method are similar to the estimates of RGI and UN. All these methods basically use linear extrapolations. As the estimation and forecasting by RGI and UN seem to have been made with the assumption of an upper limit, they may underestimate mortality in the long run. In each of the forecasts, the differences in male and female life expectancies tend to be two years or more. The life expectancies estimated by the new model of mortality are estimated to be about five years less then those of the three other sources.

7.8 Conclusion

Parameterised estimation of mortality through the new mortality model provides 83.4 years as the life expectancy for urban Indian females in 2020; this is the highest life expectancy of the six cases. Such a result would imply a higher quality of life and better health facilities along with a higher status of women. The accuracy of the forecast is contingent upon these changes actually taking place. As at 2020 a difference of seven and a half years between rural and urban males is compared to around six years between rural and urban females.

The forecasted parameters of the new mortality model, which represent the age patterns of mortality, were modelled successfully using the ARIMA technique. In some cases logistic transformation of the series was required to yield consistent results. Parameters estimated for the new mortality model in most of the cases were found to be modelled with higher orders of autoregressive and moving average components. Usually higher-order components result in some year-to-year fluctuations in the forecasts but they abide by the trends in the time series. Also some higher-order coefficients included in the models were not statistically significant, but they provided improvements in the forecasts by reducing the standard errors. The urban females series yielded the wider disturbances because the input data sets were characterised by high year-to-year fluctuations.

For the Lee-Carter method, it has been observed that ARIMA models with higher orders than (0,1,0) were adequate to model the Indian mortality. A difference of almost a year in the forecasted life expectancies was noticed between optimal ARIMA models and ARIMA (0,1,0) but substantial reduction in the 95 per cent confidence intervals was obtained with optimal models.

Theoretically, the new mortality model provides better forecasts of mortality than the Lee-Carter method, as the former incorporates changes in the age patterns of mortality through its seven parameters. Also, in comparison with Lee-Carter, the new mortality model provides relative mortality forecasts that are more coherent with the gender and living statuses in India. Rural-urban differentials are kept as wide as 6-7 years.

The parametric graduation provides smooth results with statistical authenticity. However, in the end, the accuracy of any forecast based on past trends is dependent upon the rate of change in the conditions that give rise to the past trend continuing in the future. In the past two decades, the conditions underlying mortality have improved greatly. Whether the trends continue and produce the highly favourable outcomes of the projections from the new mortality model is a question for the future.

Figure 7.1: Parameters and their forecasts, males, India, 1970-1997

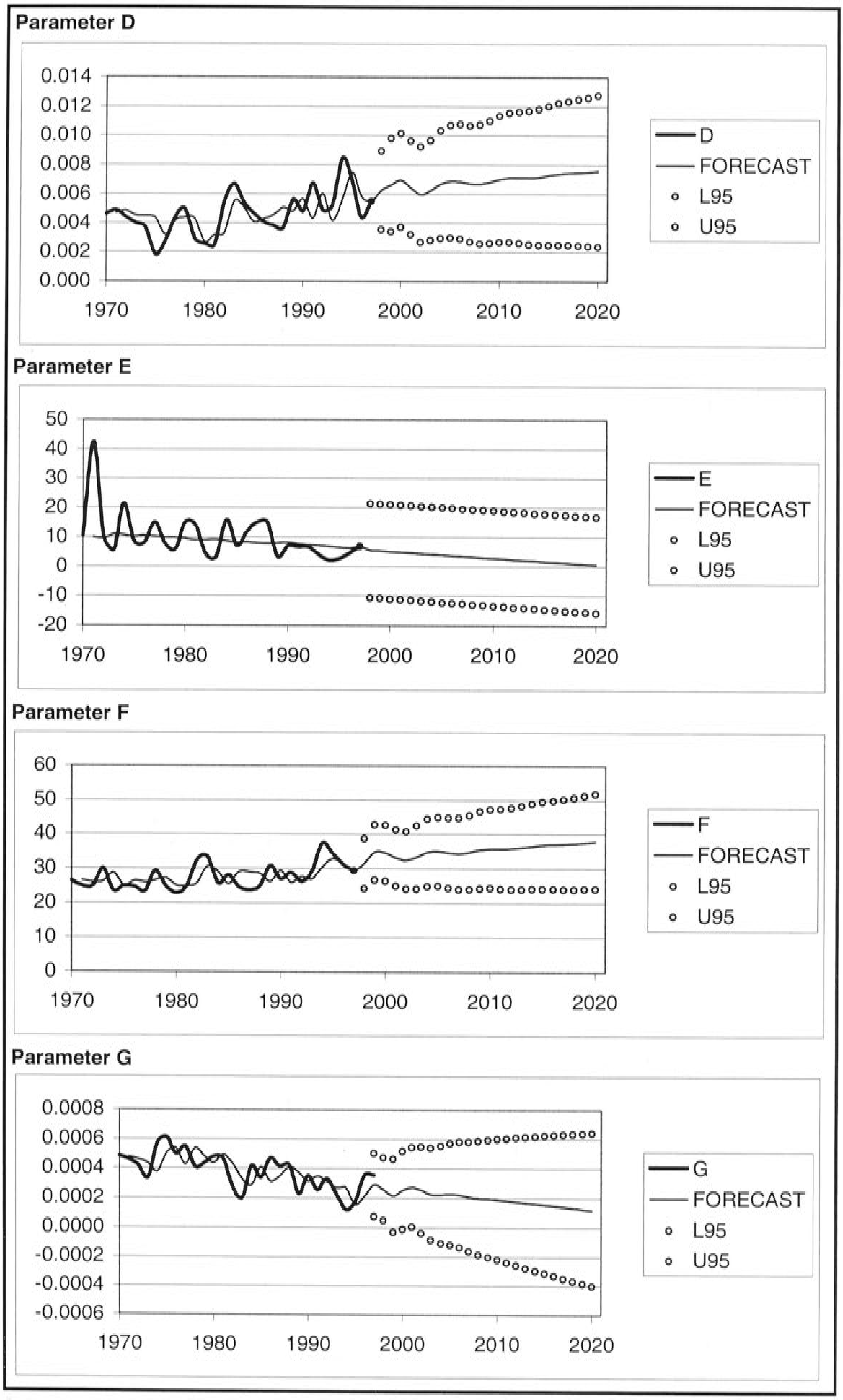


Figure 7.1 contd.: Parameters and their forecasts, males, India, 1970-1997

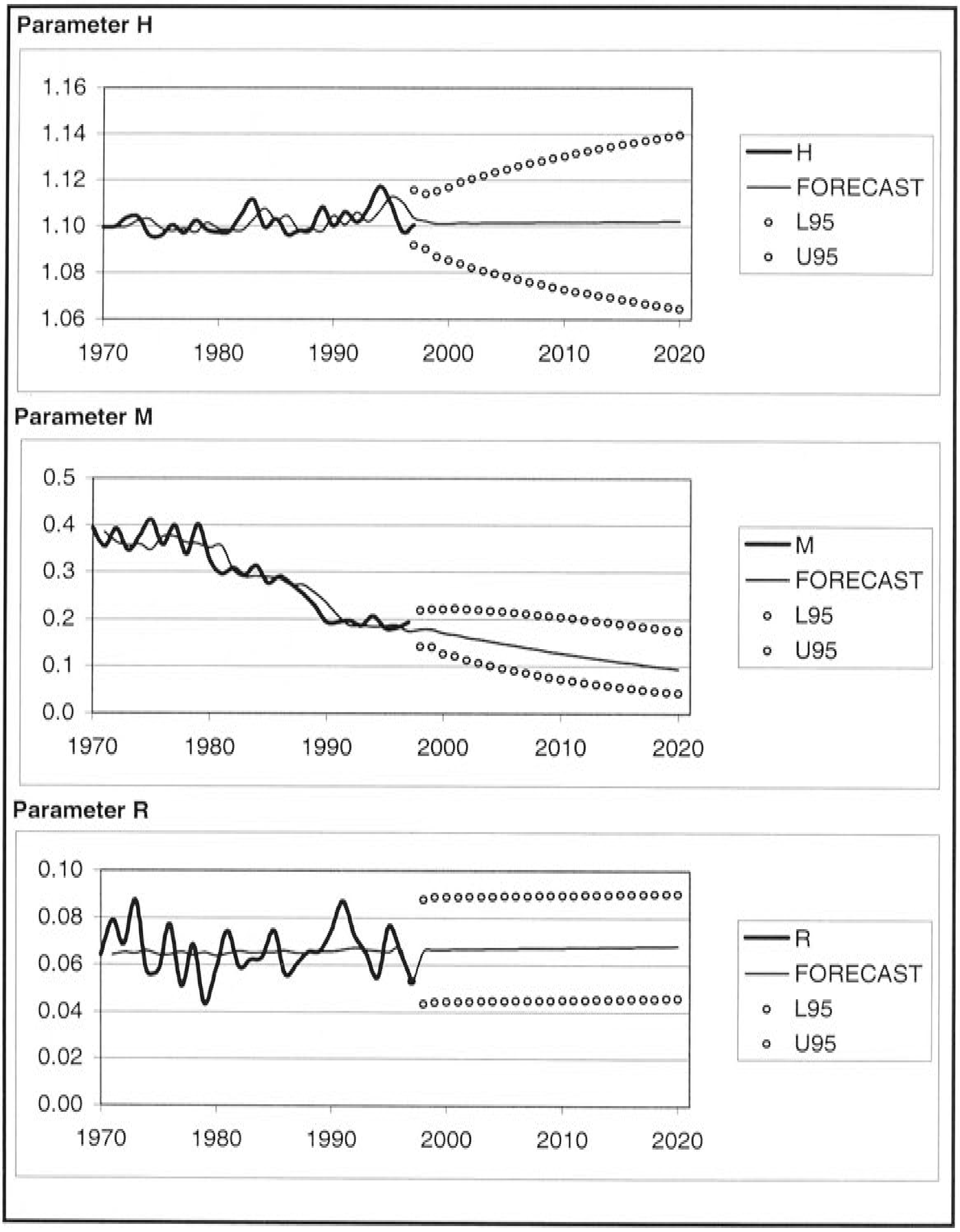


Figure 7.2: Parameters and their forecasts, females, India, 1970-1997

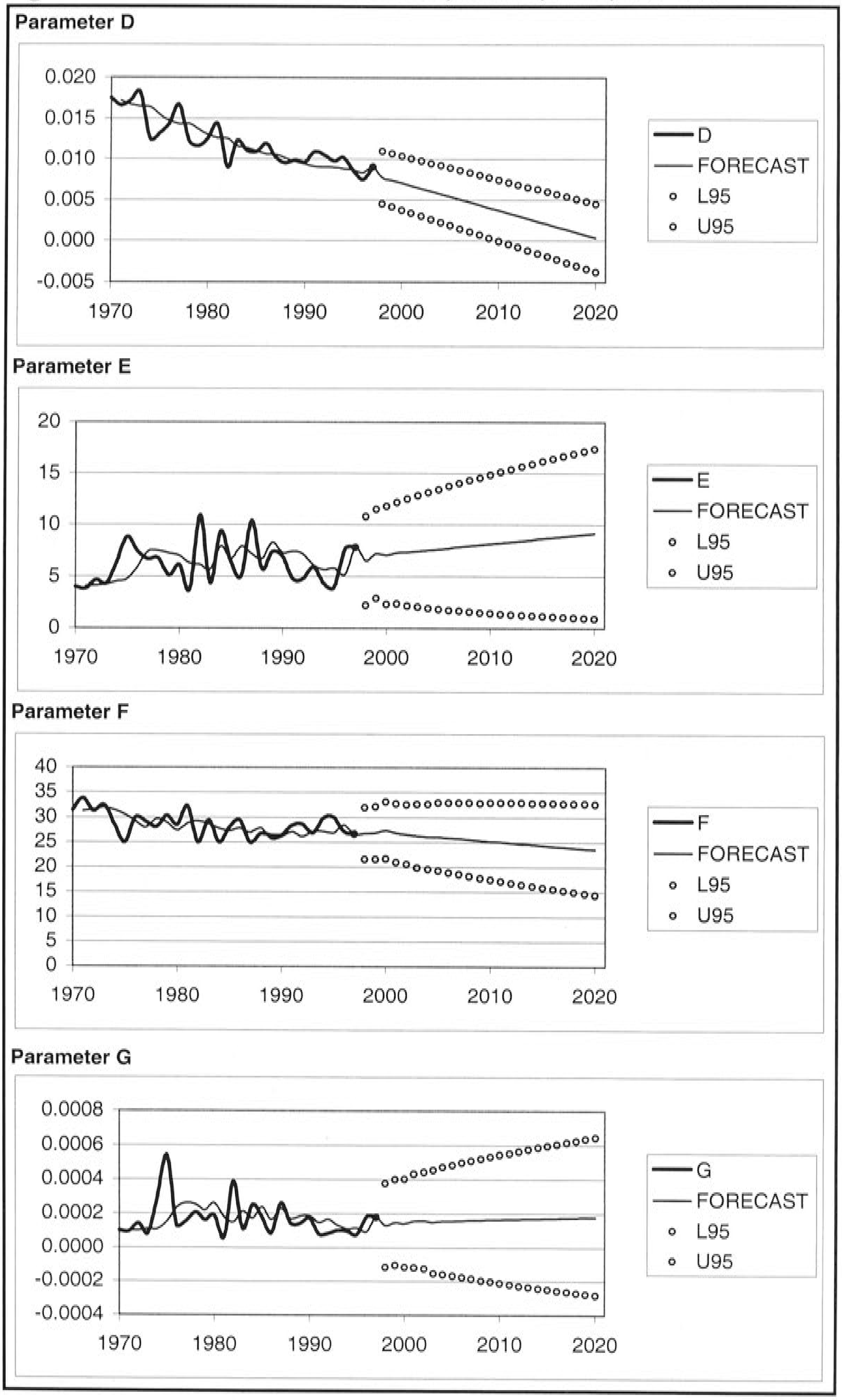


Figure 7.2 contd: Parameters and their forecasts, females, India, 1970-97

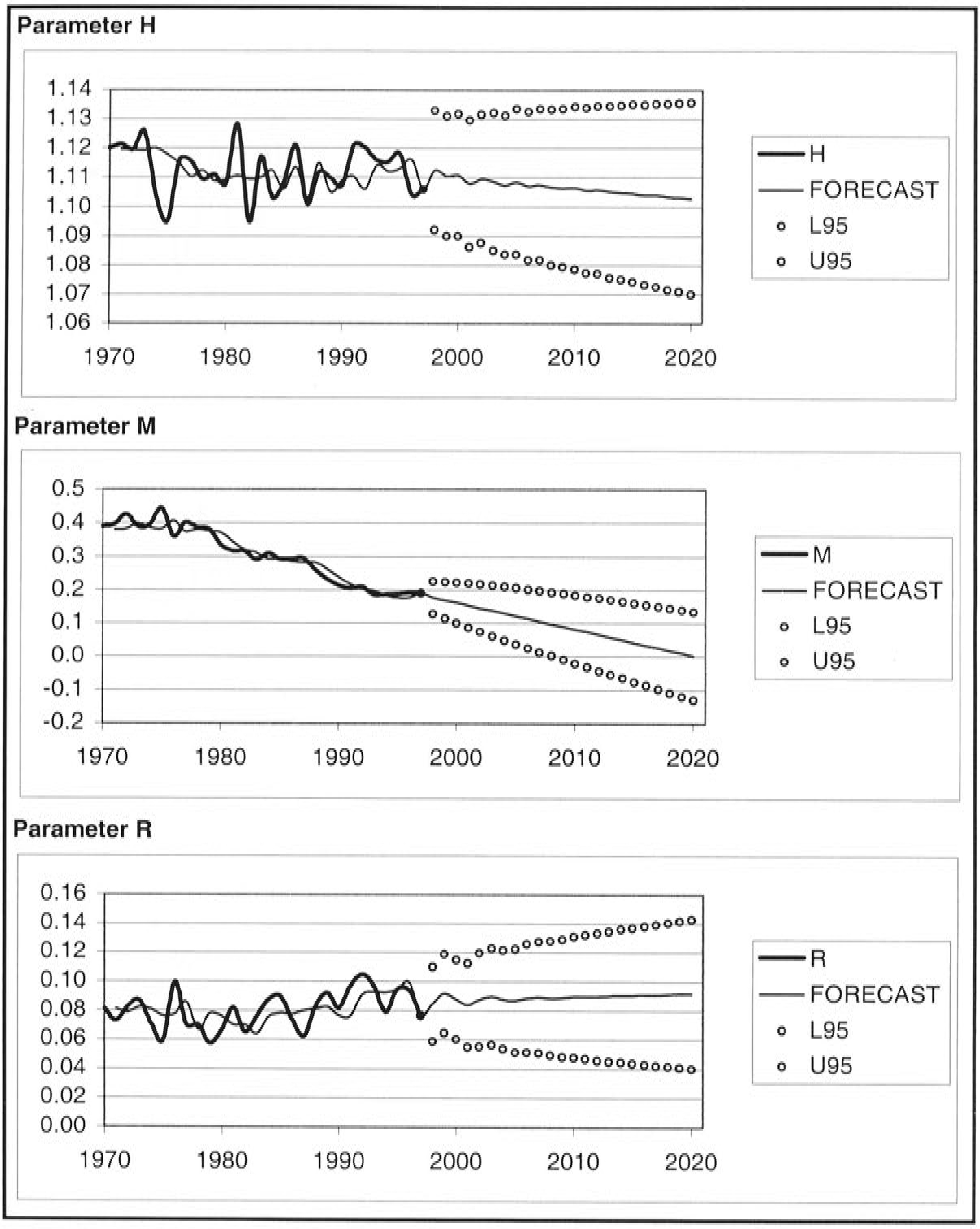


Figure 7.3: Parameters and their forecasts, rural males, India, 1970-97

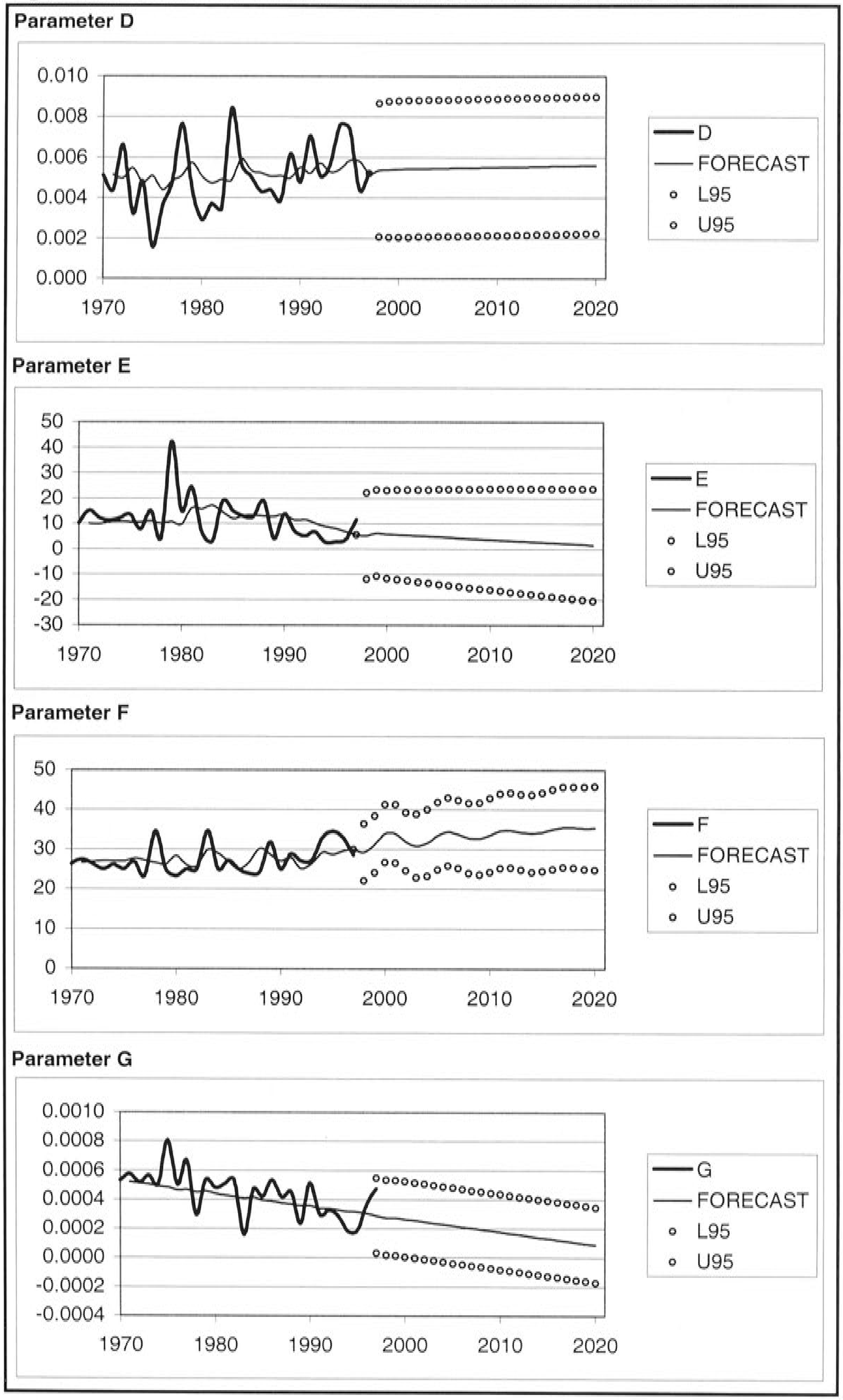


Figure 7.3 contd.: Parameters and their forecasts, rural males, India, 1970-97

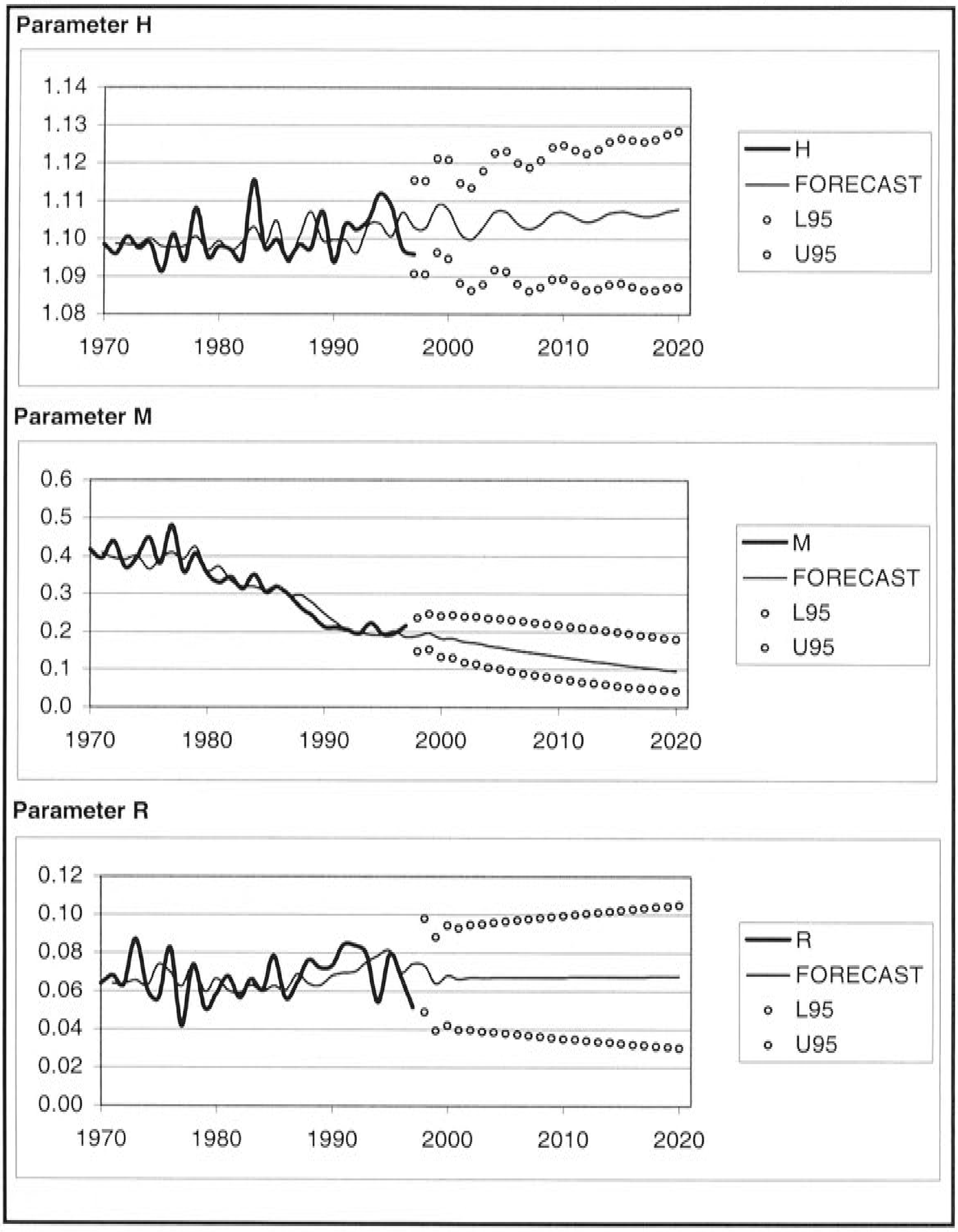


Figure 7.4: Parameters and their forecasts, rural females, India, 1970-97

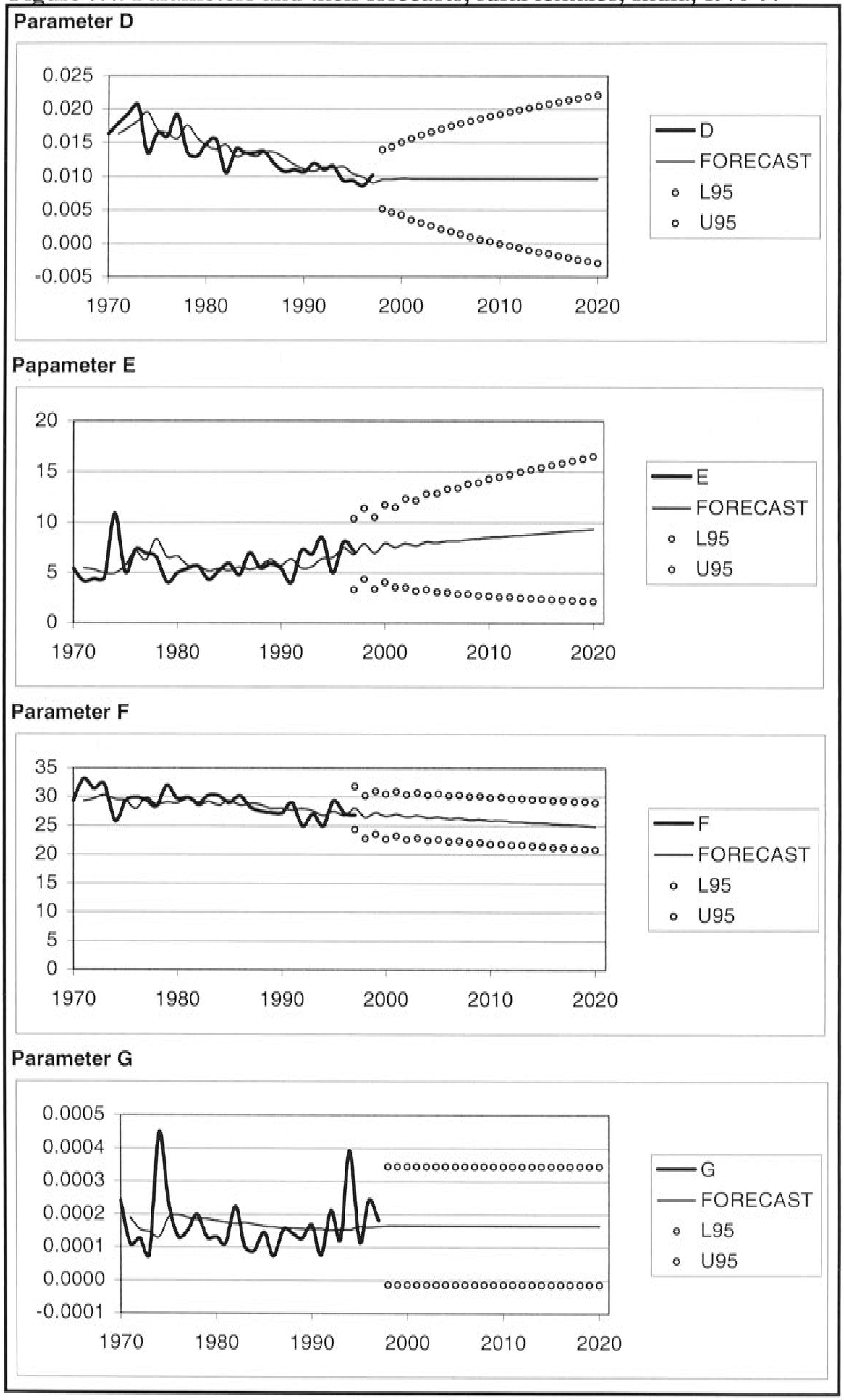


Figure 7.4 contd.: Parameters and their forecasts, rural females, India, 1970-97

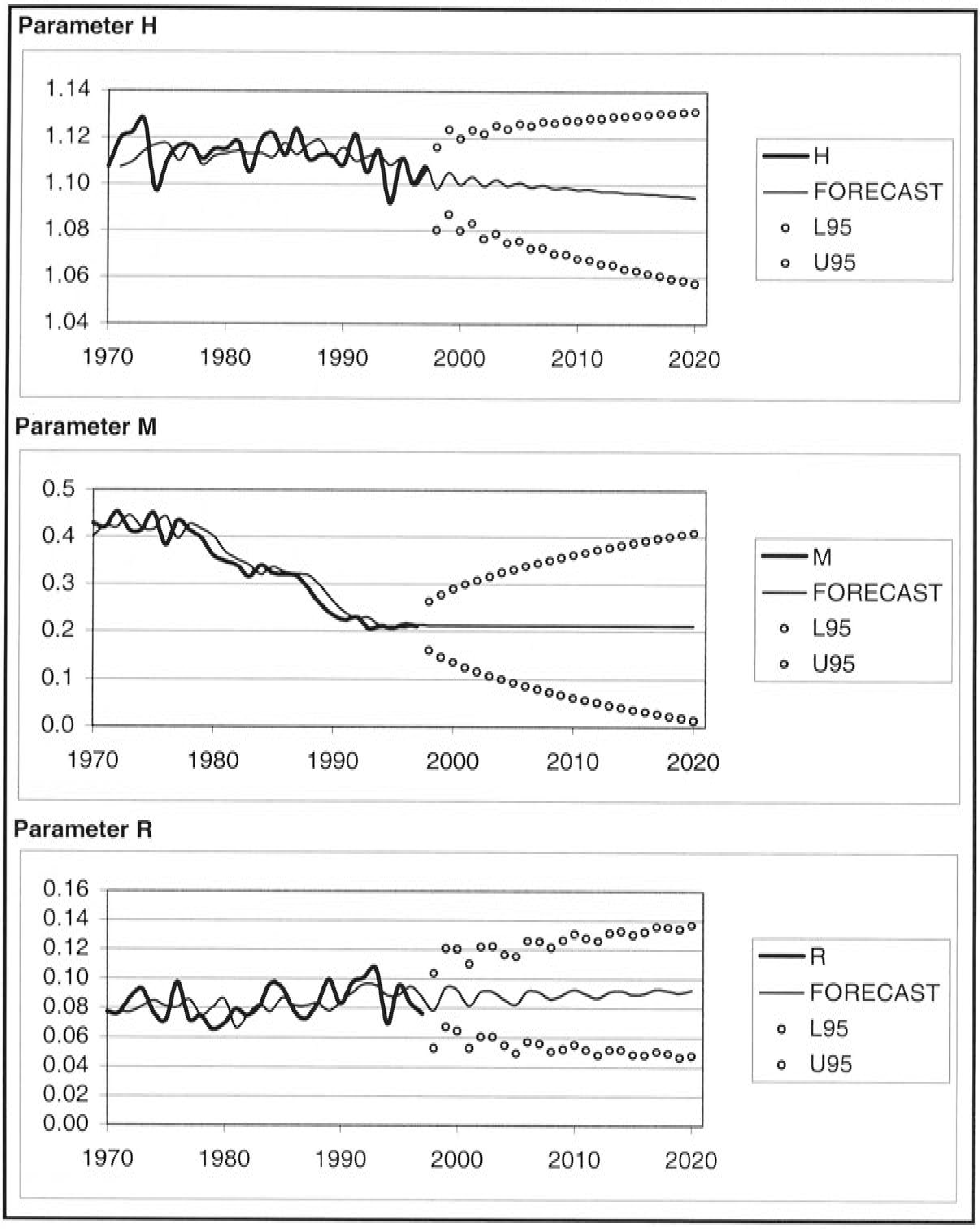
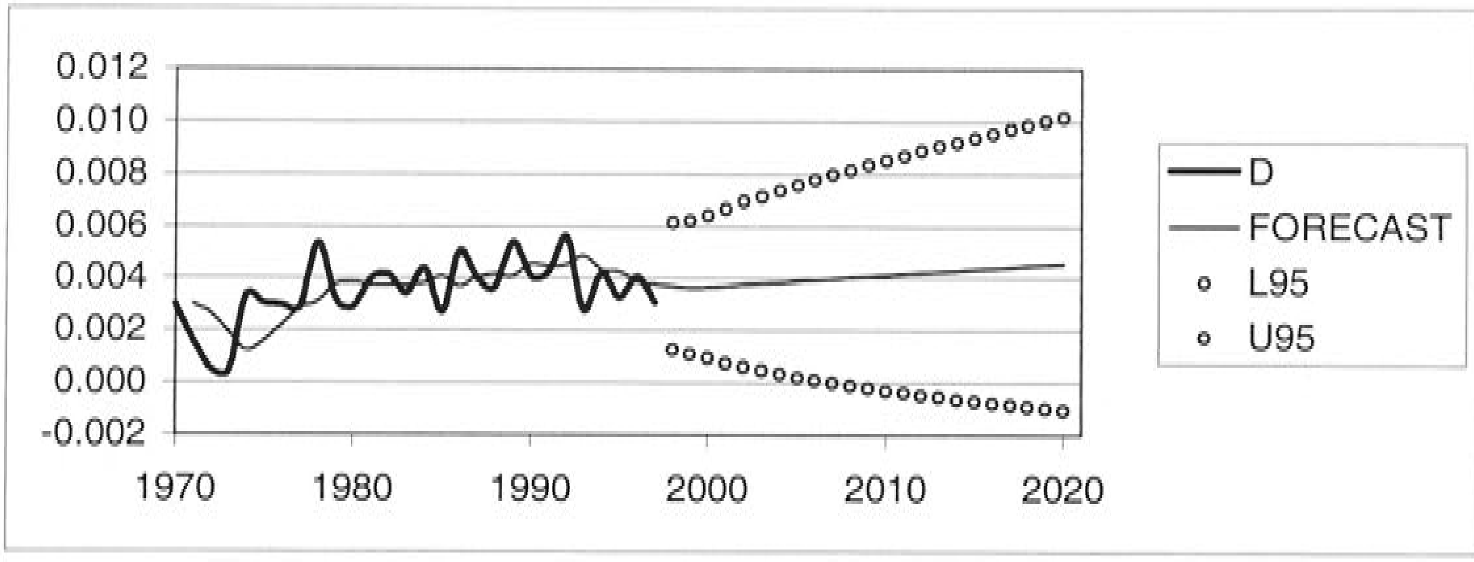
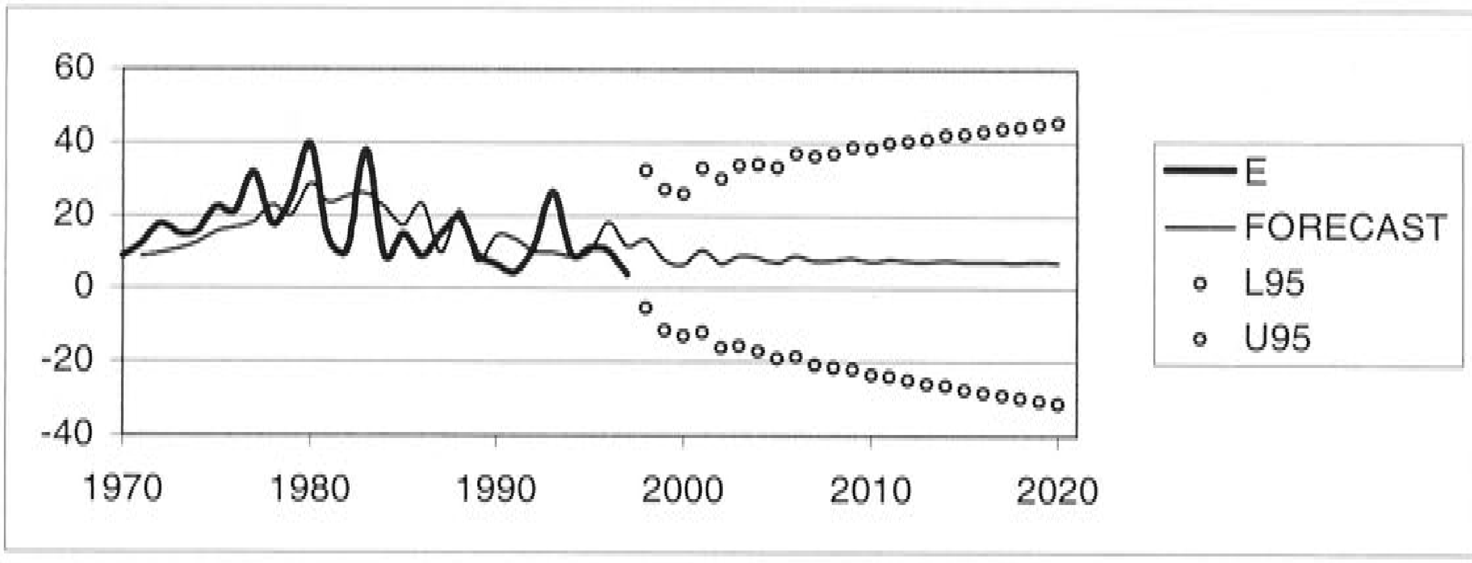


Figure 7.5: Parameters and their forecasts, urban males, India, 1970-97

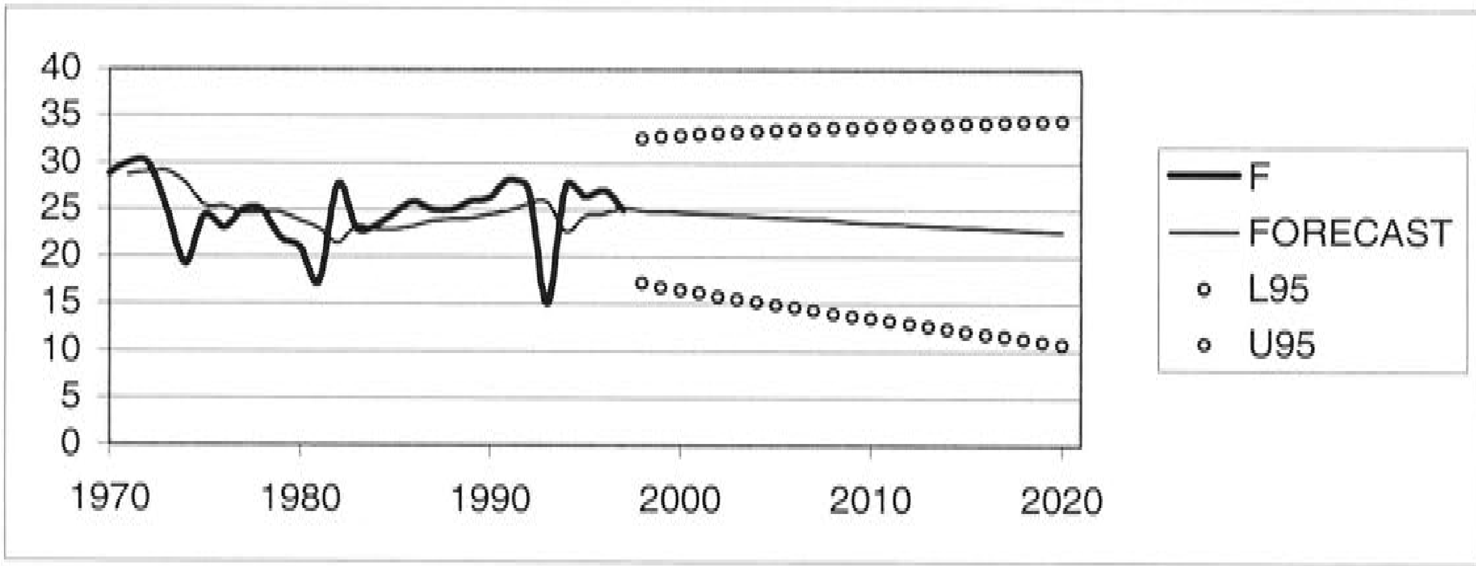
Parameter D



Parameter E



Parameter F



Parameter G

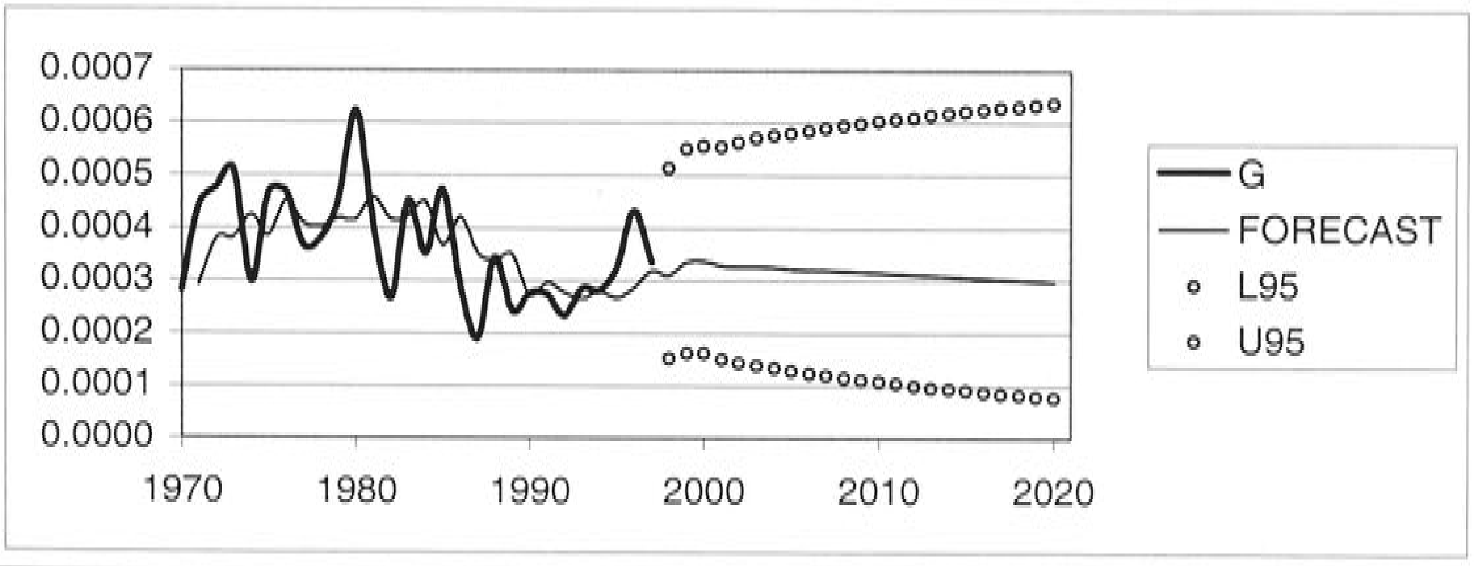
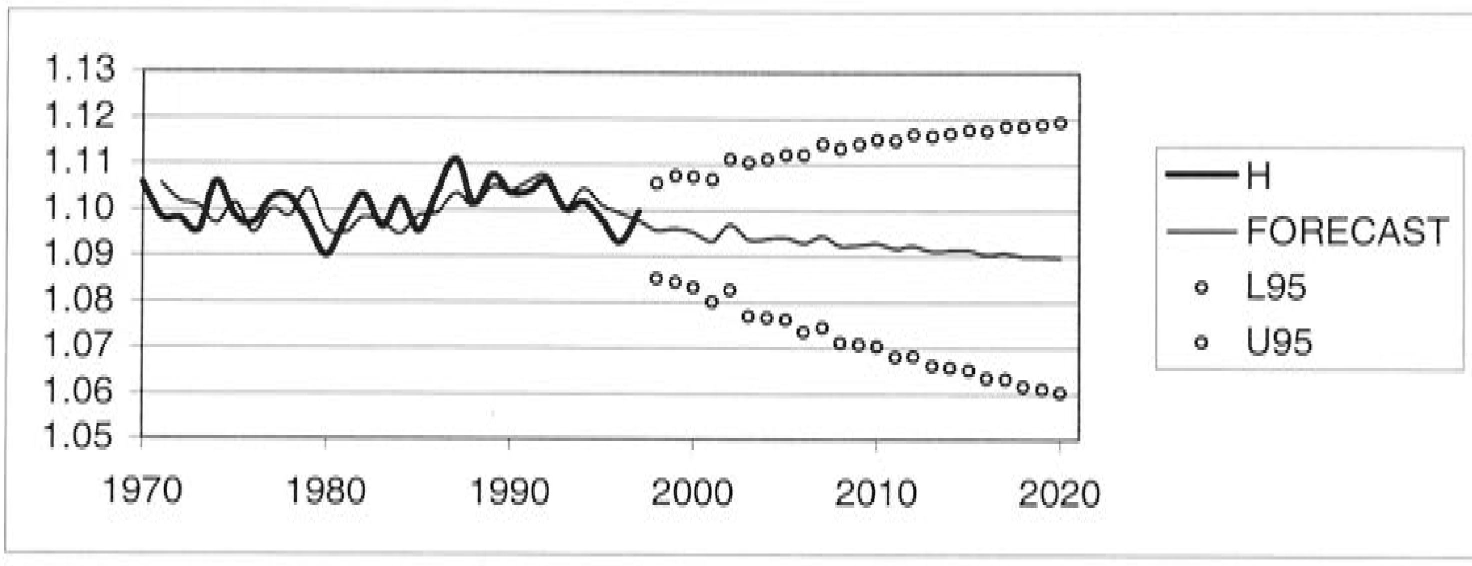
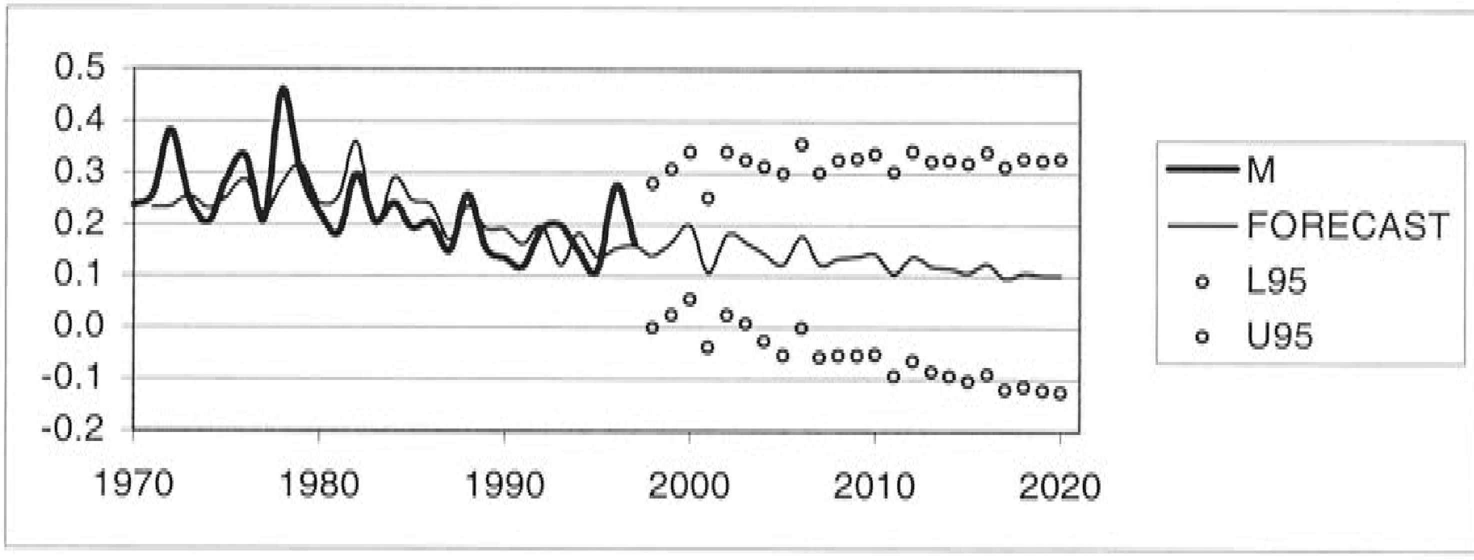


Figure 7.5 contd.: Parameters and their forecasts, urban males, India, 1970-97

Parameter H



Parameter M



Parameter R

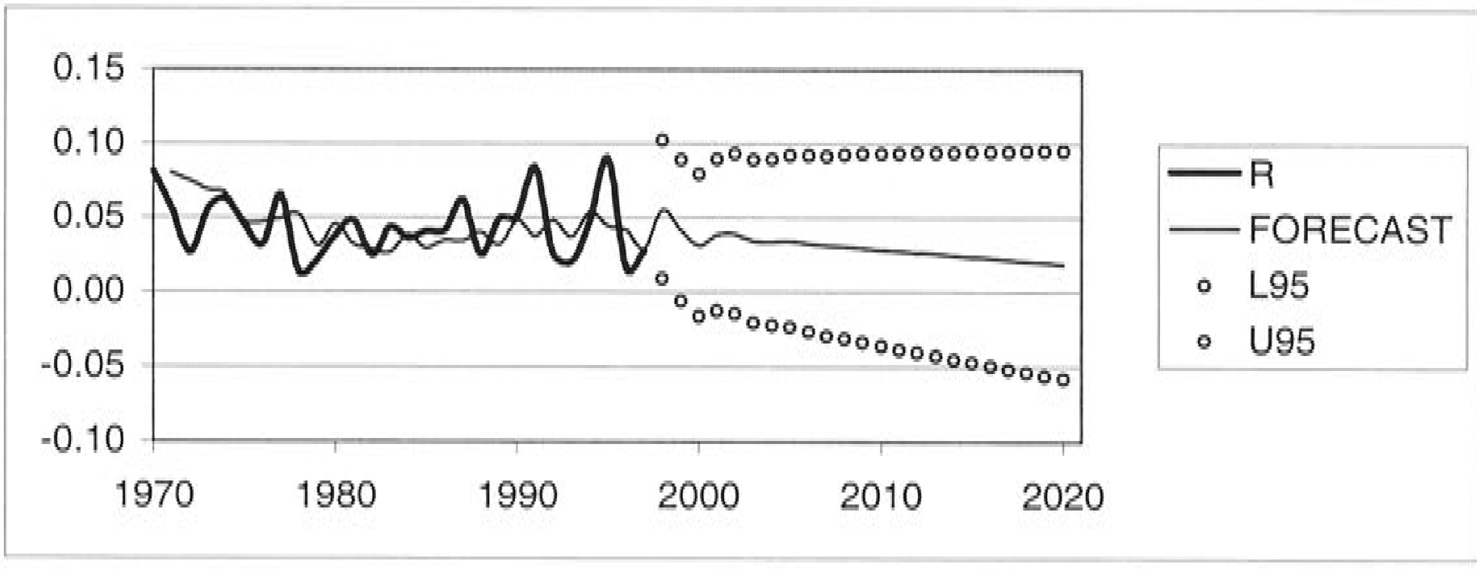


Figure 7.6: Parameters and their forecasts, urban females, India, 1970-97

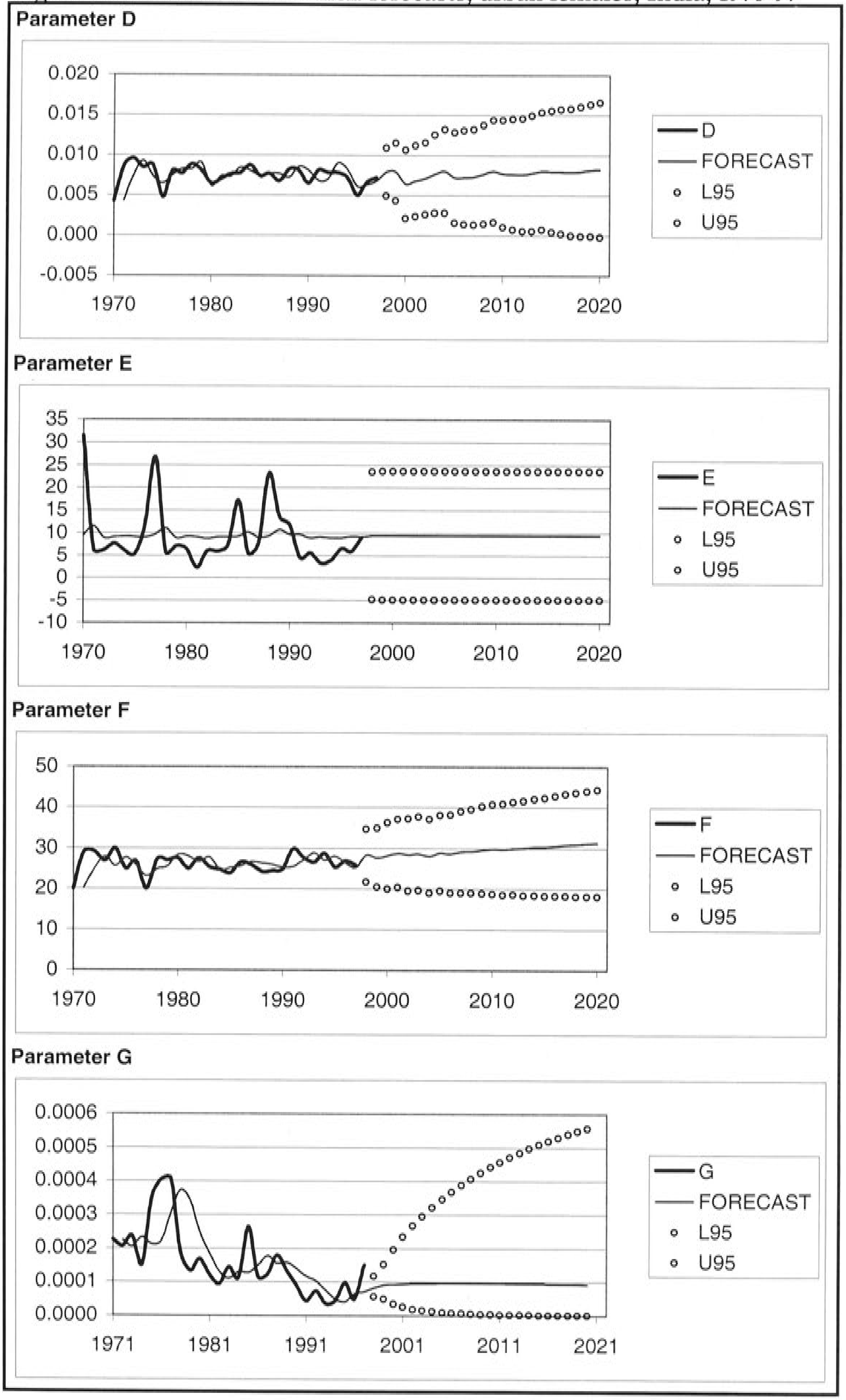
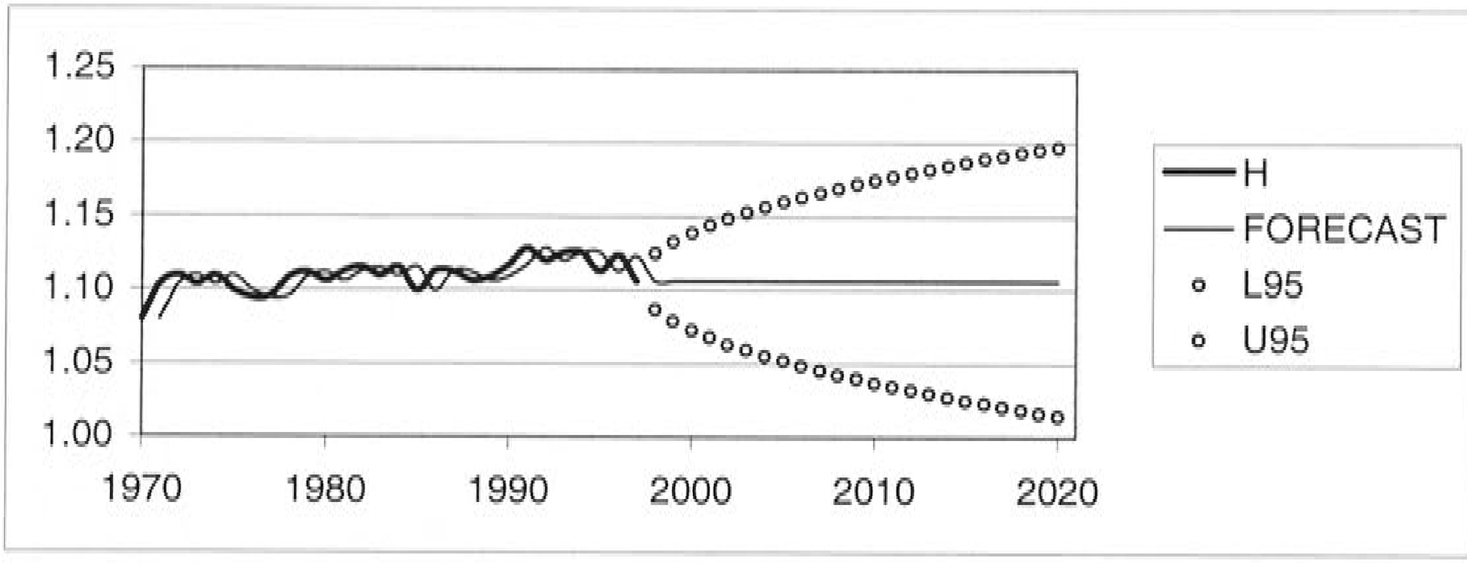
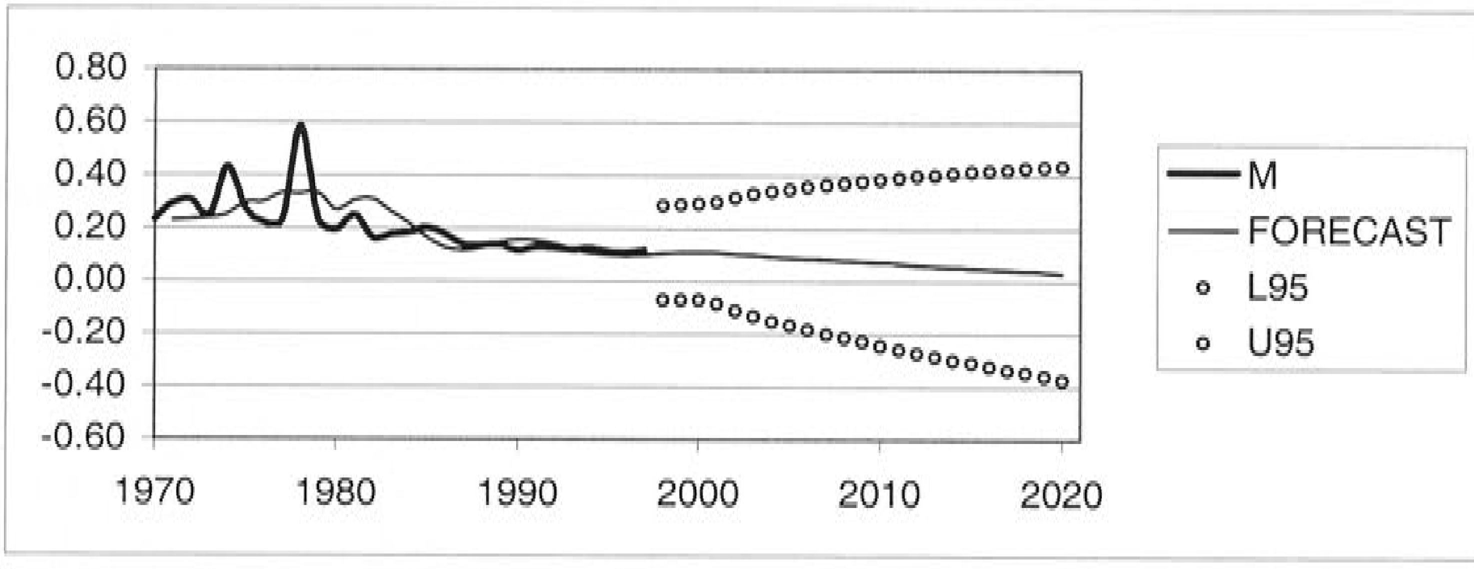


Figure 7.6 contd.: Parameters and their forecasts, urban females, India, 1970-97

Parameter H



Parameter M



Parameter R

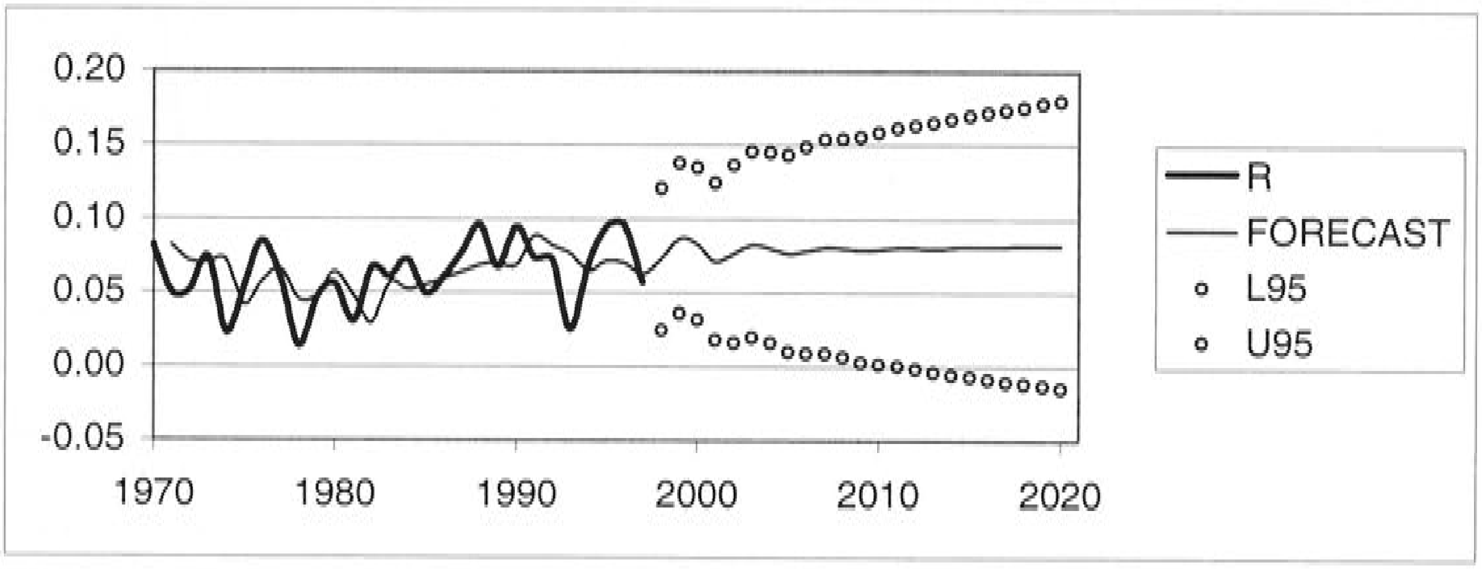


Figure7.7: Estimated/ forecasted q_x by new model, India, 1971-2020

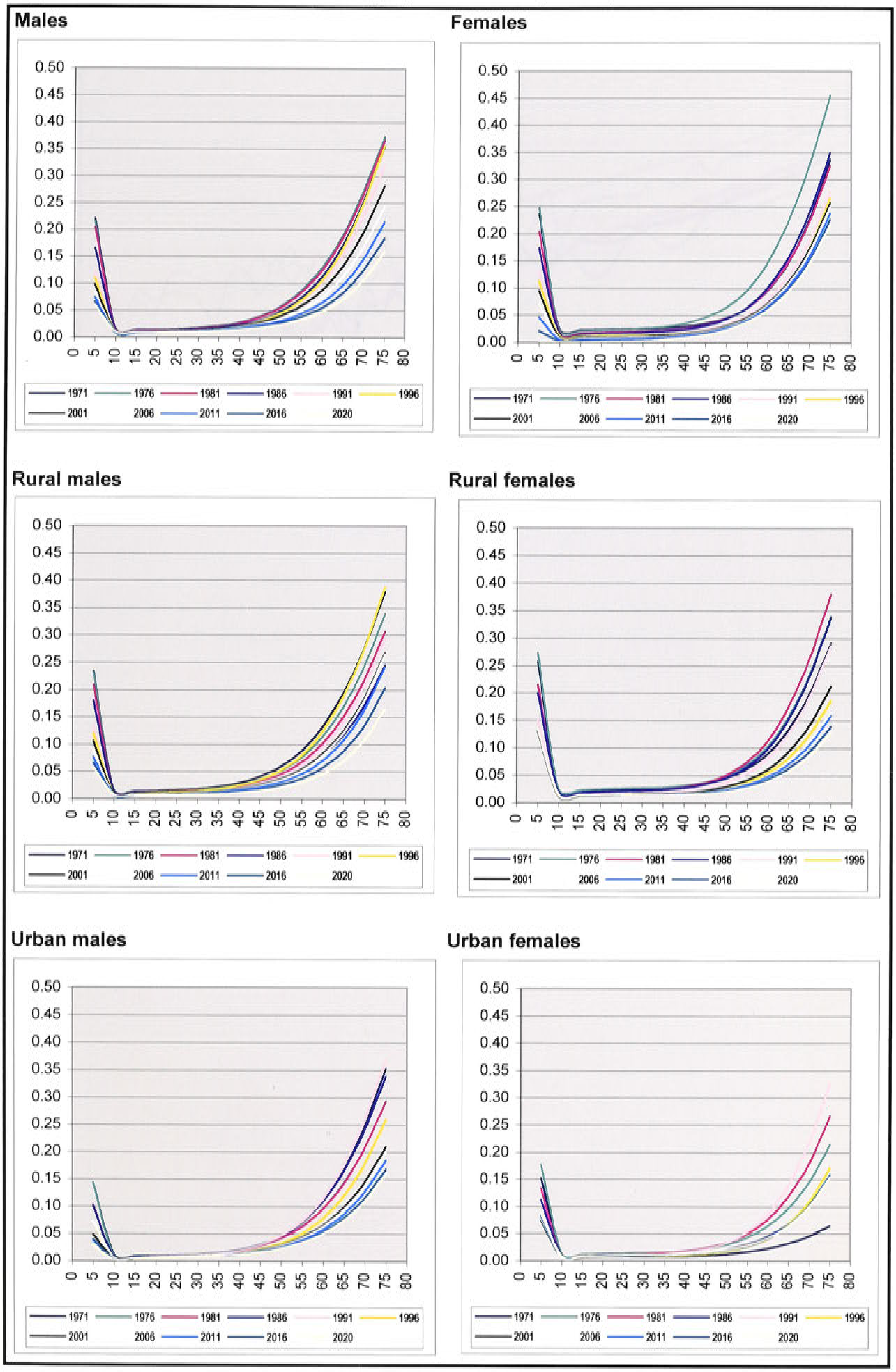


Figure 7.8: Estimated and forecast life expectancy by the new mortality model for India, six cases, 1998-2020 (in years)

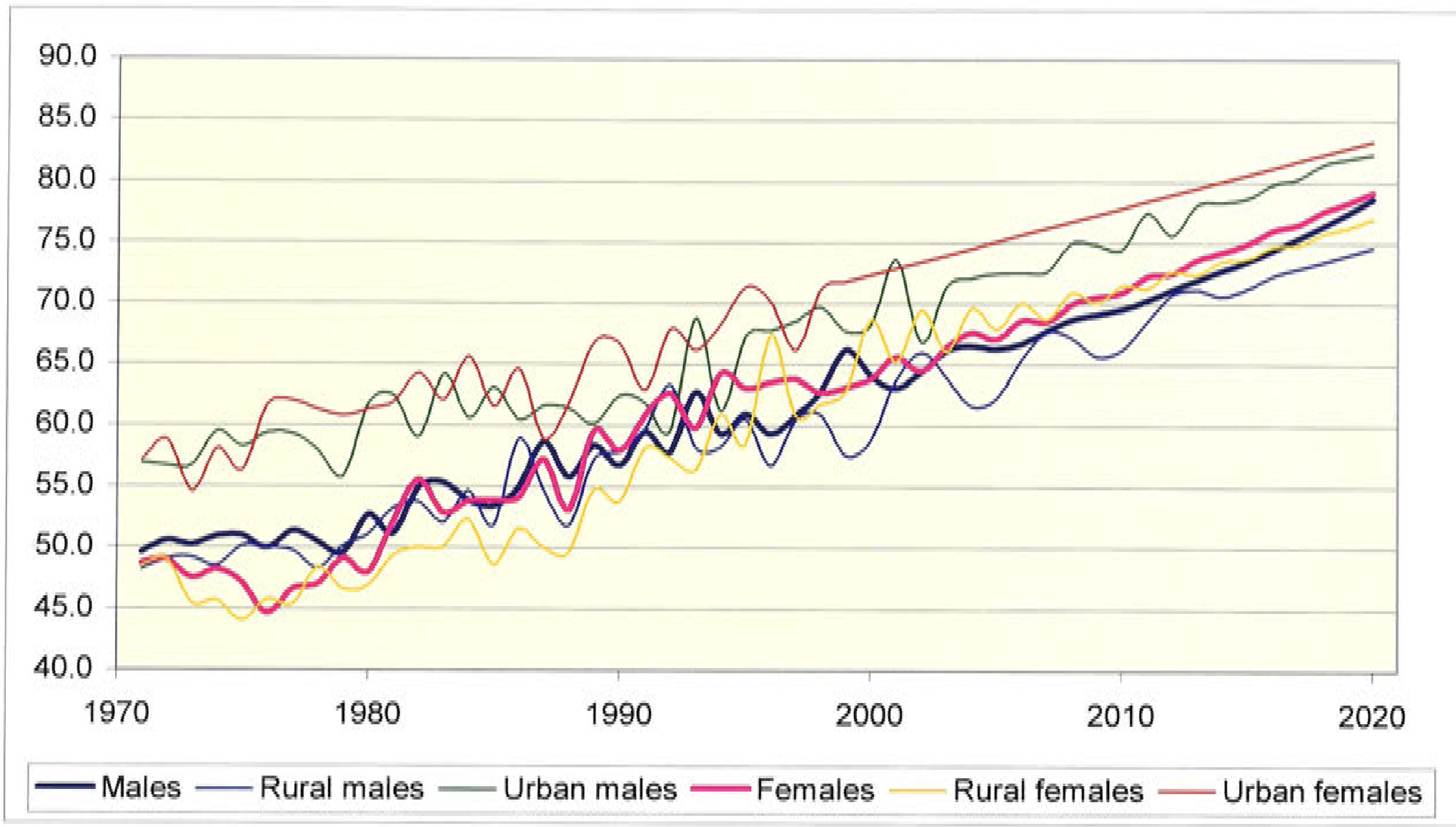


Figure 7.9: Estimated and forecasted optimal ARIMA models compared with ARIMA (0,1,0), India, males, 1970-2020

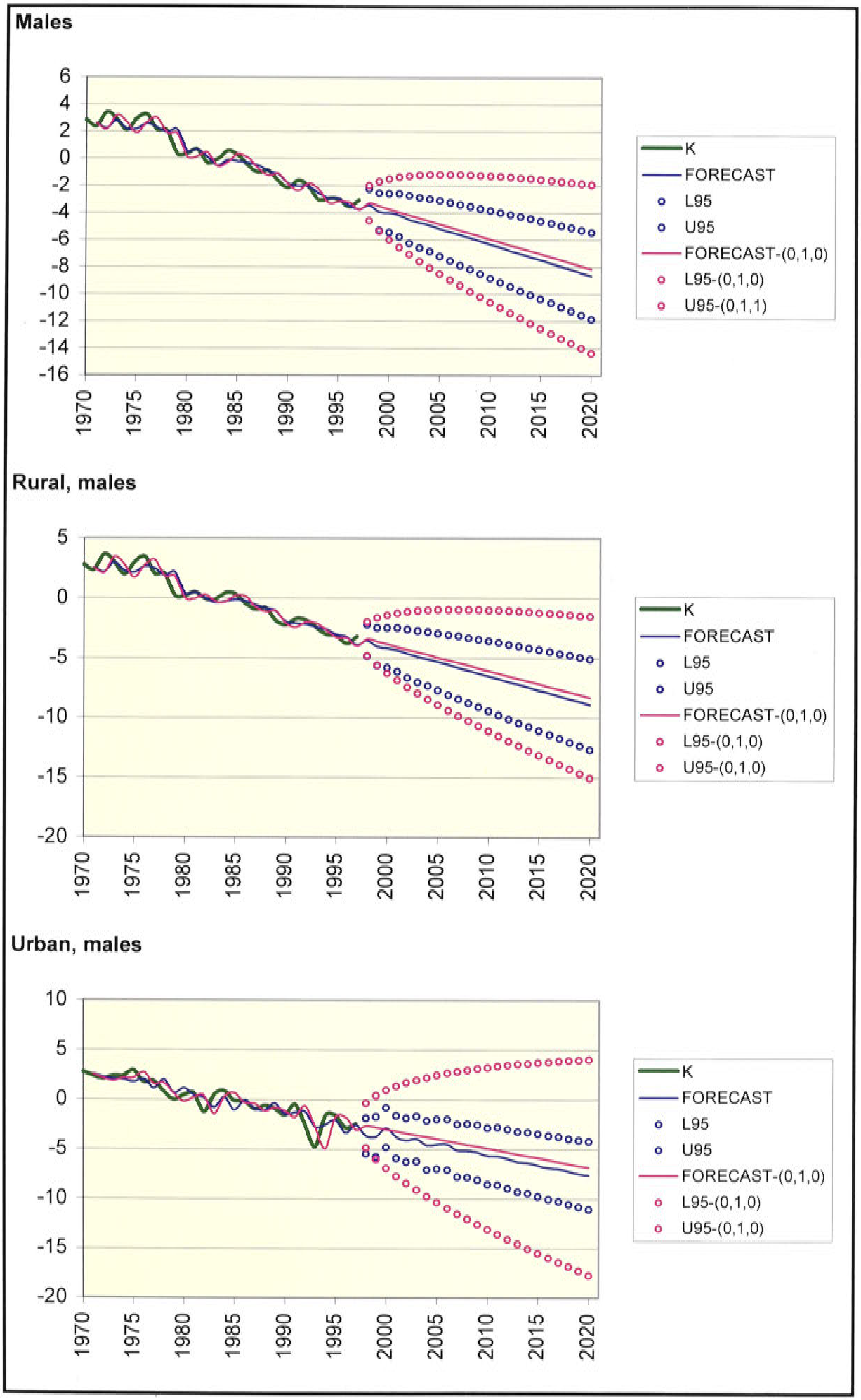


Figure 7.10: Estimated and forecasted optimal ARIMA models compared with ARIMA (0,1,0), India, females, 1970-2020

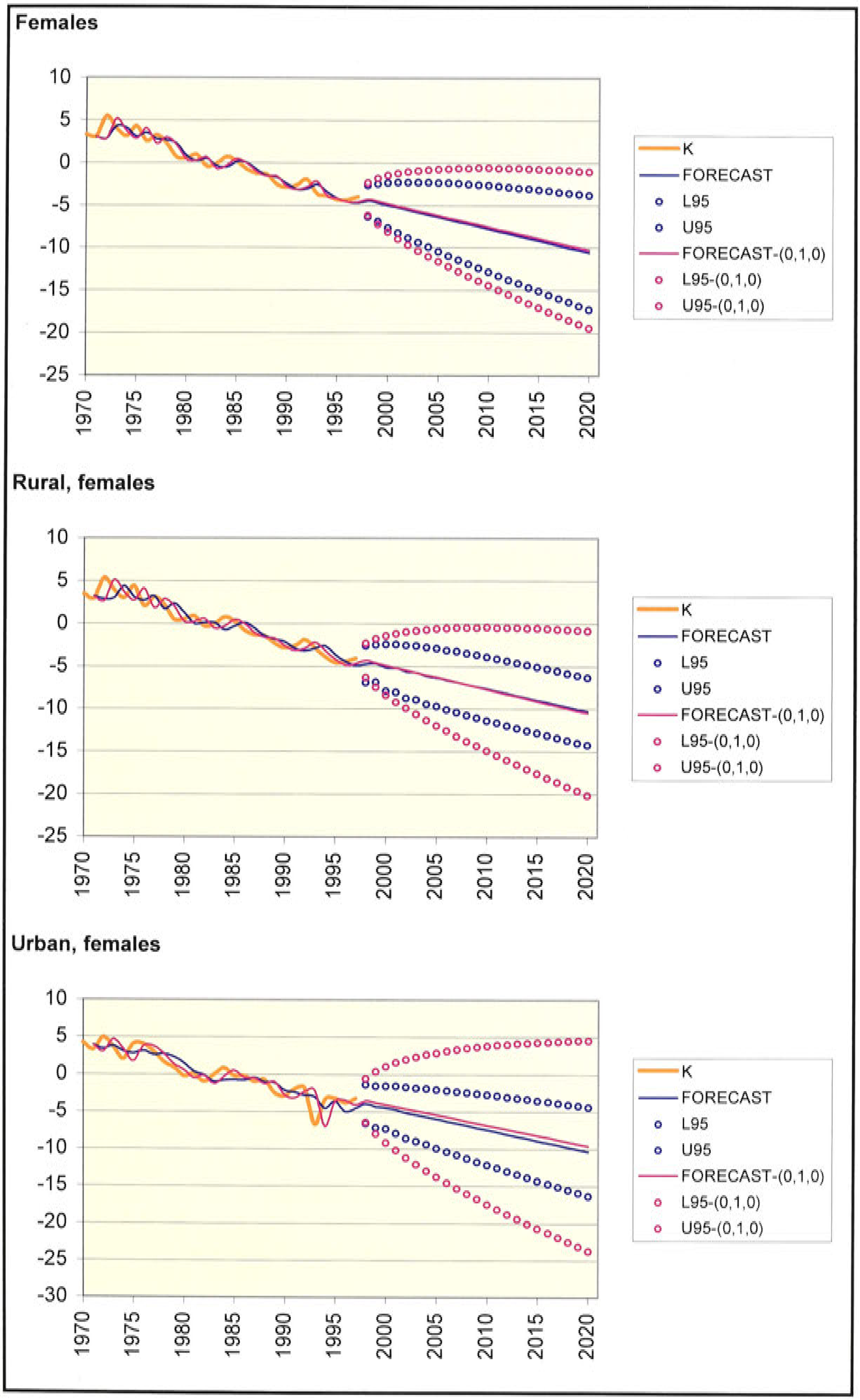


Fig. 7.11: Expectation of life at birth, estimated by Lee-Carter method, India, males and females, 1970-2020 (in years)

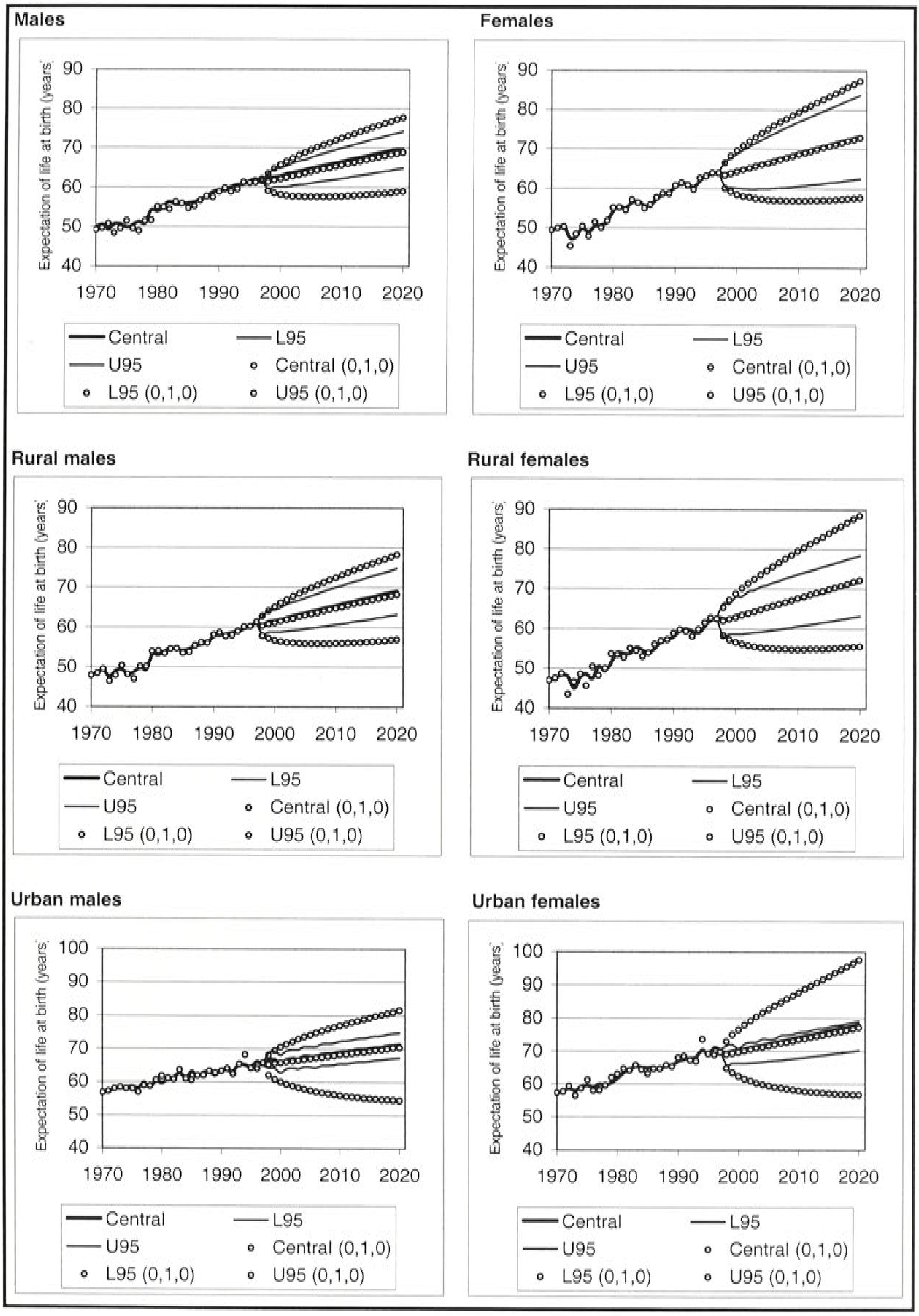


Figure 7.12: Forecasted life expectancy by the Lee-Carter model for India, six cases, 1998-2020 (in years)

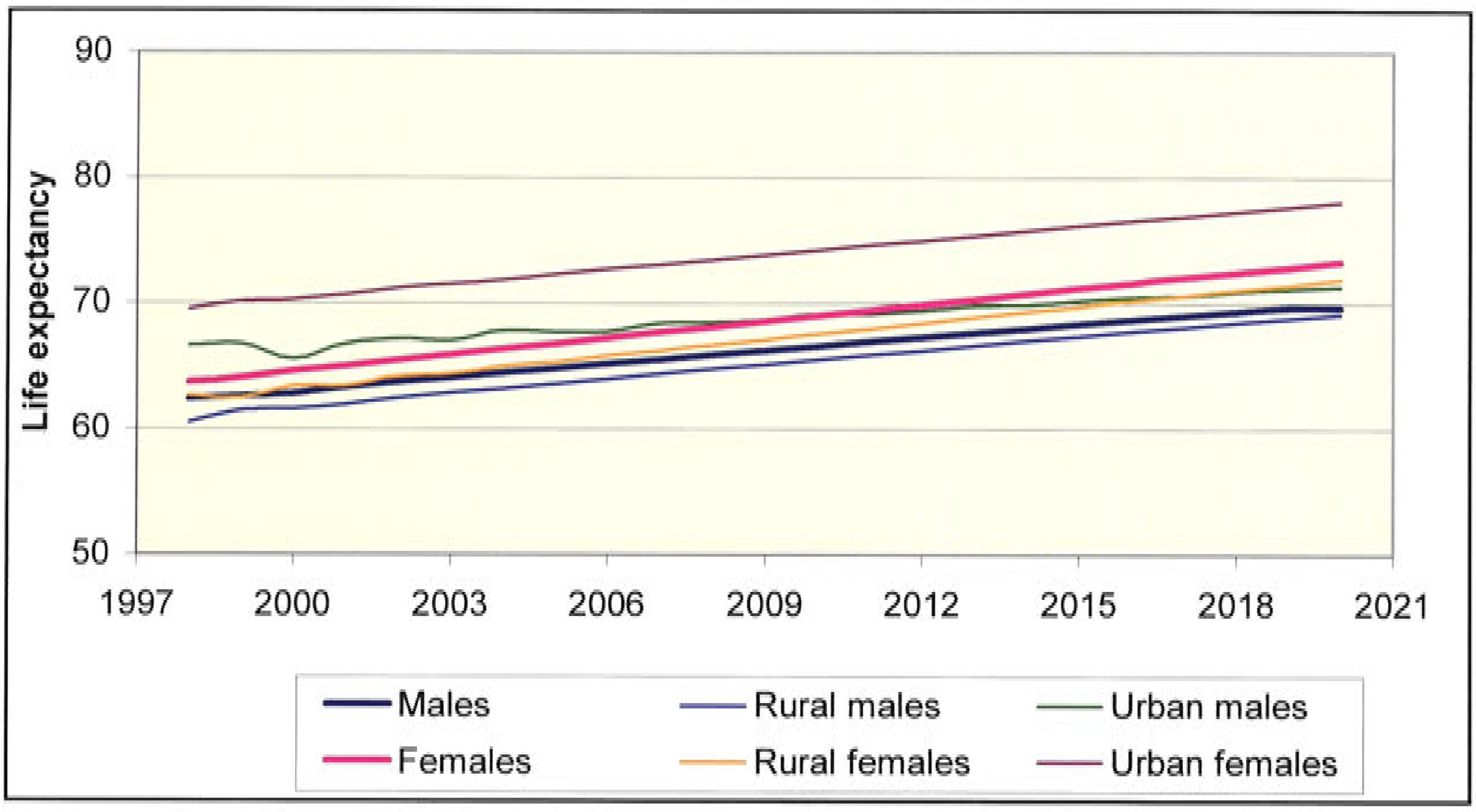
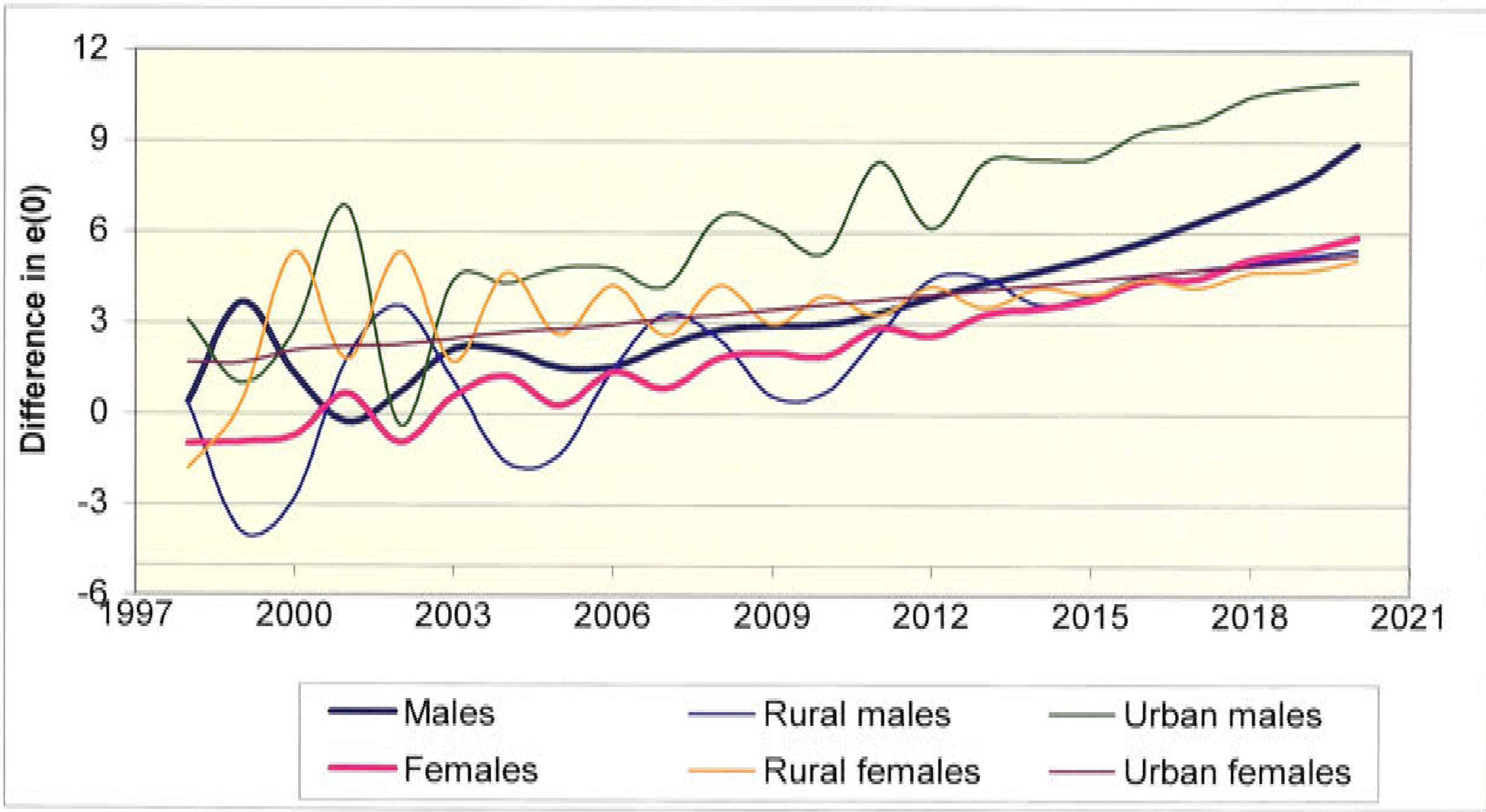


Figure 7.13: Difference in life expectancies between the new mortality model and Lee-carter model for India, six cases, 1998-2020 (new model-Lee-Carter, in years)



Chapter 8: Conclusion

When I am working on a problem I never think about beauty. I only think about how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong.
Buckminster Fuller

8.1 Need for mortality forecasts

Mortality forecasts for any society are important because policy planners need base data for health and other related planning issues. Mortality forecasts are also needed as a primary component for population forecasts. Apart from these issues, mortality forecasts can enable mortality comparisons between populations, within populations, between regions, between the sexes and according to many other demographic characteristics.

Mortality forecasts can be made mainly in three ways, by model life tables, relational methods and by the laws of mortality. As model life tables serve only as aggregated mortality schedules according to 'some defined standard', they may not incorporate any experience of the country under study. They are useful for countries where data on mortality are unavailable, for some comparable understanding of mortality. They can be adopted for mortality forecasting in the most simplistic way. Brass logit and Brass relational models of mortality also serve in the case of limited data but they suffer from the lack of fit at both ends of the mortality curves. The most appropriate way to model human mortality is by means of the laws of mortality: these laws consist of mathematical functions with parameters. The parameters themselves serve as sources of information apart from representing overall mortality. In this thesis, parameters were estimated and forecasted for Indian data and the forecasts were generated for 2020.

This study provides comprehensive mortality forecasts for India based on three age segments of mortality. No such forecasts are available from any other source for India; the available forecasts are limited to the single numerical value, that is, expectation of life at birth. This study provides estimates of the probabilities of death according to age for the years until 2020.

8.2 Data

In this thesis, most of the data used are from the sample registration system of India published annually by the Registrar General of India's Office. Data were entered in computer files. Since the data were only available in hard copy form, the electronic source constructed for this thesis may be useful for future research on Indian mortality.

8.3 Testing the new mortality model

As discussed above, models represented by the laws of mortality serve best to represent human mortality. A literature survey of various attempts to model human mortality and testing of different options revealed that the model developed by Heligman-Pollard was usable for the present study. However, this model had some limitations and required modification. For the reasons of parsimony and sharp comparison, reduction in the number of parameters was sought. The H-P model also had certain estimation anomalies, as some parameters kept attaining negative values, and constraining these parameters resulted in non-convergence of the model. The first phase of life was noted as an area for improvement.

A reliability model with finite range has been explored for this purpose; the model has already been used to represent the distribution of deaths for infancy (Chauhan, 1997) and under age five (Krishnamoorthy & Rajna, 1999). The statistical and other characteristics of this model were derived.

The new reliability finite range model was used to replace the first part of the Heligman-Pollard model. This provided stable estimates of the parameters. The new model also has one parameter less than the H-P model. The two new parameters, M and R, represent the scale and shape of early age mortality.

The seven parameters of the new mortality model were estimated by the weighted least squares technique. The estimated parameters were stable and their confidence intervals were provided. In most of the cases, the estimates were found to be highly statistically significant.

The model was successfully tested with data from Australia, Japan and Sweden. These countries were selected for reasons of the quality of data and their widely varying mortality experiences. In total, 26 data sets for these three countries were tested for the first part of model; later the same data sets were tested for the entire age range of the new mortality model. For all the countries, a comparative fitting of the unmodified H-P model has also been made. The new model fits the above data sets well. The error sum of squares were smaller than or similar to that of the H-P model. This suggests that the H-P model has been overparameterised and reduction in the number of parameters has not reduced the fitting ability with empirical data. The new model is more parsimonious and generates stable sets of parameters.

8.4 Fitting the new model to Indian data

The new mortality model when fitted to Indian data also provides good fits. Appendices 6A to 6F, showing the parameter estimates and their standard errors, suggest that the parameter estimates are reliable. In non-linear estimation, it is common that a small number of parameters are not statistically significant but they are included in the model as they contribute to the fitting. The new mortality model was fitted to 168 data sets for India, covering 28 years for males and females in rural and urban areas. Time series of parameters were generated. All the 168 fittings were statistically significant. As far as individual parameters are concerned, 88 per cent of the total of 1176 parameters were significant at the 5 per cent level of significance. If the level of significance is widened to 10 per cent, 97 per cent of the total estimated parameters become significant.

8.5 ARIMA forecasts of Indian mortality

The time series of estimated parameters were extended into the future by ARIMA techniques of time series analysis and forecasting. In fitting the ARIMA models to the parameters, most of the time series required the taking of first differences to achieve stationarity. Proper diagnostic tools were applied to decide the order selection of **p** and **q**; the basis for the selection of a particular forecasting model was more towards its capacity to forecast than its statistical characteristics. The forecasted ARIMA processes usually provided the coefficients of autoregressive and moving average components with their 95 per cent confidence limits.

The Lee-Carter method was also used to forecast Indian mortality. The mortality index of the Lee-Carter method, $\mathbf{k}_t$, was also modelled by the improved ARIMA process rather than by the ARIMA (0, 1, 0) model used by Lee-Carter. Standard ARIMA diagnostic tests and techniques were applied for model selection. The results provided by proper selection of model are robust and had smaller standard errors in every case than the (0,1,0) model.

8.6 Key findings

The new mortality model graduates mortality adequately with the use of seven parameters, it also provides stable sets of estimated parameters. It has been found that human mortality can be modelled successfully by the seven parameters. The new mortality model with seven parameters has been successful in modelling the widely ranging age patterns of mortality for Australia, India, Japan and Sweden. Accordingly, the model can be expected to be successfully applied to model mortality in other populations.

The parameters of the new mortality model can be used for forecasting mortality. The model demonstrated the utilities recommended by Keyfitz (1982), such as removing awkward irregularities of data and providing a comparable set of parameters which can be used for forecasting.

Life expectancy for India is forecasted by the new model to be 78.6 years for males and 79.2 years for females in 2020. Urban male and female life expectancy forecasts are about four years higher in 2020. The gap between rural and urban areas is 7.7 years for males and 6.3 years for females in 2020. Urban females are likely to be the longest-surviving group in India by 2020. The group with lowest expectation of life at birth is that of rural males. These 2020 rankings represent no change in the usual ranking of life expectancy in India.

In the thesis, the Lee-Carter method has been applied for the first time to data for a developing country. The Lee-Carter method provided expectations of life 5.9 years less for females and 8.9 years less for males compared to the estimates of the new mortality model. The Lee-Carter estimates for Indian mortality were comparable to those made by the United Nations and the Expert Committee on Population in India, essentially because all three methods are basically linear extrapolations.

8.7 Future prospects

A system of simultaneous estimation of the mortality parameters may provide a better landscape for mortality modelling. Independent estimation of all the parameters in a model yields values which are the outcome of the model but may not be appropriate as measures of the phenomenon. A simultaneous estimation system may provide solutions for the mortality curves that are coherent with the requirements. Multivariate ARIMA models could be used to do this, but caution must be taken with the multivariate ARIMA method, as the prerequisite of knowledge of the covariance structure and joint error structures could be crucial to the process.

Mortality parameterisation can also be tackled by the Bayesian approach to forecasting (Dellaportas *et al.*, 2001). In this approach, a prior and a current distribution of mortality supply the posterior distribution of mortality. This approach is highly forecasting-oriented, as future model estimation is a part of the theoretical basis of this approach. In the field of statistical analysis, the Bayesian approach has provided revolutionary outcomes and is widely used in reliability and other engineering life-testing analyses. In situations where the probability densities of mortality are known for societies, their future course can be determined by this approach. The statistical techniques needed for the task are highly sophisticated and the majority of work may include statistical inferences.

Epidemiological analysis of mortality for India is a basic need for mortality analysis in India. This requires cause specific death data, the non-availability of which has limited the comparison of the statistical findings in this study. Data collection in this area must be strengthened to see the true epidemiological transition in India. An integrated approach to incorporate vital registration and death certification requires political intervention and state intervention to implement it more strongly. Availability of cause-of-death data would enable a more thorough assessment of the likely reliability of statistical forecasts like those made in this thesis. It may also allow to do the multiple decrement life table analysis, by using tactical groups of cause-specific rates and fit them with Gompertz curve (Krishnamoorthy & Kulkarni, 1993).

Though SRS provides a good source of demographic data in India, we need to move on strengthening the vital registration system. In the twenty-first century, the emphasis is on human resource management and the improvement of the quality of the life. As mortality only measures the terminal end of the status of health, a systematic data collection on morbidity is also needed. Most needed in this area are good data on the 'causes of death' in India. The nature of epidemiological transition can never be determined without proper data sets. An integrated approach to the collection of morbidity data, registration of deaths and their medical certification might be initiated by an independent institution. Support from the widely available health care system, medical research institutions, population research institutions and statistical organisations may be sought for the establishment and smooth functioning of the proposed institution.

REFERENCES

Aarssen, K. and L. de Haan (1994). On the maximal life span of humans. *Mathematical Population Studies* 4: 259-281.

Andreev, K. (1999). Lexis (Version 1.1). Max Planck Institute for Demographic Research, Germany and Odense University, Denmark.

Anson, J. (1985). The parameters of death: a proposed parameterisation of the mortality curve. Ph.D. Dissertation, Department of Sociology, Brown University, Providence.

Anson, J. (1988). The parameters of death: a comparison of the quantity of information in a life table using a polynomial representation of the survivorship curve. *Statistics in Medicine* 7: 895-912.

Australia. Office of the Australian Government Actuary (1997). *Australian Life Tables 1995-97.* Canberra: Australian Government Publishing Service.

Benjamin, B. and A.S. Soliman (1993). *Mortality on the Move.* Oxford: City University.

Bhat, P.N.M., K. Navaneetham and S.I. Rajan (1995). Maternal mortality in India: estimates from a regression model. *Studies in Family Planning* 26(4): 217-232.

Biswas, S. (1995). *Applied Stochastic Processes: A Biostatistical and Population Oriented Approach.* New York: J. Wiley.

Booth, H., R.K. Chauhan, J. Maindonald and L. Smith (2000). The future of Australian mortality. Paper presented to 10th Biennial Conference of the Australian Population Association, Melbourne, 29 November - 1 December.

Booth, H., J. Maindonald and L. Smith (2001). Age-time interactions in mortality projection: applying Lee-Carter to Australia. Working Papers in Demography 85, Demography and Sociology Program, Research School of Social Sciences, The Australian National University, Canberra.

Box, G.E.P., G.N. Jenkins and G.C. Reinsel (1994). *Time Series Analysis: Forecasting and Control* (3rd edition). Englewood Cliffs, N.J.: Prentice Hall.

Brass, W. (1971). On the scale of mortality. In W. Brass (ed.), *Biological Aspects of Demography.* London: Taylor and Francis Ltd.

Carriere, J.F. (1992). Parametric models for life tables. *Transactions of the Society of Actuaries* 44: 77-99.

Carter, L.R. and R.D. Lee (1992). Modeling and forecasting US sex differentials in mortality. *International Journal of Forecasting* 8: 393-411.

Center on the Economics and Demography of Ageing (2001a). *Berkeley Mortality Database.* Department of Demography, University of California at Berkeley http://demog.berkeley.edu/wilmoth/mortality Accessed 1 January, 2001.

Center on the Economics and Demography of Ageing (2001b). *Berkeley Mortality Database.* Department of Demography, University of California at Berkeley http://demog.berkeley.edu/wilmoth/mortality Accessed 5 July, 2001.

Center on the Economics and Demography of Ageing (2001c). *Berkeley Mortality Database.* Department of Demography, University of California at Berkeley http://demog.berkeley.edu/wilmoth/mortality Accessed 19 October, 2001.

Chauhan, R.K. (1997). Graduation of infant deaths by age. *Demography India* 26(2): 261-274.

Chaurasia, A. (1993). A parametric investigation of mortality transition in India: 1970-89. *Journal of Institute of Economic Research* 28(1): 11-21.

Chiang, C.L. (1968). The life table and its construction. In C.L. Chiang (ed.), *Introduction to Stochastic Processes in Biostatistics.* New York: John Wiley & Sons.

Chiang, C.L. (1984). *The Life Table and Its Application.* Malabar: Robert E. Krieger Publishing Company.

Choe, M.K. (1981). Fitting the age pattern of infant and child mortality with the Weibull survival distribution. *Asian and Pacific Census Forum* 7(4): 10-13.

Coale, A.J. and P. Demeny (1967). *Regional Model Life Tables and Stable Populations.* Princeton: Princeton University Press.

Coale, A.J., P.G. Demeny and B. Vaughan (1983). *Regional Model Life Tables and Stable Populations* (2nd edition). New York: Academic Press.

Congdon, P. (1993). Statistical graduation in local demographic analysis and projection. *Journal of the Royal Statistical Society. Series A, Statistics in Society* 156: 237-270.

Davis, K. (1951). *The Population of India and Pakistan.* Princeton: Princeton University Press.

Dellaportas, P., A.F.M. Smith and P. Stavropoulos (2001). Bayesian analysis of mortality data. *Journal of the Royal Statistical Society. Series A, Statistics in Society* 164(2): 275-291.

Enders, W. (1995). *Applied Econometric Time Series.* New York: Wiley.

Ewbank, D.C., J.C. Gomez de Leon and M.A. Stoto (1983). A reducible four-parameter system of model life tables. *Population Studies* 37(1): 105-127.

Farnum, N.R. and L.W. Stanton (1989). *Quantitative Forecasting Methods.* Boston: PWS-Kent Pub. Co.

Forfar, D.O. and D.M. Smith (1987). The changing face of English life tables. *Transactions of the Faculty of Actuaries (Edinburgh)* 40: 98-134.

Fries, F.F. (1980). Ageing, natural death and compression of morbidity. *New England Journal of Medicine* 303: 130-135.

Gage, T.B. and C.J. Mode (1993). Some laws of mortality: how well do they fit? *Human Biology* 65(3): 445-461.

Gompertz, B. (1977). On the nature of the function expressive of the law of human mortality. In D. Smith and N. Keyfitz (eds), *Mathematical Demography: Selected Papers.* Berlin: Springer-Verlag.

Hamilton, J.D. (1994). *Time Series Analysis.* Princeton: Princeton University Press.

Hayflick, L. (1994). *How and Why We Age.* New York: Ballantine.

Heligman, L. and J.H. Pollard (1980). The age pattern of mortality. *Journal of the Institute of Actuaries* 107(1): 49-80.

Hood, G. (2001). Pop Tools (Version 2.35). Pest Animal Control Co-operative Research Centre, Wildlife and Ecology, CSIRO, Canberra.

Horiuchi, S. and J.R. Wilmoth (1998). Deceleration in the age pattern of mortality at older ages. *Demography* 35(4): 391-412.

Ihaka, R. and R. Gentleman (1996). R: A language for data analysis and graphics. *Journal of Computational and Graphical Statistics* 5(3): 299-314.

India. Office of the Registrar General (ORG) (1982). *Sample Registration System: 1976-78.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1984a). *Sample Registration System: 1979-80.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1984b). *SRS based abridged life tables 1970-75, Occasional Papers No.1.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1985a). *Sample Registration System: 1981.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1985b). *Sample Registration System: 1982.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1985c). *SRS Based Abridged Life Tables 1976-80.* Occasional Papers No.1. New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1986). *Sample Registration System: 1983.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1987). *Sample Registration System: 1984.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1988a). *Sample Registration System: 1985.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1988b). *Sample Registration System: 1986.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1989a). *Sample Registration System: 1987.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1989b). *SRS Based Abridged Life Tables 1981-85.* Occasional Papers No.1. New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1991). *Sample Registration System: 1988.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1992a). *Mortality Statistics of Causes of Death 1988.* New Delhi: Vital Statistics Division.

India. Office of the Registrar General (ORG) (1992b). *Sample Registration System: 1989.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1993a). *Sample Registration System, Fertility and Mortality Indicators: 1991.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1993b). *Sample Registration System: 1990.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1994a). *Sample Registration System, Fertility and Mortality Indicators: 1992.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1994b). *SRS based abridged life tables 1986-90.* Occasional Papers No.1. New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1995a). *Sample Registration System, Fertility and Mortality Indicators: 1993.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1995b). *Survey of Causes of Deaths (Rural).* New Delhi: Vital Statistics Division.

India. Office of the Registrar General (ORG) (1996). *Report of the Technical Group on Population Projections.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1997). *Sample Registration System, Fertility and Mortality Indicators: 1994.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1998a). *Sample Registration System Bulletin 1997.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1998b). *Sample Registration System: 1995.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1998c). *SRS Based Abridged Life Tables 1991-95.* Occasional Papers No.1. New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (1999). *Compendium of India's Fertility and Mortality Indicators (1971-1997), Based on the Sample Registration System.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (2000a). *Sample Registration System: 1998.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (2000b). *SRS Based Abridged Life Tables 1992-96 and 1993-97.* Occasional Papers No.1. New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

India. Office of the Registrar General (ORG) (2001a). *List of Tables: India.* Vital Statistics Division, Registrar General of India, Ministry of Home Affairs http://www.censusindia.net/results/2001provresults.html Accessed 1 July, 2001.

India. Office of the Registrar General (ORG) (2001b). *Provisional Population Totals: India. Census of India 2001, Paper 1.* New Delhi: Vital Statistics Division, Registrar General of India, Ministry of Home Affairs.

International Institute for Population Sciences (IIPS) and ORC Macro (1995). *National Family Health Survey (NFHS-1) 1991-92: India.* Mumbai: IIPS.

International Institute for Population Sciences (IIPS) and ORC Macro (2000). *National Family Health Survey (NFHS-2) 1998-99: India.* Mumbai: IIPS.

Ishak, M. (1992). *Mortality Models for Countries with Deficient Data.* Frankfurt am Main: Peter Lang.

Johnston, J. and J. DiNardo (1997). *Econometric Methods* (4th edition). New York: McGraw-Hill.

Kaneko, R. (1995). New relational models of age pattern of mortality and their application. Working Paper Series 21. Tokyo: Institute of Population Problems, Ministry of Health & Welfare.

Kannisto, V., J. Lauritsen, A.R. Thatcher and J.W. Vaupel (1994). Reductions in mortality at advanced ages: several decades of evidence from 27 countries. *Population and Development Review* 20(4): 793-810.

Keyfitz, N. (1982). Choice of function for mortality analysis: effective forecasting depends on a minimum parameter representation. *Theoretical Population Biology* 21(3): 329-352.

Kostaki, A. (1992). A nine-parameter version of the Heligman-Pollard formula. *Mathematical Population Studies* 3(4): 277-288.

Krishnamoorthy, S. (1982). On graduating infant and childhood mortality with mathematical curves. *Asian and Pacific Census Forum* 8(3): 14-15.

Krishnamoorthy, S. and P.M. Kulkarni (1993). Additions to Pollard's 'fun with Gompertz'. *Genus* 49(3-4): 205-208.

Krishnamoorthy, S. and P.N. Rajna (1999). Graduating childhood deaths and survivorship function with the model by Mukherjee and Islam. *Demography India* 28(1): 13-21.

Ledermann, S. (1969). *Nouvelles Tables Types de Mortalité.* Paris: Paris Presses Universitaires de France.

Lee, R.D. (2000). The Lee-Carter method for forecasting mortality, with various extensions and applications. *North American Actuarial Journal* 4(1): 80-91.

Lee, R.D. and L.R. Carter (1992). Modeling and forecasting U.S. mortality. *Journal of the American Statistical Association* 87(419): 659-671.

Lee, R.D. and T. Miller (2000). Assessing the performance of the Lee-Carter approach to modeling and forecasting mortality. Paper presented to 2000 meeting of the Population Association of America, Los Angeles, 23-25 March.

Lee, R.D. and T. Miller (2001). Evaluating the performance of the Lee-Carter method for forecasting mortality. *Demography* 38(4): 537-549.

Lee, R.D. and S. Tuljapurkar (1994). Stochastic population forecasts for the United States: beyond high, medium and low. *Journal of the Amrican Statistical Association* 89(428): 1175-1189.

Luginbuhl, R.C. and S.D. Schlotzhauer, eds (1987). *SAS/STAT Guide for Personal Computers, Version 6 Edition.* Cary, N.C.: SAS Institute.

Makeham, W.M. (1977). On the law of mortality. In D. Smith and N. Keyfitz (eds), *Mathematical Demography: Selected Papers.* Berlin: Springer-Verlag.

Mathew, A.C. (1997). Aptness and applications of Weibull law of mortality for infant and childhood ages. Ph.D. Thesis, Department of Statistics, Bharathiar University, Coimbatore, India.

McNay, K. (2000). Demographic and health status risk factors in childbearing among Indian women. Evidence from hospital data for the later stages of fertility decline. *Journal of Biosocial Science* 32(2): 191-206.

McNown, R. and A. Rogers (1989a). Forecasting mortality: a papameterized time series approach. *Demography* 26(4): 645-660.

McNown, R. and A. Rogers (1989b). Time-series analysis forecasts of a parameterised mortality schedule. In P. Congdon and P. Batey (eds), *Advances in Regional Demography: Information, Forecasts, Models.* London: Belhaven Press.

McNown, R. and A. Rogers (1992). Forecasting cause-specific mortality using time series methods. *International Journal of Forecasting* 8: 413-432.

Mitra, S. (1995). Boundary condition sets limits on Brass's model life table parameters. *Canadian Studies in Population* 22(1): 67-78.

Mitra, S. (1997). An adjustment to Brass's logit model of probabilities of survival. *Demography India* 26(1): 9-18.

Mode, C.J. and R.C. Busby (1982). An eight-parameter model of human mortality-the single decrement case. *Bulletin of Mathematical Biology* 44(5): 647-659.

Mukherjee, S.P. and A. Islam (1983). A finite range distribution of failure time. *Naval Research Logistic Quarterly* 30: 487-491.

Namboodiri, K. and C.M. Suchindran (1987). *Life Table Techniques and Their Applications.* Orlando: Academic Press.

Pathak, K.B., A. Pandey and U.S. Mishra (1991). On estimating current levels of fertility and child mortality from the data on open birth interval and survival status of the last child. *Janasamkhya* 9(1-2): 15-24.

Pathak, K.B. and F. Ram (1992). *Techniques of Demographic Analysis.* Bombay: Himalaya Publishing House.

Pathak, K.B. and F. Ram. (1994). *Completeness of Civil Registration in India and Some Major States.* IIPS Research Report Series. Bombay: International Institute for Population Sciences.

Petrioli, L. (1981). A new set of models of mortality. Paper presented to International Union for the Scientific Study of Population, Senegal, 7-10 July.

Pollard, J.H. (1987). Projection of age specific mortality rates. *Population Bulletin of the United Nations* 21/22: 55-69.

Rogers, A. (1986). Parameterized multistate population dynamics and projections. *Journal of the American Statistical Association* 81(393): 48-61.

Rogers, A. and K. Gard (1991). Application of the Heligman/Pollard Model Mortality Schedule. *Population Bulletin of the United Nations* 30: 79-105.

Ruzicka, L.T. (1984). Mortality in India: past trends and future prospects. In Tim Dyson and Nigel Crook (eds), *India's Demography: Essays on the Contemporary Population.* Atlantic Highlands: Humanities Press.

Ruzicka, L.T. (1986). Intersectoral aspects of mortality projections in developing countries. In H. Hansluwka, A.D. Lopez, Y. Porapakkham and P. Prasartkul (eds), *New Developments in the Analysis of Mortality and Causes of Death* (1st edition). Bangkok: Global Epidemiological Surveillance and Health Assessment W.H.O. ; Faculty of Public Health Institute for Population & Social Research Mahidol University.

Saboia, J.L.M. (1974). Modeling and forecasting populations by time series: the Swedish case. *Demography* 11(3): 483-492.

Sankar, C.V.R. (1996). Parametric life tables: applications of reliability models. Ph.D. Thesis, Department of Statistics, Andhra University, Visakhapatnam, India.

Seber, G.A.F. and C.J. Wild (1989). *Nonlinear Regression.* New York: Wiley.

Srivastava, S.S. (1989). Sample Registration System. In S.N. Singh, M.K. Premi, P.S. Bhatia and A. Bose (eds), *Population Transition in India.* Delhi: B.R. Publishing Corporation.

Tabeau, E., A.v.d. Berg Jeths and C. Heathcote (eds) (2001). *Forecasting Mortality in Developed Countries: Insights from a Statistical, Demographic and Epidemiological Perspective.* Dordrecht: Kluwer Academic Publishers.

Thatcher, A.R. (1999). The long-term pattern of adult mortality and the highest attained age. *Journal of the Royal Statistical Society. Series A, Statistics in Society* 162(1): 5-43.

Tsay, R.S. and G.C. Tiao (1984). Consistent estimates of autoregressive parameters and extended sample autocorrelation function for stationary and nonstationary ARMA models. *Journal of the American Statistical Association* 79(1): 84-96.

Tuljapurkar, S., N. Li and C. Boe (2000). A universal pattern of mortality decline in the G7 countries. *Nature* 15 June: 789-792.

UNAIDS (2000). *Report on the Global HIV/AIDS Epidemic.* Geneva.

United Nations (1955). *Age and Sex patterns of Mortality: Model Life Tables for Underdeveloped Countries.* Series A, Population Studies No. 22. New York.

United Nations (1982a). *Model Life Tables for Developing Countries.* ST/ESA /Ser. A/77. New York.

United Nations (1982b). *Population of India.* Country Monograph Series no.10. New York: Economic and Social Commission for Asia and the Pacific.

United Nations (1999). *World Population Prospects: the 1998 Revision; Volume I: Comprehensive Tables.* ST/ ESA/ Series A/ 177. New York: United Nations.

Vincent, P. (1951). La mortalité des vieillards. *Population* 6: 181-204.

Wei, W.W.S. (1990). *Time Series Analysis: Univariate and Multivariate Methods.* Redwood City, CA: Addison-Wesley.

Yuen, K.C. (1997). Comments on some parametric models for mortality tables. *Journal of Actuarial Practice* 5(2): 253-266.

Zaba, B. (1979). The four parameter logit life table system. *Population Studies* 33(1): 79-100.

APPENDIX 4A: FITTING THE CARRIERE MODEL TO THE USA DATA

Figure: Observed and fitted S_x by Carriere model for the USA, 1979-81

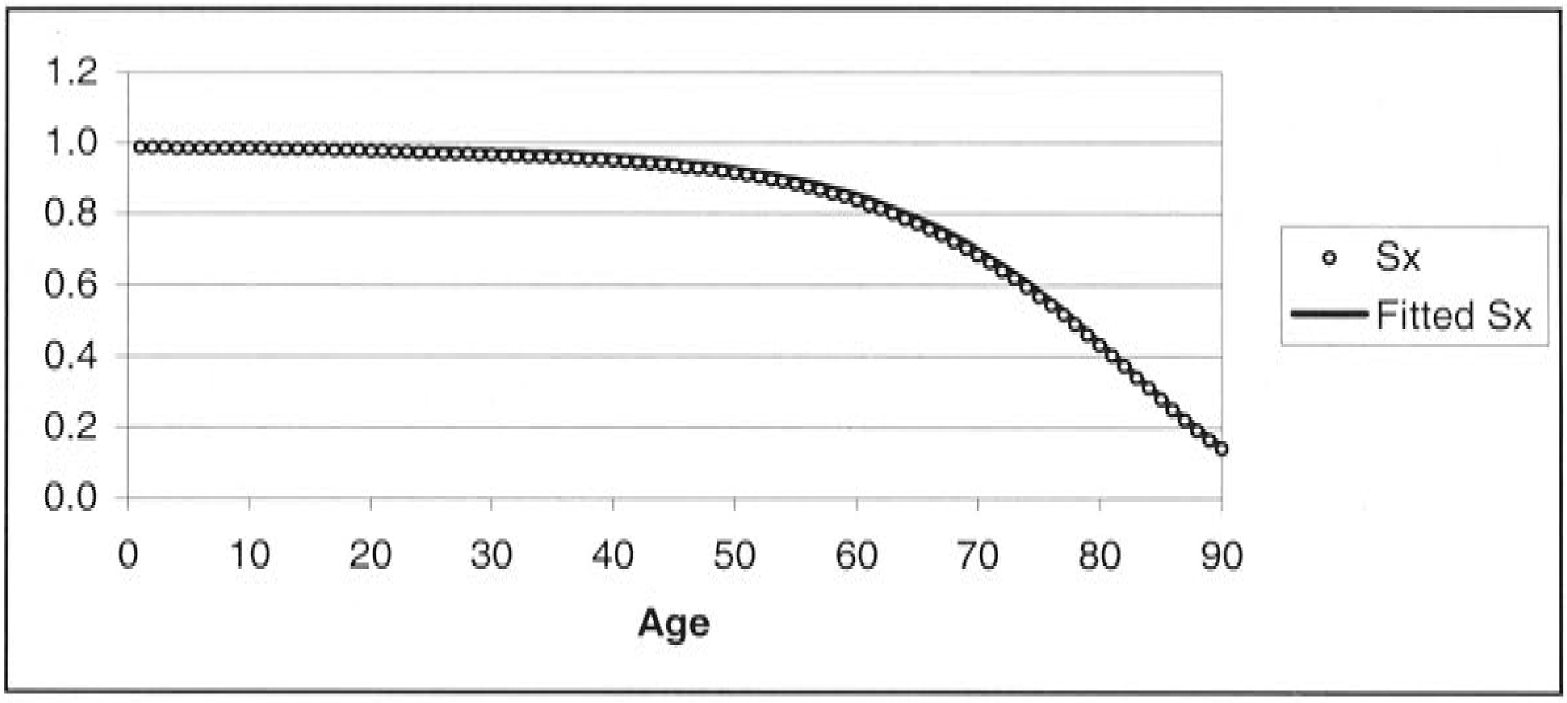
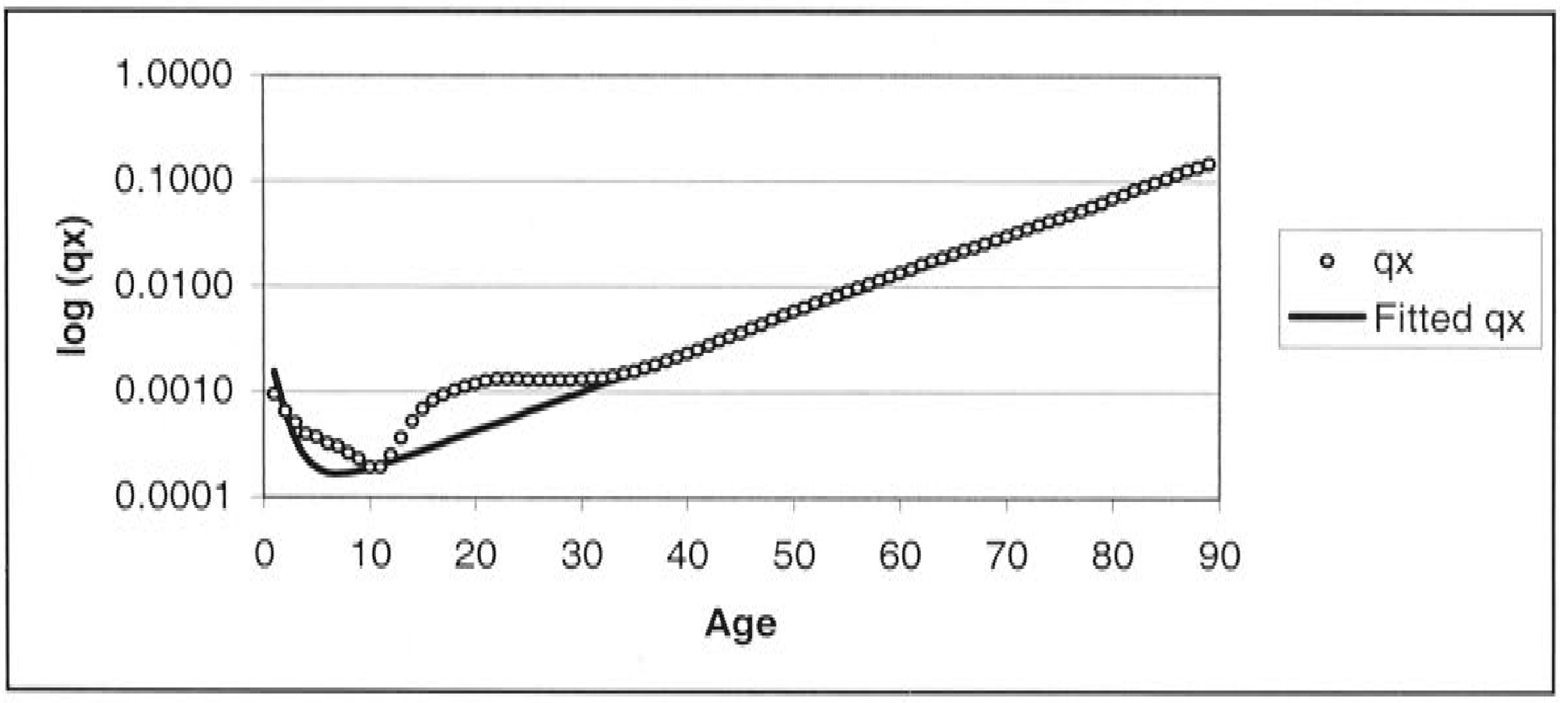


Figure: Observed and fitted q_x by Carriere model for the USA, 1979-81



APPENDIX 5A: STATISTICS OF FITTING THE NEW MODEL AND H-P MODEL FOR AUSTRALIA 1932-1992.

New Model, Australia, males, 1932-34

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	7.9328	1.1333	5453.96	<.0001
Residual	93	0.0208	0.000224		
Uncorrected Total	100	7.9536			
Corrected Total	99	7.3419			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00108	0.000170	0.000740	0.00141
E	3.5298	1.6991	0.1557	6.9039
F	27.1357	2.2107	22.7457	31.5257
G	0.000087	5.053E-6	0.000077	0.000097
H	1.0952	0.000739	1.0937	1.0966
M	0.1009	0.00501	0.0910	0.1109
R	0.1526	0.0156	0.1217	0.1836

weight $=(1/q)^{1.1}$

H-P Model, Australia, males, 1932-34

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.8560	12.4820	7971.98	<.0001
Residual	92	0.1440	0.00157		
Uncorrected Total	100	100.0			
Corrected Total	99	56.6001			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00723	0.000263	0.00670	0.00775
B	-0.9455	0.00847	-0.9624	-0.9287
C	0.1604	0.00463	0.1512	0.1696
D	0.00125	0.000036	0.00118	0.00132
E	3.3867	0.3138	2.7634	4.0100
F	25.1177	0.4056	24.3121	25.9233
G	0.000097	3.36E-6	0.000091	0.000104
H	1.0936	0.000516	1.0926	1.0947

New Model, Australia, males, 1953-55

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	99.7270	14.2467	3572.13	<.0001
Residual	93	0.2730	0.00294		
Uncorrected Total	100	100.0			
Corrected Total	99	63.1995			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00119	0.000040	0.00112	0.00127
E	10.4055	0.6950	9.0255	11.7856
F	21.3979	0.1584	21.0834	21.7124
G	0.000063	1.79E-6	0.000059	0.000066
H	1.0997	0.000468	1.0987	1.1006
M	0.0523	0.00244	0.0474	0.0571
R	0.1004	0.00723	0.0860	0.1147

weight $=(1/q)^2$

H-P Model, Australia, males, 1953-55

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.7964	12.4745	5635.66	<.0001
Residual	92	0.2036	0.00221		
Uncorrected Total	100	100.0			
Corrected Total	99	63.1995			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00292	0.000113	0.00270	0.00315
B	-0.9673	0.00590	-0.9790	-0.9556
C	0.1344	0.00402	0.1264	0.1424
D	0.00123	0.000034	0.00116	0.00129
E	9.4694	0.5452	8.3866	10.5523
F	21.3144	0.1376	21.0412	21.5877
G	0.000065	1.631E-6	0.000062	0.000068
H	1.0992	0.000412	1.0984	1.1000

New Model, Australia, males, 1970-72

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	99.2290	14.1756	1372.59	<.0001
Residual	93	0.7710	0.00829		
Uncorrected Total	100	100.0			
Corrected Total	99	69.0482			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00163	0.000091	0.00144	0.00181
E	22.0921	1.9015	18.3160	25.8682
F	20.8459	0.1497	20.5486	21.1432
G	0.000067	2.639E-6	0.000062	0.000072
H	1.0981	0.000668	1.0968	1.0994
M	0.0419	0.00567	0.0307	0.0532
R	0.0636	0.0116	0.0406	0.0867

weight $=(1/q)^2$

H-P Model, Australia, males, 1970-72

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.2674	12.4084	1558.17	<.0001
Residual	92	0.7326	0.00796		
Uncorrected Total	100	100.0			
Corrected Total	99	69.0482			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00162	0.000113	0.00139	0.00184
B	-0.9816	0.00663	-0.9948	-0.9684
C	0.1222	0.00680	0.1087	0.1357
D	0.00163	0.000088	0.00145	0.00180
E	21.4075	1.7985	17.8355	24.9795
F	20.8295	0.1468	20.5380	21.1210
G	0.000069	2.755E-6	0.000063	0.000074
H	1.0977	0.000675	1.0964	1.0990

New Model, Australia, males, 1990-92

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	99.4401	14.2057	2055.34	<.0001
Residual	93	0.5599	0.00602		
Uncorrected Total	100	100.0			
Corrected Total	99	74.8048			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00108	0.000036	0.00100	0.00115
E	9.0809	0.5610	7.9669	10.1949
F	24.2143	0.2156	23.7861	24.6425
G	0.000028	1.254E-6	0.000026	0.000031
H	1.1042	0.000702	1.1028	1.1056
M	0.0176	0.00206	0.0135	0.0217
R	0.0630	0.0100	0.0431	0.0829

weight $=(1/q)^2$

H-P Model, Australia, males, 1990-92

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.4799	12.4350	2199.46	<.0001
Residual	92	0.5201	0.00565		
Uncorrected Total	100	100.0			
Corrected Total	99	74.8048			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000694	0.000044	0.000607	0.000781
B	-0.9746	0.00750	-0.9895	-0.9597
C	0.1121	0.00565	0.1009	0.1234
D	0.00108	0.000035	0.00101	0.00115
E	8.9273	0.5349	7.8648	9.9897
F	24.1440	0.2102	23.7265	24.5616
G	0.000029	1.274E-6	0.000027	0.000032
H	1.1038	0.000693	1.1024	1.1052

New Model, Australia, females, 1932-34

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	17.4053	2.4865	5521.09	<.0001
Residual	93	0.0413	0.000444		
Uncorrected Total	100	17.4466			
Corrected Total	99	14.7386			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00239	0.000129	0.00213	0.00265
E	1.8617	0.2991	1.2678	2.4556
F	41.7977	2.6326	36.5698	47.0255
G	0.000020	1.667E-6	0.000017	0.000023
H	1.1122	0.00106	1.1101	1.1144
M	0.0834	0.00248	0.0785	0.0883
R	0.1552	0.00899	0.1374	0.1731

weight $=(1/q)^{1.5}$

H-P Model, Australia, females, 1932-34

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.8513	12.4814	7722.12	<.0001
Residual	92	0.1487	0.00162		
Uncorrected Total	100	100.0			
Corrected Total	99	59.2335			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00671	0.000246	0.00622	0.00720
B	-0.9199	0.0113	-0.9424	-0.8974
C	0.1632	0.00448	0.1543	0.1721
D	0.00233	0.000068	0.00220	0.00247
E	1.8005	0.1405	1.5214	2.0796
F	37.9755	1.1508	35.6900	40.2611
G	0.000024	1.585E-6	0.000021	0.000027
H	1.1098	0.000905	1.1080	1.1116

New Model, Australia, females, 1953-55

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	166.4	23.7647	1478.00	<.0001
Residual	93	0.9499	0.0102		
Uncorrected Total	100	167.3			
Corrected Total	99	91.5292			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000333	0.000060	0.000213	0.000453
E	2.0017	0.7350	0.5422	3.4612
F	34.4182	5.1625	24.1664	44.6700
G	0.000025	2.491E-6	0.000020	0.000030
H	1.1078	0.00147	1.1049	1.1107
M	0.0426	0.00320	0.0363	0.0490
R	0.0938	0.0105	0.0729	0.1147

weight $=(1/q)^{2.1}$

H-P Model, Australia, females, 1953-55

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.6348	12.4543	3137.43	<.0001
Residual	92	0.3652	0.00397		
Uncorrected Total	100	100.0			
Corrected Total	99	62.1119			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00260	0.000161	0.00228	0.00292
B	-0.9448	0.0145	-0.9735	-0.9160
C	0.1442	0.00808	0.1282	0.1603
D	0.000462	0.000093	0.000276	0.000647
E	1.0170	0.4206	0.1816	1.8524
F	42.1556	10.3137	21.6716	62.6395
G	0.000023	2.011E-6	0.000019	0.000027
H	1.1089	0.00122	1.1065	1.1113

New Model, Australia, females, 1970-72

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	99.5025	14.2146	2009.57	<.0001
Residual	93	0.4975	0.00535		
Uncorrected Total	100	100.0			
Corrected Total	99	64.9988			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000378	0.000025	0.000329	0.000427
E	19.4237	2.4786	14.5016	24.3457
F	19.8161	0.2038	19.4113	20.2208
G	0.000032	1.087E-6	0.000030	0.000035
H	1.1019	0.000563	1.1008	1.1030
M	0.0319	0.00317	0.0256	0.0382
R	0.0681	0.00911	0.0500	0.0862

weight $=(1/q)^2$

H-P Model, Australia, females, 1970-72

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.5369	12.4421	2471.77	<.0001
Residual	92	0.4631	0.00503		
Uncorrected Total	100	100.0			
Corrected Total	99	64.9988			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00132	0.000074	0.00117	0.00147
B	-0.9781	0.00599	-0.9900	-0.9662
C	0.1194	0.00511	0.1092	0.1295
D	0.000385	0.000023	0.000339	0.000432
E	17.4257	2.1323	13.1908	21.6607
F	19.8290	0.1989	19.4339	20.2241
G	0.000033	1.123E-6	0.000031	0.000036
H	1.1014	0.000562	1.1003	1.1026

New Model, Australia, females, 1990-92

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	99.6504	14.2358	3014.35	<.0001
Residual	93	0.3496	0.00376		
Uncorrected Total	100	100.0			
Corrected Total	99	68.3387			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000293	9.724E-6	0.000274	0.000313
E	6.7521	0.5076	5.7441	7.7601
F	23.0440	0.2600	22.5277	23.5603
G	0.000012	4.326E-7	0.000011	0.000012
H	1.1102	0.000587	1.1090	1.1113
M	0.0131	0.000965	0.0111	0.0150
R	0.0764	0.00794	0.0606	0.0922

weight $=(1/q)^2$

H-P Model, Australia, females, 1990-92

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.6688	12.4586	3460.35	<.0001
Residual	92	0.3312	0.00360		
Uncorrected Total	100	100.0			
Corrected Total	99	68.3387			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000568	0.000028	0.000513	0.000623
B	-0.9759	0.00575	-0.9873	-0.9645
C	0.1044	0.00414	0.0962	0.1127
D	0.000297	9.445E-6	0.000279	0.000316
E	6.4158	0.4857	5.4512	7.3804
F	23.0206	0.2577	22.5088	23.5323
G	0.000012	4.329E-7	0.000011	0.000013
H	1.1100	0.000580	1.1088	1.1111

APPENDIX 5B: STATISTICS OF FITTING THE NEW MODEL AND H-P MODEL FOR JAPAN 1960-1996.

New Model, Japan, males, 1960

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	10.8187	1.5455	5792.11	<.0001
Residual	94	0.0277	0.000295		
Uncorrected Total	101	10.8464			
Corrected Total	100	10.2682			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00135	0.000177	0.00100	0.00170
E	11.7799	3.3648	5.0989	18.4608
F	24.4973	0.7221	23.0635	25.9311
G	0.000068	3.011E-6	0.000062	0.000074
H	1.1001	0.000581	1.0990	1.1013
M	0.0686	0.00363	0.0614	0.0758
R	0.1219	0.0148	0.0925	0.1512

weight $=(1/q)^{1.2}$

H-P Model, Japan, males, 1960

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	100.5	12.5670	2517.79	<.0001
Residual	93	0.4642	0.00499		
Uncorrected Total	101	101.0			
Corrected Total	100	67.0352			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00501	0.000263	0.00448	0.00553
B	-0.9372	0.0134	-0.9639	-0.9105
C	0.1579	0.00537	0.1472	0.1686
D	0.00153	0.000059	0.00141	0.00165
E	9.5140	0.8120	7.9014	11.1265
F	24.7133	0.2530	24.2110	25.2157
G	0.000062	2.783E-6	0.000057	0.000068
H	1.1015	0.000718	1.1000	1.1029

New Model, Japan, males, 1969

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	34.2584	4.8941	540.19	<.0001
Residual	98	0.8816	0.00900		
Uncorrected Total	105	35.1400			
Corrected Total	104	30.0393			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000766	0.000146	0.000477	0.00106
E	12.2742	5.0050	2.3418	22.2066
F	23.0810	1.0057	21.0851	25.0768
G	0.000062	5.52E-6	0.000051	0.000073
H	1.0983	0.00128	1.0958	1.1009
M	0.0335	0.00420	0.0251	0.0418
R	0.1233	0.0286	0.0666	0.1800

weight $=(1/q)^{1.7}$

H-P Model, Japan, males, 1969

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	103.2	12.9055	713.05	<.0001
Residual	97	1.7556	0.0181		
Uncorrected Total	105	105.0			
Corrected Total	104	71.9473			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00208	0.000216	0.00165	0.00251
B	-0.9605	0.0190	-0.9982	-0.9227
C	0.1234	0.00993	0.1037	0.1431
D	0.000820	0.000075	0.000671	0.000969
E	10.8799	2.2024	6.5086	15.2511
F	23.3296	0.5383	22.2612	24.3980
G	0.000059	4.285E-6	0.000051	0.000068
H	1.0991	0.00117	1.0967	1.1014

New Model, Japan, males, 1978

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	19.8313	2.8330	1666.44	<.0001
Residual	99	0.1842	0.00186		
Uncorrected Total	106	20.0155			
Corrected Total	105	18.7866			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000612	0.000106	0.000402	0.000823
E	15.9020	5.3962	5.1947	26.6094
F	21.4428	0.7031	20.0478	22.8379
G	0.000037	2.228E-6	0.000032	0.000041
H	1.1030	0.000820	1.1014	1.1046
M	0.0198	0.00188	0.0161	0.0235
R	0.1366	0.0258	0.0854	0.1878

weight $=(1/q)^{1.5}$

H-P Model, Japan, males 1978

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	104.8	13.0981	1056.01	<.0001
Residual	98	1.2155	0.0124		
Uncorrected Total	106	106.0			
Corrected Total	105	75.7831			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00148	0.000139	0.00121	0.00176
B	-0.9179	0.0279	-0.9733	-0.8625
C	0.1303	0.00843	0.1136	0.1470
D	0.000649	0.000047	0.000556	0.000743
E	15.4358	2.1483	11.1726	19.6990
F	21.4503	0.2986	20.8577	22.0429
G	0.000038	2.067E-6	0.000034	0.000042
H	1.1023	0.000882	1.1005	1.1040

New Model, Japan, males, 1987

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.3495	1.1928	2511.78	<.0001
Residual	96	0.0519	0.000541		
Uncorrected Total	103	8.4013			
Corrected Total	102	8.1978			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000543	0.000143	0.000260	0.000827
E	14.3713	7.4019	-0.3215	29.0641
F	21.7744	1.1278	19.5357	24.0131
G	0.000028	1.851E-6	0.000025	0.000032
H	1.1038	0.000833	1.1022	1.1055
M	0.0118	0.00174	0.00837	0.0153
R	0.1507	0.0454	0.0606	0.2408

weight $=(1/q)^{1.2}$

H-P Model, Japan, males, 1987

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	101.9	12.7315	1053.85	<.0001
Residual	95	1.1477	0.0121		
Uncorrected Total	103	103.0			
Corrected Total	102	75.6931			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00112	0.000126	0.000873	0.00137
B	-0.8363	0.0496	-0.9347	-0.7379
C	0.1412	0.00962	0.1221	0.1603
D	0.000574	0.000038	0.000500	0.000649
E	13.9156	1.6629	10.6143	17.2168
F	21.8685	0.2871	21.2986	22.4384
G	0.000029	1.567E-6	0.000026	0.000032
H	1.1032	0.000877	1.1015	1.1050

New Model, Japan, males, 1996

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	17.2270	2.4610	2365.45	<.0001
Residual	98	0.1133	0.00116		
Uncorrected Total	105	17.3403			
Corrected Total	104	16.5171			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000455	0.000052	0.000351	0.000559
E	8.2142	1.9616	4.3214	12.1069
F	23.6544	0.7625	22.1411	25.1676
G	0.000017	1.032E-6	0.000015	0.000019
H	1.1094	0.000785	1.1078	1.1109
M	0.00933	0.000811	0.00772	0.0109
R	0.1493	0.0254	0.0988	0.1997

weight $=(1/q)^{1.2}$

H-P Model, Japan, males, 1996

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	104.3	13.0391	1839.76	<.0001
Residual	97	0.6875	0.00709		
Uncorrected Total	105	105.0			
Corrected Total	104	78.2093			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000905	0.000084	0.000738	0.00107
B	-0.8229	0.0414	-0.9051	-0.7406
C	0.1402	0.00776	0.1248	0.1556
D	0.000475	0.000023	0.000430	0.000520
E	11.3227	1.0380	9.2625	13.3829
F	22.3989	0.2476	21.9075	22.8903
G	0.000023	9.836E-7	0.000021	0.000025
H	1.1055	0.000687	1.1041	1.1068

New Model, Japan, females, 1960

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	12.8589	1.8370	2017.96	<.0001
Residual	97	0.0977	0.00101		
Uncorrected Total	104	12.9566			
Corrected Total	103	12.2975			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000961	0.000155	0.000652	0.00127
E	5.9792	2.5302	0.9573	11.0011
F	29.0956	1.8171	25.4891	32.7021
G	0.000029	2.403E-6	0.000024	0.000034
H	1.1074	0.00107	1.1053	1.1095
M	0.0566	0.00493	0.0468	0.0664
R	0.1182	0.0219	0.0748	0.1617

weight $=(1/q)^{1.3}$

H-P Model, Japan, females, 1960

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	103.1	12.8893	1397.29	<.0001
Residual	96	0.8856	0.00922		
Uncorrected Total	104	104.0			
Corrected Total	103	70.3296			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00461	0.000347	0.00392	0.00530
B	-0.9086	0.0245	-0.9572	-0.8601
C	0.1653	0.00743	0.1506	0.1801
D	0.00111	0.000052	0.00101	0.00122
E	5.1027	0.6415	3.8292	6.3761
F	28.9955	0.6876	27.6306	30.3604
G	0.000027	2.208E-6	0.000023	0.000032
H	1.1081	0.00121	1.1057	1.1105

New Model, Japan, females, 1969

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	9.1147	1.3021	1689.93	<.0001
Residual	96	0.0838	0.000873		
Uncorrected Total	103	9.1985			
Corrected Total	102	8.9361			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000460	0.000138	0.000186	0.000735
E	5.2550	4.3313	-3.3427	13.8527
F	29.0390	3.7299	21.6351	36.4429
G	0.000019	1.823E-6	0.000015	0.000022
H	1.1107	0.00123	1.1083	1.1132
M	0.0258	0.00343	0.0190	0.0326
R	0.1254	0.0378	0.0502	0.2005

weight $=(1/q)^{1.2}$

H-P Model, Japan, females, 1969

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	102.4	12.8001	2030.70	<.0001
Residual	95	0.5988	0.00630		
Uncorrected Total	103	103.0			
Corrected Total	102	69.3076			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00173	0.000111	0.00151	0.00195
B	-0.9448	0.0145	-0.9735	-0.9160
C	0.1277	0.00593	0.1159	0.1395
D	0.000565	0.000033	0.000499	0.000631
E	3.1802	0.4998	2.1879	4.1725
F	31.8552	1.3692	29.1369	34.5735
G	0.000015	1.165E-6	0.000013	0.000017
H	1.1136	0.00113	1.1113	1.1158

New Model, Japan, females, 1978

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	9.7482	1.3926	1336.82	<.0001
Residual	99	0.1181	0.00119		
Uncorrected Total	106	9.8663			
Corrected Total	105	9.6857			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000307	0.000119	0.000071	0.000543
E	4.4615	5.1198	-5.6975	14.6204
F	30.4194	6.0360	18.4426	42.3962
G	9.247E-6	1.09E-6	7.085E-6	0.000011
H	1.1170	0.00146	1.1142	1.1199
M	0.0156	0.00290	0.00984	0.0213
R	0.1394	0.0553	0.0297	0.2491

weight $=(1/q)^{1.2}$

H-P Model, Japan, females, 1978

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	105.0	13.1291	1330.67	<.0001
Residual	98	0.9669	0.00987		
Uncorrected Total	106	106.0			
Corrected Total	105	72.8736			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00119	0.000107	0.000975	0.00140
B	-0.9155	0.0270	-0.9691	-0.8618
C	0.1262	0.00813	0.1101	0.1424
D	0.000438	0.000047	0.000345	0.000531
E	2.0424	0.4674	1.1149	2.9698
F	38.2573	3.7514	30.8127	45.7019
G	6.482E-6	7.175E-7	5.059E-6	7.906E-6
H	1.1215	0.00155	1.1184	1.1245

New Model, Japan, females, 1987

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.7853	1.2550	1539.75	<.0001
Residual	100	0.0936	0.000936		
Uncorrected Total	107	8.8789			
Corrected Total	106	8.7423			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000275	0.000087	0.000102	0.000448
E	2.9589	3.2065	-3.4026	9.3205
F	34.1290	8.9117	16.4484	51.8096
G	5.457E-6	6.669E-7	4.134E-6	6.78E-6
H	1.1200	0.00147	1.1170	1.1229
M	0.00985	0.00197	0.00595	0.0138
R	0.1476	0.0611	0.0264	0.2689

H-P Model, Japan, females, 1987

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	105.6	13.2007	937.43	<.0001
Residual	99	1.3941	0.0141		
Uncorrected Total	107	107.0			
Corrected Total	106	74.6018			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000783	0.000104	0.000576	0.000990
B	-0.8915	0.0468	-0.9843	-0.7987
C	0.1258	0.0132	0.0996	0.1521
D	0.000616	0.000282	0.000056	0.00118
E	0.8119	0.3806	0.0567	1.5670
F	74.1710	36.9538	0.8463	147.5
G	3.374E-6	5.794E-7	2.224E-6	4.523E-6
H	1.1257	0.00226	1.1212	1.1302

New Model, Japan, females, 1996

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	104.5	14.9295	791.62	<.0001
Residual	99	1.4935	0.0151		
Uncorrected Total	106	106.0			
Corrected Total	105	73.1487			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.5089	5.9382	-11.2738	12.2916
E	0.1575	0.2427	-0.3240	0.6391
F	27811.6	296850	-561206	616830
G	1.016E-6	2.808E-7	4.588E-7	1.573E-6
H	1.1387	0.00347	1.1318	1.1455
M	0.00793	0.000556	0.00682	0.00903
R	0.1422	0.0203	0.1020	0.1824

weight $=(1/q)^2$

H-P Model, Japan, females, 1996

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	104.2	13.0271	716.05	<.0001
Residual	98	1.7829	0.0182		
Uncorrected Total	106	106.0			
Corrected Total	105	73.1487			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000637	0.000112	0.000415	0.000859
B	-0.8792	0.0743	-1.0267	-0.7317
C	0.1251	0.0229	0.0797	0.1704
D	0.0130	0.1057	-0.1968	0.2228
E	0.1550	0.3260	-0.4920	0.8019
F	3973.5	41667.8	-78715.3	86662.3
G	1.775E-6	4.403E-7	9.01E-7	2.649E-6
H	1.1315	0.00319	1.1251	1.1378

APPENDIX 5C: STATISTICS OF FITTING THE NEW MODEL AND H-P MODEL FOR SWEDEN 1891-1981.

New Model, Sweden, males, 1891

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.6278	1.2325	500.24	<.0001
Residual	93	0.2213	0.00238		
Uncorrected Total	100	8.8491			
Corrected Total	99	7.3631			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00335	0.000969	0.00142	0.00527
E	5.2231	4.3356	-3.3866	13.8328
F	30.4893	3.4718	23.5949	37.3836
G	0.000090	0.000017	0.000056	0.000124
H	1.0955	0.00242	1.0907	1.1003
M	0.3725	0.0369	0.2992	0.4457
R	0.2349	0.0310	0.1734	0.2964

weight $=(1/q)^{1.1}$

H-P Model, Sweden, males, 1891

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	98.9256	12.3657	1058.88	<.0001
Residual	92	1.0744	0.0117		
Uncorrected Total	100	100.0			
Corrected Total	99	46.1309			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.0475	0.00572	0.0362	0.0589
B	-0.7196	0.0985	-0.9152	-0.5239
C	0.2704	0.0201	0.2305	0.3102
D	0.00514	0.000236	0.00467	0.00561
E	2.4846	0.4397	1.6114	3.3579
F	30.3987	1.0871	28.2395	32.5578
G	0.000077	0.000012	0.000054	0.000100
H	1.0973	0.00211	1.0931	1.1015

New Model, Sweden, males, 1921

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.4498	1.2071	1327.73	<.0001
Residual	94	0.0876	0.000932		
Uncorrected Total	101	8.5374			
Corrected Total	100	7.5140			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00397	0.000426	0.00313	0.00482
E	2.0451	0.7713	0.5137	3.5764
F	31.1282	2.6359	25.8945	36.3618
G	0.000029	4.623E-6	0.000019	0.000038
H	1.1088	0.00201	1.1048	1.1128
M	0.1655	0.0142	0.1373	0.1937
R	0.1659	0.0270	0.1123	0.2196

weight $=(1/q)^{1.1}$

H-P Model, Sweden, males, 1921

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.9452	12.4931	1101.49	<.0001
Residual	93	1.0548	0.0113		
Uncorrected Total	101	101.0			
Corrected Total	100	50.9051			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.0106	0.000971	0.00870	0.0126
B	-0.9723	0.0141	-1.0003	-0.9442
C	0.1516	0.0126	0.1267	0.1765
D	0.00411	0.000173	0.00377	0.00445
E	3.0244	0.3852	2.2594	3.7893
F	27.9281	0.6599	26.6176	29.2385
G	0.000047	5.834E-6	0.000035	0.000058
H	1.1021	0.00178	1.0986	1.1057

New Model, Sweden, males, 1951

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	9.1729	1.3104	2241.93	<.0001
Residual	93	0.0603	0.000649		
Uncorrected Total	100	9.2333			
Corrected Total	99	8.7845			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000846	0.000149	0.000550	0.00114
E	3.6667	1.7428	0.2058	7.1276
F	26.0124	2.1929	21.6577	30.3670
G	0.000025	2.155E-6	0.000020	0.000029
H	1.1112	0.00114	1.1089	1.1134
M	0.0482	0.00467	0.0389	0.0575
R	0.1086	0.0264	0.0562	0.1611

weight $=(1/q)^{1.2}$

H-P Model, Sweden, males, 1951

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	98.8960	12.3620	1030.18	<.0001
Residual	92	1.1040	0.0120		
Uncorrected Total	100	100.0			
Corrected Total	99	61.8173			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00213	0.000170	0.00180	0.00247
B	-0.9928	0.00410	-1.0010	-0.9847
C	0.1019	0.00767	0.0866	0.1171
D	0.000839	0.000056	0.000729	0.000950
E	7.4472	1.2390	4.9864	9.9079
F	24.0647	0.5019	23.0679	25.0615
G	0.000031	2.34E-6	0.000027	0.000036
H	1.1076	0.00119	1.1052	1.1099

New Model, Sweden, males, 1981

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.0493	1.1499	5523.08	<.0001
Residual	93	0.0220	0.000237		
Uncorrected Total	100	8.0713			
Corrected Total	99	7.8594			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000628	0.000091	0.000448	0.000809
E	8.5156	2.5691	3.4139	13.6172
F	22.9752	0.8788	21.2300	24.7203
G	0.000035	1.596E-6	0.000032	0.000038
H	1.1040	0.000593	1.1028	1.1051
M	0.0159	0.00419	0.00763	0.0243
R	0.0544	0.0273	0.000248	0.1086

weight $=(1/q)^{1.2}$

H-P Model, Sweden, males, 1981

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.0316	12.3789	1176.01	<.0001
Residual	92	0.9684	0.0105		
Uncorrected Total	100	100.0			
Corrected Total	99	72.4225			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000396	0.000032	0.000332	0.000461
B	-0.9987	0.00111	-1.0009	-0.9965
C	0.0700	0.00703	0.0560	0.0840
D	0.000626	0.000040	0.000547	0.000705
E	11.0231	1.4905	8.0629	13.9834
F	22.9019	0.3568	22.1932	23.6107
G	0.000035	2.01E-6	0.000031	0.000039
H	1.1040	0.000937	1.1022	1.1059

New Model, Sweden, females, 1891

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	8.5033	1.2148	513.51	<.0001
Residual	93	0.2098	0.00226		
Uncorrected Total	100	8.7131			
Corrected Total	99	7.1614			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00320	0.00105	0.00112	0.00528
E	8.4988	7.5145	-6.4236	23.4213
F	35.5307	3.4236	28.7320	42.3294
G	0.000052	0.000011	0.000031	0.000073
H	1.1020	0.00262	1.0968	1.1072
M	0.4454	0.0389	0.3682	0.5227
R	0.3046	0.0305	0.2441	0.3651

weight $=(1/q)^{1.1}$

H-P Model, Sweden, females, 1891

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.2907	12.4113	1609.86	<.0001
Residual	92	0.7093	0.00771		
Uncorrected Total	100	100.0			
Corrected Total	99	42.3924			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.0457	0.00388	0.0380	0.0534
B	-0.7384	0.0777	-0.8926	-0.5842
C	0.2254	0.0127	0.2001	0.2506
D	0.00475	0.000236	0.00428	0.00522
E	3.2537	0.5735	2.1148	4.3926
F	34.8080	1.0805	32.6620	36.9541
G	0.000050	7.112E-6	0.000036	0.000064
H	1.1024	0.00194	1.0985	1.1063

New Model, Sweden, females, 1921

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	9.1838	1.3120	2694.06	<.0001
Residual	93	0.0449	0.000483		
Uncorrected Total	100	9.2288			
Corrected Total	99	7.8576			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00367	0.000217	0.00324	0.00410
E	1.7096	0.4133	0.8887	2.5304
F	32.5949	1.9365	28.7494	36.4403
G	0.000018	2.104E-6	0.000014	0.000022
H	1.1138	0.00145	1.1109	1.1167
M	0.1448	0.00823	0.1284	0.1611
R	0.1987	0.0183	0.1624	0.2350

weight $=(1/q)^{1.2}$

H-P Model, Sweden, females, 1921

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	99.4935	12.4367	2258.90	<.0001
Residual	92	0.5065	0.00551		
Uncorrected Total	100	100.0			
Corrected Total	99	46.2269			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00971	0.000703	0.00831	0.0111
B	-0.9580	0.0165	-0.9907	-0.9253
C	0.1486	0.0114	0.1259	0.1713
D	0.00392	0.000123	0.00368	0.00417
E	1.4326	0.1879	1.0594	1.8057
F	32.8226	1.1660	30.5068	35.1385
G	0.000017	2.177E-6	0.000013	0.000022
H	1.1142	0.00171	1.1108	1.1176

New Model, Sweden, females, 1951

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	16.2174	2.3168	1668.02	<.0001
Residual	93	0.1373	0.00148		
Uncorrected Total	100	16.3546			
Corrected Total	99	14.9118			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000558	0.000086	0.000388	0.000728
E	1.7931	0.9147	-0.0232	3.6095
F	34.8329	6.5078	21.9096	47.7562
G	0.000014	1.567E-6	0.000011	0.000017
H	1.1169	0.00147	1.1140	1.1198
M	0.0371	0.00383	0.0295	0.0447
R	0.0929	0.0186	0.0560	0.1298

weight $=(1/q)^{1.5}$

H-P Model, Sweden, females, 1951

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	98.4858	12.3107	747.99	<.0001
Residual	92	1.5142	0.0165		
Uncorrected Total	100	100.0			
Corrected Total	99	63.8405			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.00208	0.000258	0.00157	0.00260
B	-0.9551	0.0238	-1.0023	-0.9079
C	0.1408	0.0150	0.1110	0.1706
D	0.000604	0.000096	0.000413	0.000795
E	1.3031	0.5334	0.2438	2.3624
F	35.9661	6.9201	22.2221	49.7100
G	0.000014	2.235E-6	9.414E-6	0.000018
H	1.1171	0.00231	1.1125	1.1217

New Model, Sweden, females, 1981

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	7	5.1312	0.7330	2456.54	<.0001
Residual	93	0.0319	0.000343		
Uncorrected Total	100	5.1630			
Corrected Total	99	5.0841			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000325	0.000113	0.000101	0.000550
E	1.2330	1.3947	-1.5365	4.0025
F	39.9826	22.2490	-4.1998	84.1650
G	6.511E-6	8.061E-7	4.91E-6	8.112E-6
H	1.1208	0.00152	1.1178	1.1238
M	0.0259	0.0535	-0.0803	0.1321
R	0.0197	0.0514	-0.0824	0.1218

weight $=(1/q)^{1.1}$

H-P Model, Sweden, females, 1981

Source	DF	Sum of Squares	Mean Square	F Value	Approx Pr > F
Regression	8	96.4971	12.0621	316.80	<.0001
Residual	92	3.5029	0.0381		
Uncorrected Total	100	100.0			
Corrected Total	99	70.7575			

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
A	0.000191	0.000039	0.000113	0.000269
B	-1.0000	0.000021	-1.0000	-1.0000
C	0.0421	0.0179	0.00663	0.0776
D	0.000360	0.000225	-0.00009	0.000808
E	1.4718	1.2358	-0.9825	3.9262
F	49.6162	26.5122	-3.0395	102.3
G	6.032E-6	1.735E-6	2.586E-6	9.478E-6
H	1.1216	0.00397	1.1137	1.1295

APPENDIX 6A: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN MALES 1970-1997.

New Mortality Model India, males 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00461	0.00169	0.000728	0.00850
E	10.2322	11.0635	-15.2804	35.7449
F	26.3517	2.7232	20.0720	32.6314
G	0.000484	0.000167	0.000098	0.000870
H	1.0995	0.00642	1.0847	1.1143
M	0.3930	0.0807	0.2068	0.5792
R	0.0644	0.0204	0.0175	0.1114

weight $=(1/q)^2$ F Value = 81.26 Level of significance of F <0.0001

New Mortality Model India, males, 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00448	0.00429	-0.00523	0.0142
E	42.5051	93.3842	-168.7	253.8
F	25.0000	0	25.0000	25.0000
G	0.000462	0.000157	0.000108	0.000816
H	1.0997	0.00587	1.0865	1.1130
M	0.3535	0.0551	0.2289	0.4781
R	0.0789	0.0325	0.00537	0.1525

weight $=(1/q)^{1.2}$; F Value = 112.25; Level of significance of F <0.0001

New Mortality Model India, males, 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00440	0.00337	-0.00322	0.0120
E	10.3508	22.8359	-41.3079	62.0096
F	25.0000	0	25.0000	25.0000
G	0.000423	0.000150	0.000083	0.000763
H	1.1037	0.00624	1.0896	1.1179
M	0.3929	0.0800	0.2119	0.5739
R	0.0686	0.0328	-0.00574	0.1429

weight $=(1/q)^{1.3}$; F Value = 117.90; Level of significance of F <0.0001

New Mortality Model India, males, 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00399	0.00267	-0.00205	0.0100
E	5.5285	10.7163	-18.7136	29.7705
F	30.0000	0	30.0000	30.0000
G	0.000377	0.000136	0.000069	0.000684
H	1.1040	0.00614	1.0901	1.1179
M	0.3454	0.0375	0.2606	0.4302
R	0.0875	0.0275	0.0253	0.1497

weight $=(1/q)^{1.3}$; F Value = 167.73; Level of significance of F <0.0001

New Mortality Model India, males, 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00370	0.00165	-0.00010	0.00751
E	21.4071	24.3322	-34.7035	77.5177
F	23.5870	1.6634	19.7512	27.4228
G	0.000576	0.000121	0.000298	0.000854
H	1.0962	0.00402	1.0869	1.1055
M	0.3780	0.0738	0.2077	0.5483
R	0.0571	0.0171	0.0177	0.0965

weight $=(1/q)^{1.9}$; F Value = 124.80; Level of significance of F <0.0001

New Mortality Model India, males, 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00184	0.00264	-0.00413	0.00780
E	8.4810	37.2776	-75.8476	92.8096
F	25.0000	0	25.0000	25.0000
G	0.000614	0.000233	0.000088	0.00114
H	1.0960	0.00688	1.0804	1.1115
M	0.4120	0.1212	0.1378	0.6862
R	0.0577	0.0311	-0.0127	0.1281

weight $=(1/q)^{1.6}$; F Value = 78.99; Level of significance of F <0.0001

New Mortality Model India, males, 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00276	0.00183	-0.00146	0.00697
E	7.8882	16.9186	-31.1265	46.9030
F	24.6392	5.2969	12.4245	36.8539
G	0.000497	0.000169	0.000108	0.000886
H	1.1005	0.00620	1.0863	1.1148
M	0.3579	0.0575	0.2254	0.4904
R	0.0772	0.0240	0.0218	0.1327

weight $=(1/q)^{1.8}$; F Value = 88.06; Level of significance of F <0.0001

New Mortality Model India, males, 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00440	0.00279	-0.00190	0.0107
E	15.0000	0	15.0000	15.0000
F	23.4523	3.0785	16.4881	30.4165
G	0.000553	0.000135	0.000247	0.000860
H	1.0972	0.00424	1.0877	1.1068
M	0.3997	0.1100	0.1508	0.6485
R	0.0509	0.0267	-0.00943	0.1112

weight $=(1/q)^{1.2}$; F Value = 206.10; Level of significance of F <0.0001

New Mortality Model India, males, 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00496	0.00147	0.00157	0.00835
E	7.7319	6.6109	-7.5130	22.9768
F	29.2335	2.9709	22.3826	36.0844
G	0.000412	0.000119	0.000136	0.000687
H	1.1025	0.00505	1.0909	1.1142
M	0.3378	0.0386	0.2488	0.4267
R	0.0687	0.0143	0.0358	0.1016

weight $=(1/q)^{1.7}$; F Value = 206.10; Level of significance of F <0.0001

New Mortality Model India, males, 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00292	0.00259	-0.00295	0.00879
E	5.8667	16.3034	-31.0145	42.7479
F	25.0000	0	25.0000	25.0000
G	0.000438	0.000153	0.000093	0.000783
H	1.0984	0.00592	1.0850	1.1118
M	0.4025	0.1972	-0.0437	0.8487
R	0.0436	0.0376	-0.0414	0.1287

weight $=(1/q)^{1.2}$; F Value = 139.78; Level of significance of F <0.0001

New Mortality Model India, males, 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00257	0.00107	0.000142	0.00500
E	15.0000	0	15.0000	15.0000
F	22.8499	1.9905	18.3471	27.3527
G	0.000481	0.000094	0.000269	0.000693
H	1.0973	0.00383	1.0886	1.1059
M	0.3248	0.0623	0.1839	0.4658
R	0.0586	0.0167	0.0207	0.0965

weight $=(1/q)^{2}$; F Value = 145.80; Level of significance of F <0.0001

New Mortality Model India, males, 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00248	0.00111	-0.00003	0.00499
E	14.0428	15.7056	-21.4861	49.5716
F	25.0000	0	25.0000	25.0000
G	0.000467	0.000093	0.000257	0.000677
H	1.0979	0.00387	1.0891	1.1066
M	0.2949	0.0402	0.2040	0.3859
R	0.0743	0.0161	0.0379	0.1107

weight $=(1/q)^{2}$; F Value = 161.47; Level of significance of F <0.0001

New Mortality Model India, males, 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00549	0.00177	0.00140	0.00957
E	4.0392	2.9127	-2.6774	10.7559
F	32.4926	5.1101	20.7086	44.2765
G	0.000277	0.000123	-6.21E-6	0.000560
H	1.1052	0.00738	1.0882	1.1222
M	0.3086	0.0417	0.2124	0.4047
R	0.0591	0.0122	0.0310	0.0872

weight $=(1/q)^2$; F Value = 249.98; Level of significance of F <0.0001

New Mortality Model India, males, 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00670	0.00284	0.000141	0.0133
E	3.2302	3.0100	-3.7111	10.1714
F	33.4711	8.3140	14.2987	52.6434
G	0.000203	0.000142	-0.00012	0.000530
H	1.1115	0.0115	1.0849	1.1381
M	0.2936	0.0540	0.1692	0.4181
R	0.0621	0.0184	0.0196	0.1046

weight $=(1/q)^2$; F Value = 125.47; Level of significance of F <0.0001

New Mortality Model India, males, 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00538	0.00153	0.00186	0.00891
E	15.8265	10.4860	-8.3545	40.0074
F	25.6729	1.4540	22.3199	29.0259
G	0.000419	0.000116	0.000152	0.000686
H	1.0995	0.00526	1.0874	1.1117
M	0.3128	0.0630	0.1674	0.4581
R	0.0633	0.0192	0.0190	0.1076

weight $=(1/q)^2$; F Value = 89.32; Level of significance of F <0.0001

New Mortality Model India, males, 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00467	0.00165	0.000868	0.00848
E	7.0430	7.5571	-10.3840	24.4699
F	28.0977	3.6177	19.7553	36.4401
G	0.000341	0.000131	0.000038	0.000644
H	1.1031	0.00671	1.0876	1.1186
M	0.2766	0.0397	0.1852	0.3681
R	0.0747	0.0207	0.0269	0.1225

weight $=(1/q)^{1.7}$; F Value = 122.02; Level of significance of F <0.0001

New Mortality Model India, males, 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00410	0.00106	0.00165	0.00656
E	11.6694	8.7541	-8.5178	31.8565
F	24.6804	1.5612	21.0804	28.2805
G	0.000473	0.000109	0.000222	0.000725
H	1.0964	0.00437	1.0864	1.1065
M	0.2911	0.0576	0.1583	0.4239
R	0.0560	0.0166	0.0177	0.0942

weight $=(1/q)^2$; F Value = 132.69; Level of significance of F <0.0001

New Mortality Model India, males, 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00381	0.00104	0.00142	0.00620
E	14.9370	11.1263	-10.7204	40.5945
F	23.8121	1.2446	20.9419	26.6822
G	0.000416	0.000083	0.000224	0.000607
H	1.0981	0.00386	1.0892	1.1070
M	0.2737	0.0473	0.1646	0.3827
R	0.0606	0.0158	0.0241	0.0971

weight $=(1/q)^2$; F Value = 138.67; Level of significance of F <0.0001

New Mortality Model India, males, 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00370	0.00119	0.000948	0.00645
E	14.9744	12.2799	-13.3434	43.2922
F	24.9422	1.6689	21.0937	28.7907
G	0.000425	0.000106	0.000181	0.000670
H	1.0983	0.00476	1.0873	1.1093
M	0.2541	0.0461	0.1479	0.3604
R	0.0655	0.0184	0.0230	0.1081

weight $=(1/q)^2$; F Value = 106.81; Level of significance of F <0.0001

New Mortality Model India, males, 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00562	0.00131	0.00260	0.00865
E	3.2481	1.9154	-1.1689	7.6650
F	30.9002	4.2997	20.9849	40.8154
G	0.000229	0.000090	0.000021	0.000436
H	1.1081	0.00661	1.0928	1.1233
M	0.2290	0.0268	0.1672	0.2908
R	0.0660	0.0130	0.0360	0.0960

weight $=(1/q)^2$; F Value = 269.71; Level of significance of F <0.0001

New Mortality Model India, males, 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00482	0.000922	0.00270	0.00695
E	6.9170	3.9353	-2.1580	15.9919
F	26.9852	1.8602	22.6956	31.2749
G	0.000356	0.000099	0.000128	0.000585
H	1.1000	0.00500	1.0885	1.1116
M	0.1941	0.0235	0.1400	0.2483
R	0.0744	0.0153	0.0392	0.1097

weight $=(1/q)^2$; F Value = 183.14; Level of significance of F <0.0001

New Mortality Model India, males, 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00673	0.00114	0.00410	0.00936
E	6.6563	3.2911	-0.9332	14.2457
F	28.9175	1.9312	24.4642	33.3708
G	0.000256	0.000090	0.000049	0.000463
H	1.1063	0.00620	1.0920	1.1206
M	0.1940	0.0207	0.1462	0.2418
R	0.0871	0.0163	0.0496	0.1246

weight $=(1/q)^2$; F Value = 158.51; Level of significance of F <0.0001

New Mortality Model India, males, 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00490	0.00101	0.00257	0.00723
E	6.6153	4.1352	-2.9207	16.1512
F	26.3031	2.0593	21.5544	31.0518
G	0.000333	0.000104	0.000093	0.000573
H	1.1017	0.00568	1.0887	1.1148
M	0.1968	0.0290	0.1299	0.2636
R	0.0725	0.0185	0.0299	0.1151

weight $=(1/q)^2$; F Value = 134.68; Level of significance of F <0.0001

New Mortality Model India, males, 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00521	0.00110	0.00266	0.00775
E	4.0616	2.3299	-1.3113	9.4345
F	29.2414	3.3348	21.5512	36.9317
G	0.000228	0.000087	0.000027	0.000429
H	1.1077	0.00659	1.0925	1.1229
M	0.1863	0.0264	0.1255	0.2472
R	0.0654	0.0155	0.0297	0.1011

weight $=(1/q)^2$; F Value = 196.36; Level of significance of F <0.0001

New Mortality Model India, males, 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00846	0.00376	-0.00022	0.0171
E	2.0541	2.1180	-2.8301	6.9384
F	37.5156	14.3600	4.4010	70.6302
G	0.000122	0.000099	-0.00010	0.000349
H	1.1174	0.0125	1.0886	1.1462
M	0.2069	0.0521	0.0869	0.3270
R	0.0548	0.0265	-0.00625	0.1158

weight $=(1/q)^{1.6}$; F Value = 167.51; Level of significance of F <0.0001

New Mortality Model India, males, 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00689	0.00231	0.00157	0.0122
E	2.5685	1.8783	-1.7628	6.8999
F	34.5223	7.8526	16.4139	52.6307
G	0.000178	0.000108	-0.00007	0.000428
H	1.1095	0.00975	1.0870	1.1319
M	0.1817	0.0208	0.1337	0.2298
R	0.0766	0.0174	0.0365	0.1168

weight $=(1/q)^{1.9}$; F Value = 199.37; Level of significance of F <0.0001

New Mortality Model India, males, 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00440	0.00102	0.00206	0.00675
E	4.5213	3.2636	-3.0046	12.0473
F	31.1681	3.7418	22.5395	39.7967
G	0.000358	0.000082	0.000169	0.000546
H	1.0977	0.00361	1.0894	1.1061
M	0.1819	0.0172	0.1422	0.2216
R	0.0655	0.0151	0.0308	0.1003

weight $=(1/q)^{1.2}$; F Value = 877.59; Level of significance of F <0.0001

New Mortality Model India, males, 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00548	0.00111	0.00293	0.00803
E	6.8378	3.4560	-1.1317	14.8073
F	29.4778	2.1747	24.4629	34.4927
G	0.000357	0.000111	0.000100	0.000613
H	1.1004	0.00548	1.0877	1.1130
M	0.1937	0.0333	0.1170	0.2704
R	0.0533	0.0138	0.0214	0.0853

weight $=(1/q)^{2}$; F Value = 233.25; Level of significance of F <0.0001

APPENDIX 6B: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN FEMALES 1970-1997.

New Mortality Model India, females, 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0175	0.00259	0.0116	0.0235
E	3.9484	2.0044	-0.6739	8.5706
F	31.3986	2.9002	24.7107	38.0864
G	0.000099	0.000092	-0.00011	0.000310
H	1.1200	0.0156	1.0839	1.1560
M	0.3901	0.0698	0.2292	0.5510
R	0.0808	0.0256	0.0216	0.1399

weight $=(1/q)^2$; F Value = 52.60; Level of significance of F <0.0001

New Mortality Model India, females, 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0166	0.00384	0.00769	0.0254
E	3.8267	2.7266	-2.4609	10.1144
F	33.8050	5.0873	22.0734	45.5365
G	0.000095	0.000118	-0.00018	0.000368
H	1.1214	0.0204	1.0742	1.1685
M	0.3978	0.0883	0.1941	0.6014
R	0.0732	0.0285	0.00749	0.1389

weight $=(1/q)^{1.9}$; F Value = 45.23; Level of significance of F <0.0001

New Mortality Model India, females, 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0172	0.00262	0.0112	0.0233
E	4.6233	2.2563	-0.5798	9.8265
F	31.2846	2.6463	25.1823	37.3870
G	0.000142	0.000105	-0.00010	0.000384
H	1.1197	0.0127	1.0905	1.1489
M	0.4269	0.0638	0.2798	0.5741
R	0.0827	0.0213	0.0336	0.1318

weight $=(1/q)^2$; F Value = 77.01; Level of significance of F <0.0001

New Mortality Model India, females, 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0182	0.00189	0.0138	0.0225
E	4.2853	1.5096	0.8040	7.7665
F	32.4227	2.0577	27.6776	37.1677
G	0.000082	0.000052	-0.00004	0.000202
H	1.1258	0.0108	1.1008	1.1507
M	0.3907	0.0435	0.2903	0.4910
R	0.0870	0.0171	0.0475	0.1265

weight $=(1/q)^2$; F Value = 119.31; Level of significance of F <0.0001

New Mortality Model India, females, 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0125	0.00259	0.00656	0.0185
E	6.3850	4.2408	-3.3944	16.1643
F	28.4568	2.3196	23.1076	33.8059
G	0.000291	0.000192	-0.00015	0.000734
H	1.1039	0.0116	1.0771	1.1307
M	0.3946	0.0932	0.1798	0.6095
R	0.0702	0.0274	0.00693	0.1335

weight $=(1/q)^2$; F Value = 47.11; Level of significance of F <0.0001

New Mortality Model India, females, 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0132	0.00287	0.00667	0.0196
E	8.7909	5.3971	-3.4183	21.0001
F	25.0000	0	25.0000	25.0000
G	0.000541	0.000219	0.000046	0.00104
H	1.0950	0.00764	1.0777	1.1122
M	0.4447	0.1354	0.1384	0.7509
R	0.0593	0.0285	-0.00524	0.1238

weight $=(1/q)^2$; F Value = 52.74; Level of significance of F <0.0001

New Mortality Model India, females, 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0143	0.00145	0.0110	0.0177
E	7.4092	2.1873	2.3653	12.4531
F	29.9508	1.0844	27.4503	32.4514
G	0.000133	0.000051	0.000015	0.000251
H	1.1161	0.00672	1.1006	1.1316
M	0.3594	0.0320	0.2857	0.4331
R	0.0997	0.0151	0.0649	0.1344

weight $=(1/q)^2$; F Value = 120.63; Level of significance of F <0.0001

New Mortality Model India, females, 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0167	0.00191	0.0122	0.0211
E	6.6438	2.3360	1.2570	12.0306
F	29.1949	1.2560	26.2986	32.0913
G	0.000153	0.000073	-0.00002	0.000322
H	1.1157	0.00851	1.0961	1.1354
M	0.4023	0.0684	0.2445	0.5600
R	0.0698	0.0188	0.0265	0.1131

weight $=(1/q)^2$; F Value = 90.81; Level of significance of F <0.0001

New Mortality Model India, females, 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0123	0.00211	0.00741	0.0171
E	6.7479	3.6035	-1.5618	15.0577
F	28.0852	1.7545	24.0394	32.1310
G	0.000210	0.000114	-0.00005	0.000473
H	1.1094	0.00976	1.0869	1.1319
M	0.3862	0.0822	0.1967	0.5757
R	0.0699	0.0239	0.0148	0.1251

weight $=(1/q)^2$; F Value = 56.57; Level of significance of F <0.0001

New Mortality Model India, females, 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0116	0.00199	0.00698	0.0161
E	5.0571	2.9770	-1.8080	11.9222
F	30.3667	2.6394	24.2802	36.4532
G	0.000164	0.000097	-0.00006	0.000388
H	1.1110	0.00994	1.0881	1.1339
M	0.3836	0.0850	0.1876	0.5796
R	0.0572	0.0220	0.00642	0.1079

weight $=(1/q)^{1.7}$; F Value = 93.97; Level of significance of F <0.0001

New Mortality Model India, females, 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0126	0.00188	0.00829	0.0170
E	6.1233	2.8743	-0.5050	12.7516
F	28.4276	1.6794	24.5548	32.3005
G	0.000190	0.000106	-0.00005	0.000434
H	1.1080	0.00982	1.0854	1.1306
M	0.3355	0.0734	0.1663	0.5047
R	0.0668	0.0236	0.0124	0.1211

weight $=(1/q)^{1.7}$; F Value = 62.20; Level of significance of F <0.0001

New Mortality Model India, females, 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
Parameter	Estimate	Std Error	Approximate 95% Confidence Limits	
D	0.0143	0.00285	0.00772	0.0209
E	3.7343	2.9374	-3.0395	10.5081
F	32.2509	4.2589	22.4297	42.0721
G	0.000059	0.000052	-0.00006	0.000178
H	1.1282	0.0140	1.0958	1.1605
M	0.3163	0.0470	0.2080	0.4246
R	0.0817	0.0358	-0.00086	0.1642

weight $=(1/q)^{1.3}$; F Value = 85.34; Level of significance of F <0.0001

New Mortality Model India, females, 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00900	0.00199	0.00451	0.0135
E	10.9274	6.3841	-3.5147	25.3695
F	25.0000	0	25.0000	25.0000
G	0.000393	0.000133	0.000093	0.000694
H	1.0949	0.00637	1.0805	1.1093
M	0.3160	0.0749	0.1465	0.4855
R	0.0653	0.0242	0.0106	0.1200
weight $=(1/q)^2$; F Value = 64.42; Level of significance of F <0.0001				

New Mortality Model India, females, 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0123	0.00138	0.00909	0.0154
E	4.3575	1.6539	0.5436	8.1714
F	29.5024	1.8635	25.2052	33.7996
G	0.000110	0.000059	-0.00003	0.000247
H	1.1173	0.00924	1.0960	1.1386
M	0.2912	0.0427	0.1927	0.3897
R	0.0757	0.0193	0.0312	0.1202
weight $=(1/q)^2$; F Value = 100.01; Level of significance of F <0.0001				

New Mortality Model India, females, 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0110	0.00260	0.00511	0.0169
E	9.3478	5.9614	-4.1380	22.8336
F	25.0000	0	25.0000	25.0000
G	0.000255	0.000118	-0.00001	0.000521
H	1.1032	0.00872	1.0835	1.1229
M	0.3104	0.0643	0.1649	0.4559
R	0.0873	0.0321	0.0147	0.1599
weight $=(1/q)^2$; F Value = 38.96; Level of significance of F <0.0001				

New Mortality Model India, females, 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0109	0.00174	0.00691	0.0149
E	6.3764	3.1780	-0.9522	13.7049
F	27.9993	1.7042	24.0695	31.9292
G	0.000180	0.000095	-0.00004	0.000398
H	1.1086	0.00929	1.0871	1.1300
M	0.2930	0.0436	0.1925	0.3935
R	0.0898	0.0239	0.0346	0.1450
weight $=(1/q)^2$; F Value = 61.12; Level of significance of F <0.0001				

New Mortality Model India, females, 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0119	0.00154	0.00839	0.0155
E	5.0171	2.2078	-0.0740	10.1083
F	29.4839	1.8997	25.1031	33.8648
G	0.000085	0.000052	-0.00004	0.000205
H	1.1210	0.0107	1.0963	1.1457
M	0.2928	0.0538	0.1687	0.4169
R	0.0744	0.0233	0.0205	0.1282

weight $=(1/q)^2$; F Value = 66.30; Level of significance of F <0.0001

New Mortality Model India, females, 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0102	0.00222	0.00519	0.0152
E	10.3721	6.2066	-3.6682	24.4124
F	25.0000	0	25.0000	25.0000
G	0.000264	0.000110	0.000014	0.000513
H	1.1011	0.00790	1.0832	1.1190
M	0.2961	0.0908	0.0908	0.5015
R	0.0626	0.0305	-0.00635	0.1316

weight $=(1/q)^2$; F Value = 45.11; Level of significance of F <0.0001

New Mortality Model India, females, 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00955	0.00129	0.00659	0.0125
E	5.7175	2.3831	0.2221	11.2129
F	26.7335	1.4698	23.3440	30.1229
G	0.000146	0.000062	3.463E-6	0.000288
H	1.1119	0.00756	1.0944	1.1293
M	0.2606	0.0381	0.1728	0.3484
R	0.0824	0.0218	0.0320	0.1327

weight $=(1/q)^2$; F Value = 80.13; Level of significance of F <0.0001

New Mortality Model India, females, 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00989	0.00137	0.00672	0.0131
E	7.3744	2.8869	0.7172	14.0316
F	26.4114	1.1201	23.8284	28.9944
G	0.000144	0.000056	0.000014	0.000274
H	1.1099	0.00707	1.0935	1.1262
M	0.2328	0.0308	0.1618	0.3038
R	0.0920	0.0222	0.0407	0.1433

weight $=(1/q)^2$; F Value = 73.37; Level of significance of F <0.0001

New Mortality Model India, females, 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00965	0.00132	0.00660	0.0127
E	6.8765	2.6789	0.6990	13.0541
F	26.3791	1.2376	23.5253	29.2329
G	0.000173	0.000071	0.000010	0.000336
H	1.1073	0.00736	1.0903	1.1243
M	0.2133	0.0336	0.1359	0.2908
R	0.0812	0.0228	0.0285	0.1339

weight $=(1/q)^2$; F Value = 75.75; Level of significance of F <0.0001

New Mortality Model India, females, 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0109	0.00118	0.00817	0.0136
E	4.7204	1.6702	0.8689	8.5719
F	28.3873	1.4979	24.9331	31.8416
G	0.000078	0.000039	-0.00001	0.000168
H	1.1210	0.00882	1.1007	1.1413
M	0.2052	0.0238	0.1502	0.2601
R	0.0967	0.0218	0.0464	0.1469

weight $=(1/q)^2$; F Value = 86.58; Level of significance of F <0.0001

New Mortality Model India, females, 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0105	0.00134	0.00742	0.0136
E	4.7922	2.0734	0.0109	9.5735
F	28.6642	1.8575	24.3809	32.9476
G	0.000086	0.000048	-0.00003	0.000197
H	1.1203	0.00981	1.0977	1.1430
M	0.2094	0.0246	0.1526	0.2662
R	0.1046	0.0250	0.0471	0.1621

weight $=(1/q)^2$; F Value = 71.48; Level of significance of F <0.0001

New Mortality Model India, females, 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00980	0.000961	0.00758	0.0120
E	5.8673	1.6533	2.0546	9.6799
F	26.8418	0.9619	24.6236	29.0599
G	0.000102	0.000035	0.000022	0.000183
H	1.1159	0.00608	1.1019	1.1299
M	0.1859	0.0183	0.1436	0.2282
R	0.0970	0.0180	0.0556	0.1385

weight $=(1/q)^2$; F Value = 119.27; Level of significance of F <0.0001

New Mortality Model India, females, 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0102	0.00146	0.00687	0.0136
E	4.2991	1.9559	-0.2112	8.8095
F	29.8473	2.4432	24.2132	35.4813
G	0.000096	0.000068	-0.00006	0.000253
H	1.1154	0.0121	1.0874	1.1434
M	0.1881	0.0338	0.1102	0.2660
R	0.0791	0.0252	0.0211	0.1372

weight $=(1/q)^2$; F Value = 63.99; Level of significance of F <0.0001

New Mortality Model India, females, 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00859	0.00135	0.00549	0.0117
E	3.9856	2.2290	-1.1546	9.1258
F	30.0137	2.8401	23.4643	36.5631
G	0.000077	0.000050	-0.00004	0.000193
H	1.1182	0.0109	1.0930	1.1434
M	0.1903	0.0235	0.1362	0.2444
R	0.0949	0.0261	0.0348	0.1550

weight $=(1/q)^{1.8}$; F Value = 76.95; Level of significance of F <0.0001

New Mortality Model India, females, 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00749	0.000694	0.00589	0.00909
E	7.6592	2.0780	2.8673	12.4510
F	27.1634	0.8143	25.2856	29.0412
G	0.000182	0.000041	0.000088	0.000276
H	1.1043	0.00398	1.0952	1.1135
M	0.1924	0.0148	0.1584	0.2265
R	0.0939	0.0129	0.0642	0.1235

weight $=(1/q)^2$; F Value = 219.84; Level of significance of F <0.0001

New Mortality Model India, females, 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00904	0.00121	0.00625	0.0118
E	7.8185	2.9534	1.0079	14.6290
F	26.6398	1.1296	24.0350	29.2447
G	0.000180	0.000068	0.000025	0.000336
H	1.1060	0.00676	1.0904	1.1216
M	0.1918	0.0314	0.1194	0.2641
R	0.0765	0.0212	0.0276	0.1255

weight $=(1/q)^2$; F Value = 84.89; Level of significance of F <0.0001

APPENDIX 6C: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN MALES, RURAL 1970-1997.

New Mortality Model India, males, rural 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00513	0.00203	0.000439	0.00981
E	10.1527	11.8229	-17.1113	37.4167
F	26.4042	2.9599	19.5787	33.2297
G	0.000530	0.000196	0.000078	0.000983
H	1.0985	0.00678	1.0829	1.1142
M	0.4179	0.0883	0.2144	0.6215
R	0.0639	0.0218	0.0138	0.1141

weight $=(1/q)^{1.9}$; F Value = 77.28; Level of significance of F <0.0001

New Mortality Model India, males, rural 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00440	0.00203	-0.00018	0.00898
E	15.0000	0	15.0000	15.0000
F	27.5100	2.5410	21.7618	33.2582
G	0.000576	0.000171	0.000189	0.000963
H	1.0960	0.00565	1.0832	1.1087
M	0.3961	0.0803	0.2143	0.5778
R	0.0681	0.0214	0.0198	0.1164

weight $=(1/q)^{2}$; F Value = 87.28; Level of significance of F <0.0001

New Mortality Model India, males, rural 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00663	0.00223	0.00147	0.0118
E	12.1050	10.9137	-13.0622	37.2722
F	26.3144	2.1873	21.2705	31.3582
G	0.000516	0.000196	0.000064	0.000968
H	1.1006	0.00716	1.0840	1.1171
M	0.4400	0.1025	0.2036	0.6764
R	0.0634	0.0225	0.0116	0.1152

weight $=(1/q)^{2}$; F Value = 65.86; Level of significance of F <0.0001

New Mortality Model India, males, rural 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00323	0.00219	-0.00172	0.00817
E	11.2488	21.6704	-37.7734	60.2709
F	25.0000	0	25.0000	25.0000
G	0.000566	0.000155	0.000216	0.000917
H	1.0975	0.00505	1.0861	1.1089
M	0.3705	0.0455	0.2676	0.4734
R	0.0874	0.0223	0.0369	0.1379

weight $=(1/q)^{1.7}$; F Value = 121.51; Level of significance of F <0.0001

New Mortality Model India, males, rural 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00484	0.00185	0.000580	0.00910
E	12.0924	12.2704	-16.2034	40.3882
F	26.1312	2.4243	20.5406	31.7217
G	0.000505	0.000139	0.000184	0.000827
H	1.0989	0.00499	1.0874	1.1104
M	0.3990	0.0709	0.2356	0.5625
R	0.0607	0.0186	0.0178	0.1035

weight $=(1/q)^{1.7}$; F Value = 132.19; Level of significance of F <0.0001

New Mortality Model India, males, rural 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00158	0.00242	-0.00389	0.00704
E	13.5390	53.0479	-106.5	133.5
F	25.0000	0	25.0000	25.0000
G	0.000809	0.000281	0.000172	0.00145
H	1.0913	0.00665	1.0763	1.1063
M	0.4499	0.1381	0.1375	0.7623
R	0.0559	0.0263	-0.00358	0.1154

weight $=(1/q)^{1.9}$; F Value = 63.64; Level of significance of F <0.0001

New Mortality Model India, males, rural 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00359	0.00360	-0.00471	0.0119
E	7.6224	23.6686	-46.9580	62.2028
F	26.8380	9.0192	6.0395	47.6366
G	0.000500	0.000234	-0.00004	0.00104
H	1.1014	0.00802	1.0829	1.1199
M	0.3802	0.0575	0.2476	0.5127
R	0.0829	0.0349	0.00249	0.1633

weight $=(1/q)^{1.3}$; F Value = 87.48; Level of significance of F <0.0001

New Mortality Model India, males, rural 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00484	0.00162	0.00117	0.00851
E	15.0000	0	15.0000	15.0000
F	23.2574	1.5395	19.7748	26.7401
G	0.000668	0.000145	0.000339	0.000997
H	1.0942	0.00424	1.0846	1.1038
M	0.4816	0.1585	0.1231	0.8401
R	0.0417	0.0186	-0.00053	0.0838

weight $=(1/q)^{2}$; F Value = 115.28; Level of significance of F <0.0001

New Mortality Model India, males, rural 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00766	0.00318	0.000322	0.0150
E	4.2874	4.5061	-6.1038	14.6786
F	34.5595	7.5610	17.1236	51.9954
G	0.000296	0.000161	-0.00008	0.000668
H	1.1082	0.00867	1.0882	1.1282
M	0.3580	0.0357	0.2756	0.4403
R	0.0740	0.0161	0.0370	0.1110
weight $=(1/q)^{1.5}$; F Value = 266.38; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00446	0.00258	-0.00137	0.0103
E	42.1862	53.1758	-78.1068	162.5
F	25.0000	0	25.0000	25.0000
G	0.000533	0.000137	0.000223	0.000842
H	1.0953	0.00462	1.0848	1.1057
M	0.4068	0.1049	0.1694	0.6442
R	0.0509	0.0221	0.000879	0.1009
weight $=(1/q)^{1.5}$; F Value = 130.04; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00291	0.00122	0.000148	0.00568
E	15.0000	0	15.0000	15.0000
F	23.2401	1.9812	18.7583	27.7219
G	0.000480	0.000103	0.000248	0.000712
H	1.0979	0.00419	1.0884	1.1073
M	0.3547	0.0724	0.1910	0.5184
R	0.0583	0.0176	0.0186	0.0981
weight $=(1/q)^{2}$; F Value = 127.65; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00375	0.00160	0.000133	0.00737
E	24.3428	19.9223	-20.7250	69.4106
F	25.0000	0	25.0000	25.0000
G	0.000506	0.000119	0.000236	0.000776
H	1.0970	0.00463	1.0865	1.1074
M	0.3300	0.0618	0.1903	0.4697
R	0.0677	0.0193	0.0240	0.1114
weight $=(1/q)^{2}$; F Value = 105.67; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00347	0.000987	0.00124	0.00570
E	6.0209	4.9526	-5.1826	17.2245
F	25.0000	0	25.0000	25.0000
G	0.000537	0.000111	0.000287	0.000787
H	1.0945	0.00388	1.0858	1.1033
M	0.3448	0.0613	0.2062	0.4834
R	0.0564	0.0155	0.0213	0.0916

weight $=(1/q)^2$; F Value = 195.08; Level of significance of F <0.0001

New Mortality Model India, males, rural 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00843	0.00284	0.00189	0.0150
E	3.0593	2.2701	-2.1756	8.2942
F	34.6684	7.3100	17.8113	51.5255
G	0.000161	0.000110	-0.00009	0.000416
H	1.1156	0.0112	1.0897	1.1415
M	0.3148	0.0471	0.2061	0.4234
R	0.0663	0.0163	0.0288	0.1038

weight $=(1/q)^2$; F Value = 154.08; Level of significance of F <0.0001

New Mortality Model India, males, rural 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00569	0.00184	0.00152	0.00986
E	18.9032	12.5525	-9.4929	47.2992
F	25.0000	0	25.0000	25.0000
G	0.000472	0.000124	0.000191	0.000753
H	1.0975	0.00513	1.0859	1.1091
M	0.3509	0.0809	0.1678	0.5339
R	0.0607	0.0206	0.0141	0.1074

weight $=(1/q)^2$; F Value = 87.73; Level of significance of F <0.0001

New Mortality Model India, males, rural 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00505	0.00122	0.00224	0.00785
E	14.9464	9.0057	-5.8209	35.7138
F	27.2719	1.4353	23.9620	30.5818
G	0.000420	0.000098	0.000194	0.000647
H	1.0998	0.00438	1.0897	1.1099
M	0.3046	0.0362	0.2211	0.3882
R	0.0786	0.0149	0.0443	0.1130

weight $=(1/q)^2$; F Value = 149.96; Level of significance of F <0.0001

New Mortality Model India, males, rural 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00431	0.00128	0.00136	0.00725
E	12.8435	10.3947	-11.1270	36.8141
F	25.0433	1.7043	21.1131	28.9736
G	0.000533	0.000135	0.000222	0.000843
H	1.0947	0.00478	1.0837	1.1057
M	0.3194	0.0675	0.1638	0.4749
R	0.0564	0.0177	0.0155	0.0973

weight $=(1/q)^2$; F Value = 111.45; Level of significance of F <0.0001

New Mortality Model India, males, rural 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00439	0.00160	0.000711	0.00807
E	12.6393	13.0002	-17.3395	42.6182
F	23.9319	1.9358	19.4678	28.3960
G	0.000416	0.000104	0.000177	0.000656
H	1.0985	0.00456	1.0880	1.1090
M	0.2976	0.0526	0.1763	0.4188
R	0.0645	0.0209	0.0163	0.1127

weight $=(1/q)^{1.7}$; F Value = 120.17; Level of significance of F <0.0001

New Mortality Model India, males, rural 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00389	0.00143	0.000585	0.00719
E	18.8706	16.1861	-18.4550	56.1963
F	24.4304	1.6174	20.7006	28.1601
G	0.000454	0.000114	0.000192	0.000716
H	1.0973	0.00482	1.0862	1.1084
M	0.2659	0.0426	0.1676	0.3642
R	0.0764	0.0199	0.0306	0.1223

weight $=(1/q)^2$; F Value = 90.70; Level of significance of F <0.0001

New Mortality Model India, males, rural 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00619	0.00176	0.00212	0.0103
E	4.0134	2.7972	-2.4371	10.4639
F	31.7787	4.7436	20.8399	42.7175
G	0.000237	0.000120	-0.00004	0.000513
H	1.1073	0.00848	1.0878	1.1269
M	0.2427	0.0310	0.1713	0.3141
R	0.0723	0.0153	0.0370	0.1076

weight $=(1/q)^2$; F Value = 171.86; Level of significance of F <0.0001

New Mortality Model India, males, rural 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00479	0.00110	0.00231	0.00727
E	13.7083	7.5259	-3.3165	30.7332
F	25.0000	0	25.0000	25.0000
G	0.000515	0.000097	0.000295	0.000735
H	1.0938	0.00365	1.0856	1.1021
M	0.2126	0.0287	0.1477	0.2774
R	0.0734	0.0162	0.0367	0.1101

weight $=(1/q)^2$; F Value = 176.47; Level of significance of F <0.0001

New Mortality Model India, males, rural 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00707	0.00111	0.00450	0.00964
E	6.8659	3.0913	-0.2628	13.9946
F	28.8975	1.7157	24.9411	32.8539
G	0.000300	0.000095	0.000080	0.000520
H	1.1038	0.00563	1.0908	1.1168
M	0.2125	0.0213	0.1633	0.2616
R	0.0843	0.0146	0.0507	0.1180

weight $=(1/q)^2$; F Value = 192.17; Level of significance of F <0.0001

New Mortality Model India, males, rural 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00509	0.00131	0.00207	0.00812
E	5.1927	4.0408	-4.1256	14.5110
F	27.1472	3.1416	19.9025	34.3918
G	0.000329	0.000135	0.000019	0.000640
H	1.1023	0.00727	1.0856	1.1191
M	0.2064	0.0278	0.1423	0.2706
R	0.0839	0.0213	0.0349	0.1329

weight $=(1/q)^2$; F Value = 108.45; Level of significance of F <0.0001

New Mortality Model India, males, rural 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00560	0.00113	0.00299	0.00821
E	6.7181	4.1594	-2.8736	16.3098
F	27.3796	2.1242	22.4811	32.2781
G	0.000287	0.000100	0.000057	0.000517
H	1.1051	0.00628	1.0906	1.1196
M	0.1954	0.0264	0.1345	0.2563
R	0.0798	0.0190	0.0359	0.1237

weight $=(1/q)^2$; F Value = 125.14; Level of significance of F <0.0001

New Mortality Model India, males, rural 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00759	0.00200	0.00298	0.0122
E	2.5891	1.5406	-0.9635	6.1418
F	33.0149	5.9367	19.3248	46.7049
G	0.000179	0.000099	-0.00005	0.000407
H	1.1118	0.00907	1.0909	1.1327
M	0.2242	0.0403	0.1314	0.3171
R	0.0544	0.0155	0.0186	0.0903
weight $=(1/q)^2$; F Value = 203.71; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00742	0.00228	0.00216	0.0127
E	2.7884	1.7613	-1.2732	6.8501
F	34.5618	6.6024	19.3366	49.7871
G	0.000192	0.000113	-0.00007	0.000452
H	1.1086	0.00942	1.0868	1.1303
M	0.1946	0.0202	0.1481	0.2412
R	0.0794	0.0152	0.0444	0.1144
weight $=(1/q)^2$; F Value = 215.15; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00442	0.000928	0.00228	0.00656
E	3.8302	1.5670	0.2167	7.4438
F	32.9079	3.1776	25.5802	40.2356
G	0.000373	0.000080	0.000187	0.000558
H	1.0973	0.00350	1.0892	1.1054
M	0.1958	0.0109	0.1707	0.2209
R	0.0678	0.00644	0.0529	0.0826
weight $=(1/q)^2$; F Value = 1159.23; Level of significance of F <0.0001				

New Mortality Model India, males, rural 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00522	0.00111	0.00267	0.00777
E	11.6408	6.0043	-2.2053	25.4870
F	28.6412	1.5661	25.0296	32.2528
G	0.000475	0.000120	0.000198	0.000752
H	1.0959	0.00460	1.0853	1.1065
M	0.2164	0.0422	0.1190	0.3137
R	0.0516	0.0150	0.0171	0.0862
weight $=(1/q)^2$; F Value = 192.87; Level of significance of F <0.0001				

APPENDIX 6D: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN FEMALES, RURAL 1970-1997.

New Mortality Model India, females, rural 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0163	0.00256	0.0104	0.0222
E	5.4082	2.7191	-0.8620	11.6785
F	29.3601	2.0929	24.5338	34.1865
G	0.000241	0.000160	-0.00013	0.000610
H	1.1076	0.0114	1.0813	1.1340
M	0.4290	0.0758	0.2542	0.6038
R	0.0773	0.0232	0.0239	0.1307

weight $=(1/q)^2$; F Value = 62.15; Level of significance of F <0.0001

New Mortality Model India, females, rural 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0179	0.00380	0.00913	0.0267
E	4.1387	2.6966	-2.0797	10.3572
F	33.0870	4.2489	23.2889	42.8851
G	0.000110	0.000133	-0.00020	0.000416
H	1.1196	0.0202	1.0731	1.1662
M	0.4210	0.0930	0.2066	0.6355
R	0.0764	0.0283	0.0111	0.1416

weight $=(1/q)^2$; F Value = 41.78; Level of significance of F <0.0001

New Mortality Model India, females, rural 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0194	0.00266	0.0133	0.0255
E	4.4040	2.0050	-0.2196	9.0275
F	31.4738	2.5316	25.6359	37.3117
G	0.000127	0.000094	-0.00009	0.000344
H	1.1222	0.0126	1.0930	1.1513
M	0.4534	0.0606	0.3138	0.5930
R	0.0876	0.0208	0.0396	0.1357

weight $=(1/q)^2$; F Value = 81.63; Level of significance of F <0.0001

New Mortality Model India, females, rural 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0206	0.00181	0.0164	0.0247
E	4.4956	1.3511	1.3800	7.6112
F	32.2726	1.6635	28.4365	36.1086
G	0.000077	0.000045	-0.00003	0.000180
H	1.1276	0.00986	1.1049	1.1504
M	0.4145	0.0391	0.3243	0.5047
R	0.0929	0.0157	0.0568	0.1290

weight $=(1/q)^2$; F Value = 138.94; Level of significance of F <0.0001

New Mortality Model India, females, rural 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0135	0.00296	0.00664	0.0203
E	10.9104	6.2432	-3.4866	25.3074
F	25.8323	1.3733	22.6655	28.9992
G	0.000447	0.000199	-0.00001	0.000906
H	1.0976	0.00822	1.0786	1.1165
M	0.4154	0.0873	0.2141	0.6167
R	0.0768	0.0266	0.0153	0.1382

weight $=(1/q)^2$; F Value = 45.14; Level of significance of F <0.0001

New Mortality Model India, females, rural 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0165	0.00287	0.00984	0.0231
E	5.0798	2.9637	-1.7546	11.9141
F	29.2392	2.5380	23.3864	35.0920
G	0.000239	0.000164	-0.00014	0.000618
H	1.1103	0.0118	1.0831	1.1375
M	0.4520	0.0893	0.2460	0.6579
R	0.0719	0.0256	0.0129	0.1309

weight $=(1/q)^{1.9}$; F Value = 61.73; Level of significance of F <0.0001

New Mortality Model India, females, rural 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0159	0.00153	0.0124	0.0195
E	7.3491	2.0548	2.6107	12.0875
F	29.9935	1.0276	27.6238	32.3633
G	0.000132	0.000051	0.000014	0.000250
H	1.1166	0.00678	1.1009	1.1322
M	0.3836	0.0347	0.3035	0.4637
R	0.0975	0.0149	0.0631	0.1319

weight $=(1/q)^2$; F Value = 130.69; Level of significance of F <0.0001

New Mortality Model India, females, rural 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0192	0.00216	0.0142	0.0242
E	6.9636	2.2984	1.6634	12.2639
F	29.4654	1.1740	26.7580	32.1727
G	0.000150	0.000074	-0.00002	0.000321
H	1.1165	0.00873	1.0964	1.1366
M	0.4347	0.0717	0.2694	0.6000
R	0.0720	0.0187	0.0290	0.1150

weight $=(1/q)^2$; F Value = 87.33; Level of significance of F <0.0001

New Mortality Model India, females, rural 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0136	0.00247	0.00791	0.0193
E	6.5561	3.7800	-2.1606	15.2728
F	28.4200	1.9286	23.9726	32.8675
G	0.000199	0.000124	-0.00009	0.000484
H	1.1108	0.0111	1.0853	1.1363
M	0.4151	0.0880	0.2120	0.6181
R	0.0748	0.0262	0.0143	0.1353

weight $=(1/q)^2$; F Value = 47.01; Level of significance of F <0.0001

New Mortality Model India, females, rural 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0130	0.00231	0.00766	0.0183
E	4.0716	2.7002	-2.1551	10.2982
F	32.0114	3.5296	23.8720	40.1508
G	0.000129	0.000080	-0.00005	0.000313
H	1.1151	0.00982	1.0925	1.1378
M	0.3973	0.0617	0.2550	0.5395
R	0.0654	0.0236	0.0111	0.1198

weight $=(1/q)^{1.3}$; F Value = 160.57; Level of significance of F <0.0001

New Mortality Model India, females, rural 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0148	0.00231	0.00949	0.0201
E	5.0087	2.6819	-1.1757	11.1932
F	29.6554	2.2357	24.4999	34.8109
G	0.000131	0.000085	-0.00007	0.000327
H	1.1151	0.0110	1.0898	1.1404
M	0.3605	0.0682	0.2032	0.5178
R	0.0696	0.0260	0.00954	0.1296

weight $=(1/q)^{1.7}$; F Value = 72.32; Level of significance of F <0.0001

New Mortality Model India, females, rural 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0155	0.00277	0.00909	0.0218
E	5.4251	3.2563	-2.0841	12.9343
F	29.9270	2.3863	24.4242	35.4299
G	0.000112	0.000080	-0.00007	0.000297
H	1.1184	0.0120	1.0906	1.1461
M	0.3481	0.0583	0.2136	0.4825
R	0.0791	0.0296	0.0108	0.1474

weight $=(1/q)^{1.6}$; F Value = 62.93; Level of significance of F <0.0001

New Mortality Model India, females, rural 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0105	0.00193	0.00606	0.0150
E	5.6938	3.3858	-2.1141	13.5016
F	28.9243	2.3358	23.5378	34.3108
G	0.000224	0.000136	-0.00009	0.000539
H	1.1053	0.0105	1.0810	1.1296
M	0.3385	0.0639	0.1911	0.4858
R	0.0748	0.0236	0.0203	0.1293

weight $=(1/q)^2$; F Value = 60.55; Level of significance of F <0.0001

New Mortality Model India, females, rural 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0140	0.00152	0.0105	0.0175
E	4.2812	1.5979	0.5965	7.9660
F	30.3073	1.9276	25.8622	34.7524
G	0.000100	0.000058	-0.00003	0.000233
H	1.1196	0.00992	1.0967	1.1425
M	0.3155	0.0418	0.2190	0.4120
R	0.0810	0.0190	0.0371	0.1249

weight $=(1/q)^2$; F Value = 100.71; Level of significance of F <0.0001

New Mortality Model India, females, rural 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0135	0.00228	0.00825	0.0188
E	5.1895	3.1039	-1.9681	12.3471
F	30.1637	2.4953	24.4095	35.9180
G	0.000089	0.000073	-0.00008	0.000257
H	1.1218	0.0142	1.0890	1.1546
M	0.3398	0.0548	0.2133	0.4662
R	0.0970	0.0289	0.0304	0.1635

weight $=(1/q)^2$; F Value = 41.50; Level of significance of F <0.0001

New Mortality Model India, females, rural 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0135	0.00189	0.00917	0.0179
E	5.9221	2.6347	-0.1536	11.9978
F	28.9906	1.6664	25.1480	32.8333
G	0.000146	0.000084	-0.00005	0.000340
H	1.1127	0.0101	1.0894	1.1359
M	0.3227	0.0443	0.2205	0.4250
R	0.0932	0.0232	0.0399	0.1466

weight $=(1/q)^2$; F Value = 63.92; Level of significance of F <0.0001

New Mortality Model India, females, rural 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0136	0.00167	0.00974	0.0175
E	4.7925	2.0660	0.0283	9.5567
F	30.1716	1.9620	25.6471	34.6960
G	0.000073	0.000049	-0.00004	0.000186
H	1.1239	0.0115	1.0973	1.1504
M	0.3201	0.0546	0.1941	0.4461
R	0.0782	0.0231	0.0248	0.1316

weight $=(1/q)^2$; F Value = 65.59; Level of significance of F <0.0001

New Mortality Model India, females, rural 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0118	0.00197	0.00729	0.0164
E	6.9784	3.6098	-1.3458	15.3026
F	28.2205	1.6969	24.3074	32.1336
G	0.000155	0.000091	-0.00005	0.000365
H	1.1108	0.0104	1.0869	1.1347
M	0.3167	0.0695	0.1564	0.4770
R	0.0728	0.0264	0.0120	0.1337

weight $=(1/q)^2$; F Value = 48.51; Level of significance of F <0.0001

New Mortality Model India, females, rural 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0107	0.00152	0.00722	0.0142
E	5.4634	2.4889	-0.2760	11.2028
F	27.5760	1.6995	23.6569	31.4952
G	0.000141	0.000072	-0.00002	0.000306
H	1.1127	0.00895	1.0921	1.1334
M	0.2884	0.0450	0.1845	0.3922
R	0.0830	0.0237	0.0285	0.1376

weight $=(1/q)^2$; F Value = 67.77 Level of significance of F <0.0001

New Mortality Model India, females, rural 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0110	0.00136	0.00783	0.0141
E	5.8961	2.2355	0.7410	11.0512
F	27.2589	1.2674	24.3364	30.1815
G	0.000127	0.000055	9.035E-7	0.000254
H	1.1125	0.00764	1.0948	1.1301
M	0.2561	0.0288	0.1898	0.3224
R	0.0996	0.0215	0.0500	0.1491

weight $=(1/q)^2$; F Value = 80.66 Level of significance of F <0.0001

New Mortality Model India, females, rural 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0106	0.00144	0.00731	0.0140
E	5.4270	2.2315	0.2811	10.5729
F	27.2400	1.5755	23.6070	30.8731
G	0.000168	0.000082	-0.00002	0.000358
H	1.1082	0.00866	1.0882	1.1282
M	0.2338	0.0364	0.1498	0.3177
R	0.0831	0.0238	0.0281	0.1380

weight $=(1/q)^2$; F Value = 71.22 Level of significance of F <0.0001

New Mortality Model India, females, rural 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0120	0.00135	0.00885	0.0151
E	3.9917	1.5667	0.3789	7.6046
F	29.0129	1.8967	24.6391	33.3868
G	0.000076	0.000046	-0.00003	0.000181
H	1.1217	0.0104	1.0977	1.1457
M	0.2233	0.0268	0.1614	0.2852
R	0.0978	0.0236	0.0433	0.1523

weight $=(1/q)^2$; F Value = 76.42 Level of significance of F <0.0001

New Mortality Model India, females, rural 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0110	0.00197	0.00656	0.0155
E	7.2471	3.5077	-0.6879	15.1820
F	25.0000	0	25.0000	25.0000
G	0.000212	0.000086	0.000017	0.000408
H	1.1054	0.00761	1.0882	1.1226
M	0.2288	0.0350	0.1496	0.3081
R	0.1006	0.0308	0.0309	0.1704

weight $=(1/q)^2$; F Value = 53.09 Level of significance of F <0.0001

New Mortality Model India, females, rural 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0115	0.00115	0.00884	0.0141
E	6.8390	1.9271	2.3950	11.2830
F	27.0690	0.9007	24.9919	29.1461
G	0.000125	0.000042	0.000029	0.000222
H	1.1145	0.00601	1.1006	1.1283
M	0.2065	0.0184	0.1641	0.2490
R	0.1069	0.0183	0.0648	0.1490

weight $=(1/q)^2$; F Value = 119.35 Level of significance of F <0.0001

New Mortality Model India, females, rural 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00938	0.00211	0.00461	0.0141
E	8.5046	4.9209	-2.6272	19.6365
F	25.0000	0	25.0000	25.0000
G	0.000392	0.000171	5.283E-6	0.000778
H	1.0922	0.00815	1.0738	1.1106
M	0.2109	0.0586	0.0784	0.3433
R	0.0696	0.0327	-0.00451	0.1436

weight $=(1/q)^2$; F Value = 43.94 Level of significance of F <0.0001

New Mortality Model India, females, rural 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00936	0.00144	0.00603	0.0127
E	5.0009	2.5292	-0.8315	10.8333
F	29.3778	2.2308	24.2337	34.5220
G	0.000116	0.000075	-0.00006	0.000289
H	1.1114	0.0111	1.0859	1.1369
M	0.2076	0.0292	0.1402	0.2750
R	0.0958	0.0253	0.0375	0.1541

weight $=(1/q)^2$; F Value = 59.99; Level of significance of F <0.0001

New Mortality Model India, females, rural 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00857	0.000912	0.00647	0.0107
E	8.0531	2.4590	2.3826	13.7236
F	27.1042	0.8972	25.0352	29.1732
G	0.000243	0.000063	0.000097	0.000390
H	1.1001	0.00463	1.0894	1.1108
M	0.2151	0.0221	0.1641	0.2661
R	0.0831	0.0147	0.0492	0.1170

weight $=(1/q)^2$; F Value = 166.16; Level of significance of F <0.0001

New Mortality Model India, females, rural 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.0102	0.00146	0.00680	0.0135
E	7.1704	2.9879	0.2802	14.0607
F	26.9048	1.3399	23.8150	29.9945
G	0.000181	0.000081	-5.07E-6	0.000368
H	1.1066	0.00798	1.0882	1.1250
M	0.2115	0.0390	0.1216	0.3013
R	0.0764	0.0242	0.0206	0.1323

weight $=(1/q)^2$; F Value = 66.46; Level of significance of F <0.0001

APPENDIX 6E: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN MALES, URBAN 1970-1997.

New Mortality Model India, males, urban 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00298	0.00208	-0.00182	0.00779
E	8.8789	18.1666	-33.0137	50.7715
F	28.8365	6.5109	13.8221	43.8509
G	0.000282	0.000126	-8.01E-6	0.000571
H	1.1058	0.00767	1.0881	1.1235
M	0.2371	0.0381	0.1493	0.3249
R	0.0814	0.0280	0.0168	0.1461

weight $=(1/q)^{1.5}$; F Value = 90.26; Level of significance of F <0.0001

New Mortality Model India, males, urban 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00159	0.00105	-0.00079	0.00396
E	12.3546	17.9569	-28.2671	52.9763
F	30.0000	0	30.0000	30.0000
G	0.000441	0.000101	0.000213	0.000670
H	1.0983	0.00432	1.0885	1.1081
M	0.2575	0.0476	0.1498	0.3651
R	0.0566	0.0148	0.0230	0.0901

weight $=(1/q)^{2}$; F Value = 189.79; Level of significance of F <0.0001

New Mortality Model India, males, urban 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000517	0.00174	-0.00341	0.00445
E	17.5980	131.2	-279.1	314.3
F	30.0000	0	30.0000	30.0000
G	0.000478	0.000172	0.000089	0.000866
H	1.0981	0.00685	1.0826	1.1137
M	0.3831	0.3143	-0.3280	1.0942
R	0.0263	0.0263	-0.0332	0.0858

weight $=(1/q)^{2}$; F Value = 66.62; Level of significance of F <0.0001

New Mortality Model India, males, urban 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.000451	0.000896	-0.00155	0.00245
E	15.0000	0	15.0000	15.0000
F	25.0000	0	25.0000	25.0000
G	0.000512	0.000090	0.000312	0.000713
H	1.0955	0.00346	1.0878	1.1032
M	0.2475	0.0500	0.1360	0.3590
R	0.0562	0.0162	0.0200	0.0924

weight $=(1/q)^{2}$; F Value = 203.42; Level of significance of F <0.0001

New Mortality Model India, males, urban 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00336	0.00228	-0.00189	0.00861
E	15.8368	23.4773	-38.3024	69.9760
F	19.2553	2.8608	12.6583	25.8523
G	0.000300	0.000126	9.617E-6	0.000590
H	1.1060	0.00828	1.0869	1.1251
M	0.2047	0.0953	-0.0151	0.4245
R	0.0627	0.0480	-0.0480	0.1735

weight $=(1/q)^2$; F Value = 21.75; Level of significance of F <0.0001

New Mortality Model India, males, urban 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00307	0.00174	-0.00094	0.00707
E	22.4136	30.1674	-47.1532	91.9805
F	24.3739	2.5597	18.4711	30.2767
G	0.000467	0.000148	0.000125	0.000809
H	1.0986	0.00618	1.0843	1.1128
M	0.2874	0.1229	0.00410	0.5707
R	0.0455	0.0267	-0.0159	0.1070

weight $=(1/q)^2$; F Value = 51.47; Level of significance of F <0.0001

New Mortality Model India, males, urban 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00299	0.00103	0.000625	0.00536
E	21.1864	21.8760	-29.2601	71.6329
F	23.0676	1.1330	20.4550	25.6802
G	0.000468	0.000078	0.000288	0.000649
H	1.0973	0.00328	1.0897	1.1049
M	0.3348	0.1249	0.0468	0.6228
R	0.0316	0.0148	-0.00258	0.0659

weight $=(1/q)^2$; F Value = 163.58; Level of significance of F <0.0001

New Mortality Model India, males, urban 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00291	0.00119	0.000223	0.00559
E	31.9556	24.5301	-23.5358	87.4471
F	25.0000	0	25.0000	25.0000
G	0.000367	0.000077	0.000193	0.000542
H	1.1026	0.00421	1.0931	1.1121
M	0.2091	0.0418	0.1145	0.3037
R	0.0656	0.0193	0.0218	0.1093

weight $=(1/q)^2$; F Value = 115.43; Level of significance of F <0.0001

New Mortality Model India, males, urban 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00536	0.00446	-0.00493	0.0157
E	17.5796	31.5588	-55.1959	90.3550
F	25.0000	3.8444	16.1348	33.8652
G	0.000383	0.000170	-9.55E-6	0.000776
H	1.1026	0.00770	1.0848	1.1203
M	0.4605	2.0059	-4.1653	5.0862
R	0.0140	0.0713	-0.1505	0.1785

weight $=(1/q)^{1.2}$; F Value = 55.96; Level of significance of F <0.0001

New Mortality Model India, males, urban 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00320	0.00140	0.000082	0.00632
E	25.0000	0	25.0000	25.0000
F	22.0322	1.5064	18.6756	25.3887
G	0.000451	0.000111	0.000203	0.000699
H	1.0964	0.00496	1.0853	1.1075
M	0.3000	0	0.3000	0.3000
R	0.0220	0.00359	0.0140	0.0300

weight $=(1/q)^2$; F Value = 75.93; Level of significance of F <0.0001

New Mortality Model India, males, urban 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00291	0.00356	-0.00529	0.0111
E	39.7972	160.7	-330.7	410.3
F	20.9256	5.4571	8.3413	33.5099
G	0.000623	0.000120	0.000346	0.000901
H	1.0900	0.00390	1.0810	1.0990
M	0.2243	0.1046	-0.0168	0.4654
R	0.0377	0.0242	-0.0182	0.0936

weight $=(1/q)^2$; F Value = 86.69; Level of significance of F <0.0001

New Mortality Model India, males, urban 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00397	0.00138	0.000801	0.00714
E	13.8379	14.5779	-19.7791	47.4550
F	17.3088	0.9549	15.1068	19.5108
G	0.000411	0.000074	0.000239	0.000582
H	1.0978	0.00365	1.0894	1.1062
M	0.1845	0.0616	0.0424	0.3267
R	0.0484	0.0248	-0.00880	0.1057

weight $=(1/q)^2$; F Value = 169.09; Level of significance of F <0.0001

New Mortality Model India, males, urban 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00416	0.000931	0.00201	0.00631
E	10.4315	5.8761	-3.1190	23.9821
F	27.7768	1.7300	23.7873	31.7663
G	0.000267	0.000080	0.000082	0.000452
H	1.1033	0.00550	1.0906	1.1160
M	0.2982	0.1675	-0.0881	0.6845
R	0.0246	0.0165	-0.0134	0.0626
weight $=(1/q)^2$; F Value = 219.84; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00344	0.00349	-0.00461	0.0115
E	37.8879	86.0392	-160.5	236.3
F	22.9184	1.4938	19.4737	26.3632
G	0.000452	0.000112	0.000194	0.000710
H	1.0965	0.00489	1.0852	1.1078
M	0.2053	0.0816	0.0171	0.3936
R	0.0437	0.0240	-0.0117	0.0991
weight $=(1/q)^2$; F Value = 122.99; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00437	0.00109	0.00187	0.00688
E	9.0120	5.9613	-4.7350	22.7589
F	23.3270	1.6825	19.4471	27.2068
G	0.000352	0.000105	0.000110	0.000594
H	1.1025	0.00570	1.0893	1.1156
M	0.2428	0.1130	-0.0178	0.5034
R	0.0363	0.0223	-0.0152	0.0877
weight $=(1/q)^2$; F Value = 149.17; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00274	0.00153	-0.00072	0.00619
E	15.0000	0	15.0000	15.0000
F	24.7169	2.7497	18.4965	30.9373
G	0.000474	0.000134	0.000171	0.000778
H	1.0954	0.00539	1.0832	1.1076
M	0.1948	0.0978	-0.0265	0.4162
R	0.0409	0.0294	-0.0256	0.1075
weight $=(1/q)^{1.8}$; F Value = 109.95; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00506	0.00112	0.00249	0.00763
E	9.0027	5.6848	-4.1066	22.1120
F	25.8535	1.7339	21.8552	29.8518
G	0.000286	0.000096	0.000064	0.000508
H	1.1039	0.00627	1.0894	1.1183
M	0.2037	0.0799	0.0195	0.3879
R	0.0415	0.0221	-0.00952	0.0924

weight $=(1/q)^2$; F Value = 145.76; Level of significance of F <0.0001

New Mortality Model India, males, urban 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00397	0.00192	-0.00031	0.00825
E	15.0000	0	15.0000	15.0000
F	25.0000	0	25.0000	25.0000
G	0.000193	0.000054	0.000072	0.000313
H	1.1112	0.00486	1.1004	1.1220
M	0.1496	0.0407	0.0590	0.2403
R	0.0629	0.0359	-0.0172	0.1429

weight $=(1/q)^{1.2}$; F Value = 213.57; Level of significance of F <0.0001

New Mortality Model India, males, urban 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00367	0.00127	0.000801	0.00653
E	19.3954	12.6206	-9.1546	47.9453
F	25.0000	0	25.0000	25.0000
G	0.000345	0.000092	0.000137	0.000553
H	1.1014	0.00529	1.0895	1.1134
M	0.2605	0.1992	-0.1902	0.7111
R	0.0255	0.0237	-0.0281	0.0791

weight $=(1/q)^2$; F Value = 135.00; Level of significance of F <0.0001

New Mortality Model India, males, urban 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00539	0.000747	0.00367	0.00712
E	9.5061	3.7511	0.8560	18.1563
F	25.9255	1.0669	23.4652	28.3858
G	0.000243	0.000056	0.000113	0.000373
H	1.1076	0.00436	1.0975	1.1176
M	0.1524	0.0331	0.0760	0.2288
R	0.0496	0.0155	0.0139	0.0852

weight $=(1/q)^2$; F Value = 310.23; Level of significance of F <0.0001

New Mortality Model India, males, urban 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00409	0.000788	0.00227	0.00590
E	6.6363	3.3785	-1.1547	14.4273
F	26.3277	1.9323	21.8718	30.7836
G	0.000276	0.000079	0.000093	0.000459
H	1.1037	0.00525	1.0916	1.1158
M	0.1352	0.0312	0.0632	0.2071
R	0.0511	0.0177	0.0102	0.0921

weight $=(1/q)^2$; F Value = 275.32; Level of significance of F <0.0001

New Mortality Model India, males, urban 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00424	0.00126	0.00133	0.00716
E	4.4714	3.4613	-3.5103	12.4532
F	28.1962	3.9759	19.0276	37.3649
G	0.000275	0.000119	7.461E-7	0.000550
H	1.1041	0.00755	1.0867	1.1216
M	0.1178	0.0165	0.0797	0.1559
R	0.0845	0.0220	0.0339	0.1351

weight $=(1/q)^2$; F Value = 214.08; Level of significance of F <0.0001

New Mortality Model India, males, urban 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00563	0.00116	0.00301	0.00824
E	11.4855	6.0748	-2.2567	25.2278
F	27.1458	1.4731	23.8133	30.4784
G	0.000235	0.000080	0.000054	0.000416
H	1.1063	0.00632	1.0920	1.1206
M	0.1900	0	0.1900	0.1900
R	0.0259	0.00336	0.0183	0.0335

weight $=(1/q)^2$; F Value = 145.57; Level of significance of F <0.0001

New Mortality Model India, males, urban 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00280	0.00254	-0.00285	0.00846
E	26.4338	88.3163	-170.3	223.2
F	15.0000	0	15.0000	15.0000
G	0.000287	0.000103	0.000057	0.000517
H	1.1001	0.00624	1.0862	1.1140
M	0.2000	0	0.2000	0.2000
R	0.0202	0.00459	0.00999	0.0304

weight $=(1/q)^2$; F Value = 84.27; Level of significance of F <0.0001

New Mortality Model India, males, urban 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00422	0.00152	0.000714	0.00774
E	9.4771	9.9142	-13.3853	32.3395
F	27.6225	3.3105	19.9884	35.2566
G	0.000283	0.000150	-0.00006	0.000630
H	1.1019	0.00969	1.0795	1.1242
M	0.1498	0.0691	-0.00957	0.3093
R	0.0475	0.0314	-0.0250	0.1200
weight $=(1/q)^2$; F Value = 76.38; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00326	0.000838	0.00133	0.00519
E	11.3606	7.6089	-6.1858	28.9069
F	26.4515	1.7278	22.4672	30.4357
G	0.000327	0.000085	0.000130	0.000524
H	1.0978	0.00484	1.0866	1.1089
M	0.1118	0.0136	0.0804	0.1432
R	0.0903	0.0193	0.0459	0.1348
weight $=(1/q)^2$; F Value = 237.24; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00409	0.000714	0.00245	0.00574
E	10.3700	4.5206	-0.0547	20.7947
F	27.0462	1.2616	24.1369	29.9554
G	0.000433	0.000088	0.000229	0.000637
H	1.0931	0.00372	1.0846	1.1017
M	0.2775	0.2297	-0.2522	0.8073
R	0.0159	0.0152	-0.0191	0.0509
weight $=(1/q)^2$; F Value = 391.02; Level of significance of F <0.0001				

New Mortality Model India, males, urban 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00310	0.000944	0.000967	0.00524
E	3.9778	3.2150	-3.2950	11.2506
F	25.0000	0	25.0000	25.0000
G	0.000336	0.000107	0.000094	0.000579
H	1.0997	0.00613	1.0858	1.1136
M	0.1639	0.1456	-0.1654	0.4933
R	0.0280	0.0325	-0.0454	0.1015
weight $=(1/q)^2$; F Value = 127.05; Level of significance of F <0.0001				

APPENDIX 6F: STATISTICS OF FITTING NEW MORTALITY MODEL FOR INDIAN FEMALES, URBAN 1970-1997.

New Mortality Model India, females, urban 1970

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00431	0.00355	-0.00372	0.0124
E	31.5038	70.0464	-127.0	190.0
F	20.0000	0	20.0000	20.0000
G	0.000840	0.000426	-0.00012	0.00180
H	1.0799	0.00995	1.0574	1.1024
M	0.2339	0.0852	0.0411	0.4267
R	0.0823	0.0542	-0.0402	0.2049

weight $=(1/q)^2$; F Value = 38.33; Level of significance of F <0.0001

New Mortality Model India, females, urban 1971

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00862	0.00205	0.00390	0.0133
E	6.2194	4.5494	-4.2717	16.7105
F	29.0314	2.8759	22.3995	35.6633
G	0.000226	0.000162	-0.00015	0.000600
H	1.1038	0.0126	1.0749	1.1328
M	0.2925	0.1165	0.0240	0.5611
R	0.0502	0.0284	-0.0154	0.1157

weight $=(1/q)^2$; F Value = 85.61; Level of significance of F <0.0001

New Mortality Model India, females, urban 1972

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00958	0.00223	0.00444	0.0147
E	6.2231	4.1105	-3.2558	15.7021
F	29.2412	2.8455	22.6794	35.8030
G	0.000207	0.000147	-0.00013	0.000546
H	1.1097	0.0125	1.0809	1.1385
M	0.3072	0.1112	0.0507	0.5637
R	0.0511	0.0259	-0.00852	0.1108

weight $=(1/q)^2$; F Value = 96.18; Level of significance of F <0.0001

New Mortality Model India, females, urban 1973

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00845	0.00173	0.00447	0.0124
E	7.7140	4.6689	-3.0527	18.4807
F	26.9061	1.7430	22.8867	30.9254
G	0.000238	0.000118	-0.00003	0.000510
H	1.1035	0.00897	1.0828	1.1242
M	0.2509	0.0542	0.1259	0.3759
R	0.0746	0.0259	0.0147	0.1344

weight $=(1/q)^2$; F Value = 113.73; Level of significance of F <0.0001

New Mortality Model India, females, urban 1974

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00878	0.00269	0.00258	0.0150
E	6.0052	5.9181	-7.6421	19.6524
F	29.9165	4.1677	20.3057	39.5273
G	0.000151	0.000167	-0.00024	0.000537
H	1.1100	0.0192	1.0657	1.1544
M	0.4335	0.6537	-1.0740	1.9410
R	0.0230	0.0417	-0.0732	0.1191

weight $=(1/q)^2$; F Value = 39.67; Level of significance of F <0.0001

New Mortality Model India, females, urban 1975

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00480	0.00194	0.000420	0.00918
E	5.3756	6.1302	-8.4920	19.2433
F	25.0000	0	25.0000	25.0000
G	0.000347	0.000171	-0.00004	0.000734
H	1.0993	0.00935	1.0782	1.1205
M	0.2750	0.1257	-0.00946	0.5595
R	0.0536	0.0373	-0.0309	0.1380

weight $=(1/q)^2$; F Value = 71.58; Level of significance of F <0.0001

New Mortality Model India, females, urban 1976

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00805	0.00201	0.00342	0.0127
E	12.0419	7.3829	-4.9834	29.0671
F	26.7994	1.5391	23.2503	30.3486
G	0.000405	0.000169	0.000014	0.000795
H	1.0946	0.00765	1.0770	1.1123
M	0.2211	0.0413	0.1257	0.3164
R	0.0852	0.0263	0.0246	0.1458

weight $=(1/q)^2$; F Value = 110.72; Level of significance of F <0.0001

New Mortality Model India, females, urban 1977

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00770	0.00328	0.000277	0.0151
E	26.7682	22.3637	-23.8225	77.3588
F	20.0000	0	20.0000	20.0000
G	0.000406	0.000175	9.882E-6	0.000802
H	1.0958	0.00869	1.0761	1.1155
M	0.2277	0.1162	-0.0351	0.4905
R	0.0591	0.0470	-0.0473	0.1654

weight $=(1/q)^2$; F Value = 43.89; Level of significance of F <0.0001

New Mortality Model India, females, urban 1978

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00889	0.00148	0.00547	0.0123
E	5.7298	2.6515	-0.3845	11.8442
F	27.2466	1.8935	22.8802	31.6130
G	0.000188	0.000100	-0.00004	0.000420
H	1.1085	0.00952	1.0865	1.1304
M	0.5876	0.8612	-1.3984	2.5735
R	0.0140	0.0227	-0.0384	0.0664

weight $=(1/q)^2$; F Value = 115.12; Level of significance of F <0.0001

New Mortality Model India, females, urban 1979

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00806	0.00180	0.00390	0.0122
E	7.1543	4.6906	-3.6622	17.9709
F	27.1226	2.1213	22.2308	32.0145
G	0.000134	0.000088	-0.00007	0.000337
H	1.1122	0.0120	1.0845	1.1400
M	0.2337	0.1254	-0.0554	0.5227
R	0.0467	0.0355	-0.0353	0.1286

weight $=(1/q)^2$; F Value = 58.19; Level of significance of F <0.0001

New Mortality Model India, females, urban 1980

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00646	0.00139	0.00327	0.00966
E	6.4949	4.2183	-3.2326	16.2224
F	27.5642	2.2731	22.3224	32.8060
G	0.000169	0.000096	-0.00005	0.000390
H	1.1063	0.0101	1.0829	1.1296
M	0.1964	0.0642	0.0484	0.3444
R	0.0563	0.0267	-0.00530	0.1179

weight $=(1/q)^2$; F Value = 96.62; Level of significance of F <0.0001

New Mortality Model India, females, urban 1981

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00706	0.00122	0.00430	0.00982
E	2.3033	1.9185	-2.0367	6.6433
F	25.0000	0	25.0000	25.0000
G	0.000121	0.000054	-2.44E-6	0.000244
H	1.1130	0.00780	1.0954	1.1307
M	0.2508	0.3014	-0.4310	0.9326
R	0.0310	0.0542	-0.0916	0.1536

weight $=(1/q)^{1.8}$; F Value = 144.67; Level of significance of F <0.0001

New Mortality Model India, females, urban 1982

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00763	0.00102	0.00528	0.00998
E	6.0058	2.3376	0.6151	11.3964
F	27.5286	1.4338	24.2223	30.8349
G	0.000095	0.000044	-6.55E-6	0.000197
H	1.1156	0.00830	1.0964	1.1347
M	0.1654	0.0350	0.0847	0.2462
R	0.0670	0.0223	0.0156	0.1184

weight $=(1/q)^2$; F Value = 155.51; Level of significance of F <0.0001

New Mortality Model India, females, urban 1983

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00779	0.000927	0.00566	0.00993
E	5.9639	1.8776	1.6340	10.2938
F	25.3792	1.0873	22.8718	27.8866
G	0.000145	0.000049	0.000033	0.000257
H	1.1098	0.00612	1.0957	1.1239
M	0.1798	0.0370	0.0944	0.2652
R	0.0613	0.0197	0.0158	0.1068

weight $=(1/q)^2$; F Value = 210.98; Level of significance of F <0.0001

New Mortality Model India, females, urban 1984

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00873	0.00127	0.00579	0.0117
E	7.3354	2.6651	1.1895	13.4813
F	24.7726	1.0602	22.3277	27.2175
G	0.000113	0.000043	0.000014	0.000212
H	1.1159	0.00709	1.0995	1.1322
M	0.1850	0.0379	0.0976	0.2724
R	0.0726	0.0244	0.0163	0.1289

weight $=(1/q)^2$; F Value = 138.91; Level of significance of F <0.0001

New Mortality Model India, females, urban 1985

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00738	0.00188	0.00305	0.0117
E	17.2718	9.7933	-5.3119	39.8555
F	23.8683	0.9515	21.6741	26.0626
G	0.000265	0.000089	0.000060	0.000470
H	1.0994	0.00645	1.0845	1.1142
M	0.2008	0.0798	0.0169	0.3847
R	0.0497	0.0275	-0.0137	0.1132

weight $=(1/q)^2$; F Value = 85.36; Level of significance of F <0.0001

New Mortality Model India, females, urban 1986

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00770	0.000875	0.00568	0.00971
E	5.4858	1.7995	1.3361	9.6355
F	26.5587	1.2702	23.6296	29.4878
G	0.000116	0.000043	0.000017	0.000216
H	1.1135	0.00667	1.0981	1.1288
M	0.1787	0.0364	0.0948	0.2626
R	0.0611	0.0193	0.0165	0.1057

weight $=(1/q)^2$; F Value = 216.57; Level of significance of F <0.0001

New Mortality Model India, females, urban 1987

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00680	0.00114	0.00416	0.00944
E	7.7837	3.7081	-0.7672	16.3346
F	26.0069	1.4220	22.7277	29.2861
G	0.000124	0.000053	1.382E-6	0.000247
H	1.1121	0.00790	1.0939	1.1304
M	0.1420	0.0297	0.0736	0.2104
R	0.0775	0.0276	0.0138	0.1412

weight $=(1/q)^2$; F Value = 113.49; Level of significance of F <0.0001

New Mortality Model India, females, urban 1988

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00824	0.00191	0.00383	0.0127
E	23.1616	8.8255	2.8097	43.5134
F	24.3194	0.6642	22.7876	25.8511
G	0.000181	0.000054	0.000058	0.000305
H	1.1064	0.00574	1.0931	1.1196
M	0.1377	0.0226	0.0856	0.1898
R	0.0967	0.0259	0.0370	0.1565

weight $=(1/q)^2$; F Value = 111.92; Level of significance of F <0.0001

New Mortality Model India, females, urban 1989

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00815	0.00123	0.00531	0.0110
E	13.6999	4.5977	3.0976	24.3022
F	24.5217	0.6985	22.9110	26.1323
G	0.000132	0.000039	0.000041	0.000223
H	1.1093	0.00569	1.0962	1.1224
M	0.1395	0.0296	0.0711	0.2078
R	0.0680	0.0221	0.0172	0.1189

weight $=(1/q)^2$; F Value = 144.08; Level of significance of F <0.0001

New Mortality Model India, females, urban 1990

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00648	0.00107	0.00400	0.00895
E	11.8723	4.5848	1.2996	22.4449
F	24.8637	0.8119	22.9914	26.7361
G	0.000089	0.000028	0.000025	0.000152
H	1.1174	0.00589	1.1038	1.1309
M	0.1155	0.0169	0.0766	0.1545
R	0.0949	0.0233	0.0413	0.1486

weight $=(1/q)^2$; F Value = 146.08; Level of significance of F <0.0001

New Mortality Model India, females, urban 1991

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00815	0.00107	0.00567	0.0106
E	4.5068	1.9660	-0.0269	9.0405
F	29.9097	2.3244	24.5497	35.2698
G	0.000044	0.000029	-0.00002	0.000111
H	1.1289	0.0116	1.1021	1.1557
M	0.1317	0.0282	0.0666	0.1968
R	0.0736	0.0258	0.0141	0.1331

weight $=(1/q)^2$; F Value = 121.98; Level of significance of F <0.0001

New Mortality Model India, females, urban 1992

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00782	0.00112	0.00524	0.0104
E	5.6603	2.4028	0.1194	11.2012
F	27.7673	1.7803	23.6619	31.8727
G	0.000074	0.000042	-0.00002	0.000171
H	1.1200	0.0101	1.0967	1.1432
M	0.1270	0.0295	0.0589	0.1951
R	0.0732	0.0289	0.00657	0.1397

weight $=(1/q)^2$; F Value = 106.54; Level of significance of F <0.0001

New Mortality Model India, females, urban 1993

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00782	0.00144	0.00455	0.0111
E	3.2910	2.1532	-1.5799	8.1619
F	26.5565	2.7727	20.2842	32.8288
G	0.000036	0.000041	-0.00006	0.000128
H	1.1256	0.0197	1.0810	1.1703
M	0.1200	0	0.1200	0.1200
R	0.0254	0.00893	0.00524	0.0456

weight $=(1/q)^2$; F Value = 39.99; Level of significance of F <0.0001

New Mortality Model India, females, urban 1994

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00720	0.00102	0.00484	0.00956
E	4.0221	1.8859	-0.3269	8.3711
F	28.7630	2.4874	23.0269	34.4991
G	0.000044	0.000032	-0.00003	0.000117
H	1.1259	0.0126	1.0968	1.1550
M	0.1256	0.0313	0.0536	0.1977
R	0.0707	0.0294	0.00290	0.1386

weight $=(1/q)^2$; F Value = 102.02; Level of significance of F <0.0001

New Mortality Model India, females, urban 1995

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00501	0.000808	0.00315	0.00687
E	6.5715	2.9148	-0.1500	13.2930
F	25.3417	1.3355	22.2621	28.4214
G	0.000099	0.000036	0.000015	0.000184
H	1.1131	0.00674	1.0976	1.1287
M	0.1144	0.0160	0.0776	0.1512
R	0.0955	0.0245	0.0391	0.1519

weight $=(1/q)^2$; F Value = 159.64; Level of significance of F <0.0001

New Mortality Model India, females, urban 1996

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00658	0.00112	0.00400	0.00917
E	5.8950	3.0272	-1.0858	12.8758
F	26.9351	1.8283	22.7190	31.1512
G	0.000050	0.000029	-0.00002	0.000117
H	1.1235	0.0103	1.0997	1.1473
M	0.1118	0.0221	0.0609	0.1627
R	0.0954	0.0336	0.0179	0.1728

weight $=(1/q)^2$; F Value = 78.80; Level of significance of F <0.0001

New Mortality Model India, females, urban 1997

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits	
D	0.00724	0.00118	0.00453	0.00996
E	9.0273	3.6611	0.5846	17.4699
F	25.7983	1.1735	23.0922	28.5045
G	0.000152	0.000067	-2.07E-6	0.000306
H	1.1060	0.00807	1.0874	1.1246
M	0.1206	0.0379	0.0331	0.2080
R	0.0572	0.0270	-0.00495	0.1194

weight $=(1/q)^2$; F Value = 103.45; Level of significance of F <0.0001

APPENDIX 7A: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, MALES, INDIA

Parameter D, males, India

Year	ARIMA model (5, 1, 0)				
	D	Est/ Forecast	Std errors	L95	U95
1970	0.00461				
1971	0.00488	0.00467	0.00137	0.00199	0.00736
1972	0.00440	0.00487	0.00137	0.00219	0.00755
1973	0.00399	0.00453	0.00137	0.00184	0.00721
1974	0.00370	0.00448	0.00137	0.00180	0.00716
1975	0.00184	0.00434	0.00137	0.00166	0.00702
1976	0.00276	0.00318	0.00137	0.00050	0.00587
1977	0.00440	0.00415	0.00137	0.00147	0.00684
1978	0.00496	0.00439	0.00137	0.00171	0.00707
1979	0.00292	0.00427	0.00137	0.00159	0.00695
1980	0.00257	0.00273	0.00137	0.00005	0.00541
1981	0.00248	0.00317	0.00137	0.00048	0.00585
1982	0.00549	0.00331	0.00137	0.00063	0.00599
1983	0.00670	0.00550	0.00137	0.00282	0.00818
1984	0.00538	0.00505	0.00137	0.00237	0.00773
1985	0.00467	0.00408	0.00137	0.00140	0.00676
1986	0.00410	0.00430	0.00137	0.00162	0.00699
1987	0.00381	0.00456	0.00137	0.00187	0.00724
1988	0.00370	0.00507	0.00137	0.00239	0.00775
1989	0.00562	0.00483	0.00137	0.00215	0.00752
1990	0.00482	0.00565	0.00137	0.00297	0.00833
1991	0.00673	0.00432	0.00137	0.00164	0.00701
1992	0.00490	0.00602	0.00137	0.00334	0.00870
1993	0.00521	0.00416	0.00137	0.00147	0.00684
1994	0.00846	0.00563	0.00137	0.00295	0.00831
1995	0.00689	0.00748	0.00137	0.00480	0.01016
1996	0.00440	0.00583	0.00137	0.00315	0.00852
1997	0.00548	0.00526	0.00137	0.00258	0.00794
1998		0.00624	0.00137	0.00356	0.00893
1999		0.00661	0.00163	0.00342	0.00980
2000		0.00692	0.00164	0.00371	0.01013
2001		0.00643	0.00165	0.00320	0.00966
2002		0.00598	0.00167	0.00270	0.00926
2003		0.00624	0.00175	0.00280	0.00968
2004		0.00665	0.00189	0.00294	0.01036
2005		0.00685	0.00198	0.00298	0.01073
2006		0.00685	0.00201	0.00291	0.01078
2007		0.00671	0.00204	0.00272	0.01070
2008		0.00666	0.00208	0.00259	0.01074
2009		0.00682	0.00215	0.00261	0.01102
2010		0.00701	0.00221	0.00267	0.01134
2011		0.00710	0.00226	0.00267	0.01154
2012		0.00711	0.00230	0.00260	0.01162
2013		0.00710	0.00234	0.00252	0.01169
2014		0.00715	0.00239	0.00247	0.01182
2015		0.00725	0.00244	0.00248	0.01203
2016		0.00736	0.00248	0.00249	0.01223
2017		0.00742	0.00253	0.00247	0.01237
2018		0.00746	0.00256	0.00243	0.01249
2019		0.00750	0.00260	0.00240	0.01261
2020		0.00757	0.00265	0.00238	0.01275

Parameter E, males, India

Year	ARIMA model (1, 1, 1)				
	E	Est/ Forecast	Std errors	L95	U95
1970	10.23				
1971	42.51	10.01	8.18	-6.03	26.05
1972	10.35	9.46	8.18	-6.58	25.50
1973	5.53	10.92	8.18	-5.12	26.96
1974	21.41	10.71	8.18	-5.34	26.75
1975	8.48	10.10	8.18	-5.94	26.14
1976	7.89	10.46	8.18	-5.58	26.50
1977	15.00	10.15	8.18	-5.89	26.19
1978	7.73	9.75	8.18	-6.29	25.79
1979	5.87	9.81	8.18	-6.24	25.85
1980	15.00	9.50	8.18	-6.54	25.54
1981	14.04	9.03	8.18	-7.01	25.07
1982	4.04	9.05	8.18	-6.99	25.09
1983	3.23	9.12	8.18	-6.92	25.16
1984	15.83	8.68	8.18	-7.36	24.72
1985	7.04	8.10	8.18	-7.94	24.14
1986	11.67	8.27	8.18	-7.77	24.31
1987	14.94	7.94	8.18	-8.10	23.98
1988	14.97	7.83	8.18	-8.22	23.87
1989	3.25	7.89	8.18	-8.16	23.93
1990	6.92	8.06	8.18	-7.98	24.10
1991	6.66	7.59	8.18	-8.45	23.63
1992	6.62	7.33	8.18	-8.71	23.37
1993	4.06	7.07	8.18	-8.97	23.11
1994	2.05	6.84	8.18	-9.20	22.88
1995	2.57	6.51	8.18	-9.53	22.55
1996	4.52	6.08	8.18	-9.96	22.12
1997	6.84	5.68	8.18	-10.36	21.72
1998		5.38	8.18	-10.66	21.42
1999		5.22	8.18	-10.82	21.26
2000		4.99	8.19	-11.06	21.05
2001		4.77	8.20	-11.30	20.84
2002		4.54	8.20	-11.54	20.63
2003		4.32	8.21	-11.77	20.41
2004		4.10	8.22	-12.01	20.20
2005		3.87	8.22	-12.24	19.99
2006		3.65	8.23	-12.48	19.78
2007		3.43	8.24	-12.72	19.57
2008		3.20	8.24	-12.95	19.36
2009		2.98	8.25	-13.19	19.15
2010		2.76	8.26	-13.42	18.94
2011		2.53	8.26	-13.66	18.73
2012		2.31	8.27	-13.89	18.52
2013		2.09	8.27	-14.13	18.30
2014		1.86	8.28	-14.37	18.09
2015		1.64	8.29	-14.60	17.88
2016		1.42	8.29	-14.84	17.67
2017		1.19	8.30	-15.07	17.46
2018		0.97	8.31	-15.31	17.25
2019		0.75	8.31	-15.54	17.04
2020		0.52	8.32	-15.78	16.83

Parameter F, males, India

Year	ARIMA model (4, 1, 0)				
	F	Est/ Forecast	Std errors	L95	U95
1970	26.35				
1971	25.00	26.59	3.68	19.38	33.80
1972	25.00	26.05	3.68	18.84	33.26
1973	30.00	26.29	3.68	19.08	33.50
1974	23.59	28.68	3.68	21.47	35.89
1975	25.00	25.12	3.68	17.90	32.33
1976	24.64	26.39	3.68	19.18	33.60
1977	23.45	25.92	3.68	18.71	33.14
1978	29.23	26.56	3.68	19.35	33.77
1979	25.00	27.34	3.68	20.12	34.55
1980	22.85	25.16	3.68	17.95	32.38
1981	25.00	24.84	3.68	17.63	32.06
1982	32.49	25.79	3.68	18.58	33.00
1983	33.47	30.58	3.68	23.37	37.80
1984	25.67	29.04	3.68	21.83	36.25
1985	28.10	25.54	3.68	18.33	32.75
1986	24.68	29.02	3.68	21.81	36.24
1987	23.81	28.99	3.68	21.78	36.21
1988	24.94	28.56	3.68	21.34	35.77
1989	30.90	26.30	3.68	19.09	33.52
1990	26.99	29.49	3.68	22.28	36.70
1991	28.92	25.98	3.68	18.77	33.20
1992	26.30	27.74	3.68	20.53	34.95
1993	29.24	26.94	3.68	19.72	34.15
1994	37.52	30.44	3.68	23.22	37.65
1995	34.52	32.80	3.68	25.59	40.02
1996	31.17	31.46	3.68	24.25	38.68
1997	29.48	30.45	3.68	23.23	37.66
1998		31.49	3.68	24.27	38.70
1999		34.76	4.10	26.73	42.78
2000		34.57	4.14	26.45	42.68
2001		33.11	4.18	24.91	41.31
2002		32.43	4.28	24.05	40.82
2003		33.27	4.70	24.05	42.49
2004		34.68	4.99	24.89	44.46
2005		34.99	5.10	24.99	44.99
2006		34.55	5.19	24.38	44.72
2007		34.34	5.31	23.94	44.74
2008		34.76	5.51	23.96	45.56
2009		35.46	5.70	24.29	46.63
2010		35.80	5.83	24.38	47.23
2011		35.79	5.94	24.16	47.43
2012		35.82	6.05	23.95	47.69
2013		36.10	6.20	23.95	48.25
2014		36.52	6.34	24.09	48.96
2015		36.83	6.47	24.15	49.51
2016		36.98	6.58	24.08	49.88
2017		37.12	6.69	24.00	50.24
2018		37.37	6.82	24.01	50.73
2019		37.67	6.94	24.07	51.28
2020		37.95	7.05	24.12	51.78

Parameter G, males, India

Year	ARIMA model (3, 1, 0)				
	G	Est/ Forecast	Std errors	L95	U95
1970	0.000484				
1971	0.000462	0.000477	0.000109	0.000263	0.000691
1972	0.000423	0.000461	0.000109	0.000248	0.000675
1973	0.000337	0.000434	0.000109	0.000221	0.000648
1974	0.000576	0.000379	0.000109	0.000165	0.000593
1975	0.000614	0.000506	0.000109	0.000292	0.000720
1976	0.000497	0.000538	0.000109	0.000324	0.000752
1977	0.000553	0.000427	0.000109	0.000213	0.000641
1978	0.000412	0.000539	0.000109	0.000325	0.000753
1979	0.000438	0.000482	0.000109	0.000268	0.000696
1980	0.000481	0.000438	0.000109	0.000224	0.000651
1981	0.000467	0.000494	0.000109	0.000280	0.000708
1982	0.000277	0.000434	0.000109	0.000220	0.000648
1983	0.000203	0.000329	0.000109	0.000115	0.000543
1984	0.000419	0.000287	0.000109	0.000073	0.000501
1985	0.000341	0.000412	0.000109	0.000198	0.000626
1986	0.000473	0.000316	0.000109	0.000102	0.000530
1987	0.000416	0.000347	0.000109	0.000133	0.000561
1988	0.000425	0.000411	0.000109	0.000197	0.000625
1989	0.000229	0.000375	0.000109	0.000161	0.000589
1990	0.000356	0.000314	0.000109	0.000101	0.000528
1991	0.000256	0.000350	0.000109	0.000136	0.000564
1992	0.000333	0.000316	0.000109	0.000102	0.000530
1993	0.000228	0.000271	0.000109	0.000057	0.000485
1994	0.000122	0.000270	0.000109	0.000056	0.000484
1995	0.000178	0.000156	0.000109	-0.000057	0.000370
1996	0.000358	0.000215	0.000109	0.000002	0.000429
1997	0.000357	0.000291	0.000109	0.000077	0.000505
1998		0.000262	0.000109	0.000048	0.000476
1999		0.000218	0.000126	-0.000030	0.000466
2000		0.000253	0.000135	-0.000011	0.000517
2001		0.000275	0.000137	0.000006	0.000544
2002		0.000257	0.000149	-0.000036	0.000549
2003		0.000229	0.000160	-0.000085	0.000543
2004		0.000223	0.000170	-0.000109	0.000556
2005		0.000227	0.000175	-0.000116	0.000571
2006		0.000223	0.000182	-0.000134	0.000581
2007		0.000211	0.000190	-0.000161	0.000583
2008		0.000201	0.000197	-0.000185	0.000588
2009		0.000196	0.000204	-0.000203	0.000595
2010		0.000192	0.000210	-0.000219	0.000603
2011		0.000184	0.000216	-0.000239	0.000607
2012		0.000176	0.000222	-0.000259	0.000611
2013		0.000169	0.000228	-0.000277	0.000616
2014		0.000163	0.000234	-0.000295	0.000621
2015		0.000156	0.000239	-0.000312	0.000625
2016		0.000149	0.000244	-0.000330	0.000628
2017		0.000142	0.000250	-0.000348	0.000632
2018		0.000135	0.000255	-0.000365	0.000635
2019		0.000128	0.000260	-0.000381	0.000638
2020		0.000122	0.000265	-0.000398	0.000641

Parameter H, males, India

Year	ARIMA model (2, 1, 0)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.0995				
1971	1.0997	1.0995	0.0061	1.0876	1.1115
1972	1.1037	1.0997	0.0061	1.0878	1.1116
1973	1.1040	1.1023	0.0061	1.0904	1.1143
1974	1.0962	1.1031	0.0061	1.0912	1.1150
1975	1.0960	1.0989	0.0061	1.0870	1.1109
1976	1.1005	1.0978	0.0061	1.0859	1.1098
1977	1.0972	1.0990	0.0061	1.0871	1.1110
1978	1.1025	1.0974	0.0061	1.0855	1.1094
1979	1.0984	1.1014	0.0061	1.0895	1.1134
1980	1.0973	1.0987	0.0061	1.0868	1.1107
1981	1.0979	1.0987	0.0061	1.0867	1.1106
1982	1.1052	1.0980	0.0061	1.0861	1.1099
1983	1.1115	1.1026	0.0061	1.0906	1.1145
1984	1.0995	1.1078	0.0061	1.0958	1.1197
1985	1.1031	1.1024	0.0061	1.0905	1.1143
1986	1.0964	1.1045	0.0061	1.0926	1.1165
1987	1.0981	1.0980	0.0061	1.0861	1.1100
1988	1.0983	1.0990	0.0061	1.0871	1.1110
1989	1.1081	1.0979	0.0061	1.0860	1.1099
1990	1.1000	1.1047	0.0061	1.0928	1.1166
1991	1.1063	1.1008	0.0061	1.0888	1.1127
1992	1.1017	1.1059	0.0061	1.0940	1.1179
1993	1.1077	1.1020	0.0061	1.0901	1.1139
1994	1.1174	1.1067	0.0061	1.0947	1.1186
1995	1.1095	1.1128	0.0061	1.1008	1.1247
1996	1.0977	1.1102	0.0061	1.0983	1.1222
1997	1.1004	1.1036	0.0061	1.0917	1.1156
1998		1.1021	0.0061	1.0902	1.1140
1999		1.1010	0.0073	1.0868	1.1152
2000		1.1011	0.0080	1.0854	1.1168
2001		1.1014	0.0090	1.0838	1.1189
2002		1.1013	0.0098	1.0821	1.1205
2003		1.1013	0.0105	1.0807	1.1219
2004		1.1014	0.0112	1.0794	1.1234
2005		1.1014	0.0119	1.0782	1.1247
2006		1.1015	0.0125	1.0770	1.1259
2007		1.1015	0.0131	1.0759	1.1271
2008		1.1016	0.0136	1.0749	1.1283
2009		1.1016	0.0142	1.0738	1.1294
2010		1.1017	0.0147	1.0729	1.1305
2011		1.1017	0.0152	1.0719	1.1315
2012		1.1017	0.0157	1.0710	1.1325
2013		1.1018	0.0162	1.0701	1.1335
2014		1.1018	0.0166	1.0693	1.1344
2015		1.1019	0.0171	1.0684	1.1353
2016		1.1019	0.0175	1.0676	1.1362
2017		1.1020	0.0179	1.0668	1.1371
2018		1.1020	0.0183	1.0661	1.1379
2019		1.1021	0.0187	1.0653	1.1388
2020		1.1021	0.0191	1.0646	1.1396

Parameter M, males, India

Year	Logistic ARIMA model (1, 1, 1)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.3930				
1971	0.3535	0.3850	0.0386	0.3115	0.4626
1972	0.3929	0.3635	0.0306	0.3053	0.4249
1973	0.3454	0.3567	0.0303	0.2991	0.4175
1974	0.3780	0.3590	0.0304	0.3012	0.4200
1975	0.4120	0.3465	0.0299	0.2897	0.4067
1976	0.3579	0.3755	0.0309	0.3164	0.4375
1977	0.3997	0.3752	0.0309	0.3162	0.4372
1978	0.3378	0.3624	0.0305	0.3044	0.4237
1979	0.4025	0.3600	0.0304	0.3021	0.4211
1980	0.3248	0.3514	0.0301	0.2942	0.4119
1981	0.2949	0.3554	0.0302	0.2979	0.4162
1982	0.3086	0.3041	0.0279	0.2515	0.3608
1983	0.2936	0.2898	0.0272	0.2387	0.3451
1984	0.3128	0.2903	0.0272	0.2392	0.3456
1985	0.2766	0.2896	0.0272	0.2386	0.3449
1986	0.2911	0.2859	0.0270	0.2353	0.3408
1987	0.2737	0.2722	0.0262	0.2233	0.3257
1988	0.2541	0.2723	0.0262	0.2233	0.3257
1989	0.2290	0.2558	0.0251	0.2089	0.3074
1990	0.1941	0.2349	0.0237	0.1908	0.2837
1991	0.1940	0.2069	0.0217	0.1669	0.2518
1992	0.1968	0.1871	0.0201	0.1501	0.2288
1993	0.1863	0.1866	0.0201	0.1497	0.2282
1994	0.2069	0.1841	0.0199	0.1476	0.2253
1995	0.1817	0.1859	0.0200	0.1491	0.2274
1996	0.1819	0.1874	0.0201	0.1503	0.2291
1997	0.1937	0.1747	0.0191	0.1397	0.2144
1998		0.1781	0.0194	0.1426	0.2184
1999		0.1785	0.0208	0.1406	0.2219
2000		0.1707	0.0241	0.1274	0.2217
2001		0.1674	0.0257	0.1216	0.2221
2002		0.1618	0.0275	0.1135	0.2208
2003		0.1576	0.0288	0.1075	0.2198
2004		0.1529	0.0300	0.1012	0.2181
2005		0.1486	0.0309	0.0958	0.2163
2006		0.1443	0.0317	0.0905	0.2142
2007		0.1402	0.0324	0.0857	0.2120
2008		0.1361	0.0330	0.0812	0.2097
2009		0.1321	0.0335	0.0770	0.2072
2010		0.1283	0.0338	0.0730	0.2046
2011		0.1245	0.0341	0.0692	0.2019
2012		0.1208	0.0343	0.0657	0.1991
2013		0.1173	0.0345	0.0624	0.1963
2014		0.1138	0.0346	0.0592	0.1934
2015		0.1104	0.0346	0.0563	0.1905
2016		0.1071	0.0346	0.0534	0.1875
2017		0.1040	0.0345	0.0508	0.1845
2018		0.1008	0.0344	0.0483	0.1815
2019		0.0978	0.0342	0.0459	0.1785
2020		0.0949	0.0341	0.0437	0.1754

Parameter R, males, India

Year	ARIMA model (1, 1, 1)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0644				
1971	0.0789	0.0645	0.0113	0.0423	0.0866
1972	0.0686	0.0654	0.0113	0.0433	0.0875
1973	0.0875	0.0649	0.0113	0.0427	0.0870
1974	0.0571	0.0661	0.0113	0.0439	0.0882
1975	0.0577	0.0643	0.0113	0.0422	0.0864
1976	0.0772	0.0644	0.0113	0.0423	0.0866
1977	0.0509	0.0657	0.0113	0.0435	0.0878
1978	0.0687	0.0641	0.0113	0.0420	0.0863
1979	0.0436	0.0653	0.0113	0.0431	0.0874
1980	0.0586	0.0639	0.0113	0.0417	0.0860
1981	0.0743	0.0648	0.0113	0.0427	0.0869
1982	0.0591	0.0658	0.0113	0.0437	0.0880
1983	0.0621	0.0650	0.0113	0.0429	0.0871
1984	0.0633	0.0652	0.0113	0.0431	0.0874
1985	0.0747	0.0654	0.0113	0.0432	0.0875
1986	0.0560	0.0661	0.0113	0.0440	0.0883
1987	0.0606	0.0651	0.0113	0.0429	0.0872
1988	0.0655	0.0654	0.0113	0.0433	0.0876
1989	0.0660	0.0658	0.0113	0.0436	0.0879
1990	0.0744	0.0659	0.0113	0.0437	0.0880
1991	0.0871	0.0665	0.0113	0.0443	0.0886
1992	0.0725	0.0673	0.0113	0.0451	0.0894
1993	0.0654	0.0665	0.0113	0.0443	0.0886
1994	0.0548	0.0661	0.0113	0.0440	0.0883
1995	0.0766	0.0656	0.0113	0.0434	0.0877
1996	0.0655	0.0669	0.0113	0.0448	0.0891
1997	0.0533	0.0663	0.0113	0.0442	0.0885
1998		0.0657	0.0113	0.0435	0.0878
1999		0.0665	0.0113	0.0443	0.0887
2000		0.0666	0.0113	0.0444	0.0888
2001		0.0667	0.0113	0.0445	0.0888
2002		0.0667	0.0113	0.0446	0.0889
2003		0.0668	0.0113	0.0446	0.0890
2004		0.0669	0.0113	0.0447	0.0891
2005		0.0670	0.0113	0.0448	0.0891
2006		0.0670	0.0113	0.0449	0.0892
2007		0.0671	0.0113	0.0449	0.0893
2008		0.0672	0.0113	0.0450	0.0894
2009		0.0673	0.0113	0.0451	0.0894
2010		0.0673	0.0113	0.0452	0.0895
2011		0.0674	0.0113	0.0452	0.0896
2012		0.0675	0.0113	0.0453	0.0897
2013		0.0676	0.0113	0.0454	0.0897
2014		0.0676	0.0113	0.0455	0.0898
2015		0.0677	0.0113	0.0455	0.0899
2016		0.0678	0.0113	0.0456	0.0899
2017		0.0678	0.0113	0.0457	0.0900
2018		0.0679	0.0113	0.0457	0.0901
2019		0.0680	0.0113	0.0458	0.0902
2020		0.0681	0.0113	0.0459	0.0902

APPENDIX 7B: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, FEMALES, INDIA

Parameter D, females, India

Year	ARIMA model (0, 1, 1)				
	D	Est/ Forecast	Std errors	L95	U95
1970	0.01750				
1971	0.01660	0.01717	0.00164	0.01396	0.02038
1972	0.01720	0.01673	0.00164	0.01352	0.01994
1973	0.01820	0.01648	0.00164	0.01327	0.01969
1974	0.01250	0.01644	0.00164	0.01323	0.01965
1975	0.01320	0.01543	0.00164	0.01222	0.01864
1976	0.01430	0.01471	0.00164	0.01150	0.01792
1977	0.01670	0.01430	0.00164	0.01109	0.01751
1978	0.01230	0.01438	0.00164	0.01117	0.01759
1979	0.01160	0.01369	0.00164	0.01048	0.01690
1980	0.01260	0.01299	0.00164	0.00978	0.01620
1981	0.01430	0.01259	0.00164	0.00938	0.01580
1982	0.00900	0.01255	0.00164	0.00934	0.01576
1983	0.01230	0.01160	0.00164	0.00839	0.01481
1984	0.01100	0.01139	0.00164	0.00818	0.01460
1985	0.01090	0.01099	0.00164	0.00778	0.01420
1986	0.01190	0.01064	0.00164	0.00743	0.01385
1987	0.01020	0.01052	0.00164	0.00731	0.01373
1988	0.00955	0.01013	0.00164	0.00692	0.01334
1989	0.00989	0.00970	0.00164	0.00649	0.01291
1990	0.00965	0.00939	0.00164	0.00618	0.01260
1991	0.01090	0.00910	0.00164	0.00589	0.01231
1992	0.01050	0.00908	0.00164	0.00587	0.01229
1993	0.00980	0.00899	0.00164	0.00578	0.01220
1994	0.01020	0.00879	0.00164	0.00558	0.01200
1995	0.00859	0.00870	0.00164	0.00549	0.01191
1996	0.00749	0.00835	0.00164	0.00514	0.01156
1997	0.00904	0.00786	0.00164	0.00465	0.01108
1998		0.00773	0.00164	0.00452	0.01094
1999		0.00740	0.00166	0.00414	0.01066
2000		0.00706	0.00169	0.00376	0.01037
2001		0.00673	0.00171	0.00338	0.01008
2002		0.00639	0.00173	0.00300	0.00979
2003		0.00606	0.00176	0.00262	0.00950
2004		0.00572	0.00178	0.00224	0.00921
2005		0.00539	0.00180	0.00186	0.00892
2006		0.00505	0.00182	0.00148	0.00863
2007		0.00472	0.00184	0.00111	0.00833
2008		0.00438	0.00187	0.00073	0.00804
2009		0.00405	0.00189	0.00035	0.00775
2010		0.00372	0.00191	-0.00002	0.00745
2011		0.00338	0.00193	-0.00040	0.00716
2012		0.00305	0.00195	-0.00077	0.00687
2013		0.00271	0.00197	-0.00115	0.00657
2014		0.00238	0.00199	-0.00152	0.00628
2015		0.00204	0.00201	-0.00190	0.00598
2016		0.00171	0.00203	-0.00227	0.00568
2017		0.00137	0.00205	-0.00264	0.00539
2018		0.00104	0.00207	-0.00302	0.00509
2019		0.00070	0.00209	-0.00339	0.00479
2020		0.00037	0.00211	-0.00376	0.00450

Parameter E, females, India

Year	ARIMA model (1, 1, 1)				
	E	Est/ Forecast	Std errors	L95	U95
1970	3.95				
1971	3.83	4.05	2.19	-0.24	8.34
1972	4.62	4.13	2.19	-0.15	8.42
1973	4.29	4.19	2.19	-0.09	8.48
1974	6.39	4.52	2.19	0.23	8.81
1975	8.79	4.73	2.19	0.44	9.01
1976	7.41	5.90	2.19	1.62	10.19
1977	6.64	7.37	2.19	3.08	11.66
1978	6.75	7.47	2.19	3.18	11.75
1979	5.06	7.21	2.19	2.92	11.49
1980	6.12	6.98	2.19	2.69	11.27
1981	3.73	6.25	2.19	1.97	10.54
1982	10.93	6.13	2.19	1.84	10.41
1983	4.36	5.69	2.19	1.40	9.97
1984	9.35	7.89	2.19	3.60	12.17
1985	6.38	6.70	2.19	2.41	10.98
1986	5.02	7.91	2.19	3.62	12.20
1987	10.37	7.17	2.19	2.89	11.46
1988	5.72	6.70	2.19	2.41	10.98
1989	7.37	8.28	2.19	3.99	12.56
1990	6.88	7.29	2.19	3.00	11.57
1991	4.72	7.43	2.19	3.14	11.72
1992	4.79	7.11	2.19	2.83	11.40
1993	5.87	6.07	2.19	1.78	10.36
1994	4.30	5.67	2.19	1.38	9.95
1995	3.99	5.78	2.19	1.49	10.06
1996	7.66	5.16	2.19	0.87	9.44
1997	7.82	5.03	2.19	0.74	9.32
1998		6.50	2.19	2.21	10.79
1999		7.19	2.19	2.89	11.49
2000		7.05	2.42	2.31	11.79
2001		7.25	2.51	2.33	12.17
2002		7.31	2.64	2.14	12.48
2003		7.43	2.74	2.05	12.81
2004		7.52	2.85	1.93	13.11
2005		7.63	2.95	1.84	13.42
2006		7.73	3.05	1.74	13.71
2007		7.83	3.15	1.66	14.00
2008		7.93	3.24	1.57	14.28
2009		8.03	3.33	1.50	14.56
2010		8.13	3.42	1.43	14.84
2011		8.23	3.51	1.36	15.11
2012		8.33	3.59	1.30	15.37
2013		8.44	3.67	1.24	15.64
2014		8.54	3.75	1.18	15.89
2015		8.64	3.83	1.13	16.15
2016		8.74	3.91	1.08	16.40
2017		8.84	3.98	1.03	16.65
2018		8.94	4.06	0.99	16.90
2019		9.04	4.13	0.95	17.14
2020		9.15	4.20	0.91	17.38

Parameter F, females, India

Year	ARIMA model (6, 1, 1)				
	F	Est/ Forecast	Std errors	L95	U95
1970	31.40				
1971	33.81	31.23	2.64	26.06	36.41
1972	31.28	31.57	2.64	26.40	36.75
1973	32.42	31.94	2.64	26.77	37.12
1974	28.46	31.40	2.64	26.22	36.57
1975	25.00	30.58	2.64	25.41	35.76
1976	29.95	29.26	2.64	24.08	34.43
1977	29.19	27.94	2.64	22.76	33.11
1978	28.09	29.64	2.64	24.47	34.82
1979	30.37	28.94	2.64	23.76	34.11
1980	28.43	27.33	2.64	22.15	32.51
1981	32.25	28.75	2.64	23.57	33.92
1982	25.00	29.28	2.64	24.11	34.46
1983	29.50	28.64	2.64	23.46	33.82
1984	25.00	27.74	2.64	22.56	32.92
1985	28.00	27.28	2.64	22.10	32.45
1986	29.48	27.87	2.64	22.69	33.04
1987	25.00	27.00	2.64	21.82	32.18
1988	26.73	27.83	2.64	22.65	33.01
1989	26.41	25.63	2.64	20.45	30.80
1990	26.38	26.05	2.64	20.87	31.23
1991	28.39	27.02	2.64	21.84	32.20
1992	28.66	26.22	2.64	21.04	31.40
1993	26.84	27.12	2.64	21.95	32.30
1994	29.85	27.15	2.64	21.98	32.33
1995	30.01	26.91	2.64	21.73	32.09
1996	27.16	28.41	2.64	23.24	33.59
1997	26.64	28.59	2.64	23.41	33.77
1998		26.77	2.64	21.59	31.95
1999		26.90	2.69	21.62	32.17
2000		27.39	2.92	21.67	33.11
2001		26.85	3.00	20.96	32.73
2002		26.55	3.04	20.59	32.50
2003		26.31	3.25	19.95	32.68
2004		26.10	3.34	19.56	32.65
2005		26.11	3.47	19.30	32.91
2006		25.92	3.58	18.90	32.93
2007		25.74	3.65	18.58	32.90
2008		25.56	3.76	18.20	32.92
2009		25.35	3.84	17.83	32.88
2010		25.22	3.93	17.51	32.92
2011		25.05	4.02	17.17	32.92
2012		24.88	4.09	16.86	32.91
2013		24.72	4.17	16.54	32.90
2014		24.54	4.25	16.21	32.86
2015		24.37	4.32	15.91	32.84
2016		24.21	4.39	15.60	32.82
2017		24.04	4.46	15.30	32.78
2018		23.87	4.53	15.00	32.75
2019		23.70	4.59	14.70	32.71
2020		23.54	4.66	14.41	32.66

Parameter G, females, India

Year	ARIMA model (4, 1, 0)				
	G	Est/ Forecast	Std errors	L95	U95
1970	0.000099				
1971	0.000095	0.000100	0.000126	-0.000147	0.000348
1972	0.000142	0.000101	0.000126	-0.000147	0.000348
1973	0.000082	0.000111	0.000126	-0.000136	0.000358
1974	0.000291	0.000107	0.000126	-0.000141	0.000354
1975	0.000541	0.000153	0.000126	-0.000094	0.000400
1976	0.000133	0.000234	0.000126	-0.000013	0.000482
1977	0.000153	0.000263	0.000126	0.000016	0.000510
1978	0.000210	0.000254	0.000126	0.000007	0.000502
1979	0.000164	0.000221	0.000126	-0.000026	0.000469
1980	0.000190	0.000262	0.000126	0.000015	0.000510
1981	0.000059	0.000179	0.000126	-0.000069	0.000426
1982	0.000393	0.000151	0.000126	-0.000096	0.000399
1983	0.000110	0.000215	0.000126	-0.000033	0.000462
1984	0.000255	0.000175	0.000126	-0.000072	0.000422
1985	0.000180	0.000241	0.000126	-0.000006	0.000488
1986	0.000085	0.000164	0.000126	-0.000083	0.000412
1987	0.000264	0.000231	0.000126	-0.000016	0.000479
1988	0.000146	0.000171	0.000126	-0.000076	0.000419
1989	0.000144	0.000185	0.000126	-0.000062	0.000432
1990	0.000173	0.000187	0.000126	-0.000060	0.000435
1991	0.000078	0.000149	0.000126	-0.000099	0.000396
1992	0.000086	0.000169	0.000126	-0.000079	0.000416
1993	0.000102	0.000131	0.000126	-0.000116	0.000379
1994	0.000096	0.000111	0.000126	-0.000137	0.000358
1995	0.000077	0.000116	0.000126	-0.000131	0.000363
1996	0.000182	0.000093	0.000126	-0.000154	0.000340
1997	0.000180	0.000113	0.000126	-0.000134	0.000360
1998		0.000132	0.000126	-0.000116	0.000379
1999		0.000148	0.000129	-0.000105	0.000401
2000		0.000143	0.000133	-0.000117	0.000404
2001		0.000157	0.000141	-0.000120	0.000434
2002		0.000160	0.000145	-0.000124	0.000444
2003		0.000152	0.000156	-0.000154	0.000457
2004		0.000158	0.000161	-0.000158	0.000473
2005		0.000158	0.000166	-0.000167	0.000483
2006		0.000160	0.000172	-0.000177	0.000498
2007		0.000163	0.000177	-0.000184	0.000509
2008		0.000162	0.000183	-0.000196	0.000520
2009		0.000165	0.000188	-0.000203	0.000533
2010		0.000166	0.000192	-0.000211	0.000543
2011		0.000167	0.000197	-0.000220	0.000554
2012		0.000169	0.000202	-0.000227	0.000565
2013		0.000170	0.000207	-0.000235	0.000576
2014		0.000172	0.000211	-0.000242	0.000586
2015		0.000174	0.000216	-0.000249	0.000596
2016		0.000175	0.000220	-0.000256	0.000606
2017		0.000177	0.000224	-0.000263	0.000616
2018		0.000178	0.000228	-0.000270	0.000626
2019		0.000179	0.000232	-0.000276	0.000635
2020		0.000181	0.000236	-0.000283	0.000644

Parameter H, females, India

Year	ARIMA model (6, 1, 0)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.1200				
1971	1.1214	1.1197	0.0104	1.0992	1.1401
1972	1.1197	1.1195	0.0104	1.0991	1.1400
1973	1.1258	1.1193	0.0104	1.0989	1.1398
1974	1.1039	1.1200	0.0104	1.0995	1.1404
1975	1.0950	1.1179	0.0104	1.0975	1.1384
1976	1.1161	1.1147	0.0104	1.0943	1.1352
1977	1.1157	1.1103	0.0104	1.0899	1.1308
1978	1.1094	1.1124	0.0104	1.0920	1.1328
1979	1.1110	1.1088	0.0104	1.0884	1.1293
1980	1.1080	1.1092	0.0104	1.0888	1.1297
1981	1.1282	1.1107	0.0104	1.0903	1.1312
1982	1.0949	1.1096	0.0104	1.0891	1.1300
1983	1.1173	1.1102	0.0104	1.0897	1.1306
1984	1.1032	1.1125	0.0104	1.0920	1.1329
1985	1.1086	1.1062	0.0104	1.0857	1.1266
1986	1.1210	1.1137	0.0104	1.0932	1.1341
1987	1.1011	1.1028	0.0104	1.0823	1.1232
1988	1.1119	1.1150	0.0104	1.0945	1.1354
1989	1.1099	1.1049	0.0104	1.0845	1.1254
1990	1.1073	1.1090	0.0104	1.0886	1.1294
1991	1.1210	1.1104	0.0104	1.0900	1.1308
1992	1.1203	1.1061	0.0104	1.0857	1.1266
1993	1.1159	1.1143	0.0104	1.0938	1.1347
1994	1.1154	1.1120	0.0104	1.0916	1.1325
1995	1.1182	1.1132	0.0104	1.0928	1.1337
1996	1.1043	1.1162	0.0104	1.0958	1.1367
1997	1.1060	1.1130	0.0104	1.0926	1.1335
1998		1.1125	0.0104	1.0921	1.1329
1999		1.1105	0.0105	1.0899	1.1311
2000		1.1109	0.0107	1.0900	1.1319
2001		1.1079	0.0111	1.0862	1.1297
2002		1.1095	0.0112	1.0875	1.1315
2003		1.1086	0.0120	1.0850	1.1322
2004		1.1074	0.0121	1.0837	1.1311
2005		1.1086	0.0127	1.0837	1.1335
2006		1.1072	0.0130	1.0817	1.1327
2007		1.1076	0.0132	1.0817	1.1335
2008		1.1067	0.0136	1.0800	1.1334
2009		1.1064	0.0138	1.0794	1.1335
2010		1.1065	0.0142	1.0787	1.1343
2011		1.1056	0.0144	1.0773	1.1339
2012		1.1058	0.0147	1.0770	1.1346
2013		1.1051	0.0150	1.0757	1.1345
2014		1.1049	0.0152	1.0751	1.1347
2015		1.1046	0.0155	1.0742	1.1350
2016		1.1041	0.0157	1.0732	1.1349
2017		1.1040	0.0160	1.0726	1.1353
2018		1.1034	0.0163	1.0716	1.1353
2019		1.1032	0.0165	1.0709	1.1355
2020		1.1029	0.0167	1.0700	1.1357

Parameter M, females, India

Year	ARIMA model (0, 1, 1)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.3901				
1971	0.3978	0.3822	0.0249	0.3334	0.4309
1972	0.4269	0.3826	0.0249	0.3339	0.4313
1973	0.3907	0.3984	0.0249	0.3497	0.4471
1974	0.3946	0.3863	0.0249	0.3376	0.4351
1975	0.4447	0.3828	0.0249	0.3341	0.4316
1976	0.3594	0.4080	0.0249	0.3593	0.4568
1977	0.4023	0.3740	0.0249	0.3253	0.4228
1978	0.3862	0.3812	0.0249	0.3325	0.4300
1979	0.3836	0.3760	0.0249	0.3272	0.4247
1980	0.3355	0.3721	0.0249	0.3234	0.4209
1981	0.3163	0.3446	0.0249	0.2958	0.3933
1982	0.3160	0.3215	0.0249	0.2727	0.3702
1983	0.2912	0.3106	0.0249	0.2619	0.3593
1984	0.3104	0.2923	0.0249	0.2435	0.3410
1985	0.2930	0.2940	0.0249	0.2453	0.3428
1986	0.2928	0.2855	0.0249	0.2368	0.3343
1987	0.2961	0.2815	0.0249	0.2328	0.3302
1988	0.2606	0.2814	0.0249	0.2326	0.3301
1989	0.2328	0.2623	0.0249	0.2136	0.3110
1990	0.2133	0.2386	0.0249	0.1898	0.2873
1991	0.2052	0.2171	0.0249	0.1683	0.2658
1992	0.2094	0.2028	0.0249	0.1540	0.2515
1993	0.1859	0.1984	0.0249	0.1496	0.2471
1994	0.1881	0.1838	0.0249	0.1350	0.2325
1995	0.1903	0.1781	0.0249	0.1294	0.2269
1996	0.1924	0.1767	0.0249	0.1280	0.2255
1997	0.1918	0.1772	0.0249	0.1284	0.2259
1998		0.1771	0.0249	0.1283	0.2258
1999		0.1691	0.0282	0.1138	0.2244
2000		0.1612	0.0312	0.1000	0.2223
2001		0.1533	0.0339	0.0868	0.2197
2002		0.1453	0.0364	0.0739	0.2167
2003		0.1374	0.0388	0.0613	0.2134
2004		0.1294	0.0410	0.0490	0.2098
2005		0.1215	0.0431	0.0369	0.2060
2006		0.1135	0.0452	0.0251	0.2020
2007		0.1056	0.0471	0.0133	0.1979
2008		0.0977	0.0489	0.0018	0.1936
2009		0.0897	0.0507	-0.0097	0.1891
2010		0.0818	0.0524	-0.0210	0.1845
2011		0.0738	0.0541	-0.0322	0.1799
2012		0.0659	0.0557	-0.0433	0.1751
2013		0.0580	0.0573	-0.0543	0.1702
2014		0.0500	0.0588	-0.0652	0.1653
2015		0.0421	0.0603	-0.0761	0.1603
2016		0.0341	0.0618	-0.0869	0.1552
2017		0.0262	0.0632	-0.0976	0.1500
2018		0.0183	0.0646	-0.1083	0.1448
2019		0.0103	0.0659	-0.1189	0.1395
2020		0.0024	0.0673	-0.1295	0.1342

Parameter R, females, India

Year	ARIMA model (4, 1, 0)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0808				
1971	0.0732	0.0810	0.0131	0.0553	0.1068
1972	0.0827	0.0788	0.0131	0.0530	0.1045
1973	0.0870	0.0820	0.0131	0.0563	0.1078
1974	0.0702	0.0807	0.0131	0.0550	0.1065
1975	0.0593	0.0763	0.0131	0.0505	0.1020
1976	0.0997	0.0773	0.0131	0.0516	0.1031
1977	0.0698	0.0859	0.0131	0.0601	0.1116
1978	0.0699	0.0669	0.0131	0.0411	0.0926
1979	0.0572	0.0778	0.0131	0.0521	0.1035
1980	0.0668	0.0757	0.0131	0.0500	0.1015
1981	0.0817	0.0700	0.0131	0.0442	0.0957
1982	0.0653	0.0698	0.0131	0.0441	0.0956
1983	0.0757	0.0639	0.0131	0.0381	0.0896
1984	0.0873	0.0755	0.0131	0.0498	0.1012
1985	0.0898	0.0782	0.0131	0.0524	0.1039
1986	0.0744	0.0776	0.0131	0.0519	0.1033
1987	0.0626	0.0798	0.0131	0.0541	0.1056
1988	0.0824	0.0808	0.0131	0.0551	0.1066
1989	0.0920	0.0827	0.0131	0.0570	0.1085
1990	0.0812	0.0766	0.0131	0.0509	0.1024
1991	0.0967	0.0762	0.0131	0.0505	0.1020
1992	0.1046	0.0908	0.0131	0.0651	0.1166
1993	0.0970	0.0927	0.0131	0.0670	0.1184
1994	0.0791	0.0924	0.0131	0.0667	0.1182
1995	0.0949	0.0943	0.0131	0.0686	0.1200
1996	0.0939	0.0995	0.0131	0.0738	0.1252
1997	0.0765	0.0904	0.0131	0.0647	0.1161
1998		0.0847	0.0131	0.0589	0.1104
1999		0.0918	0.0138	0.0648	0.1189
2000		0.0878	0.0138	0.0606	0.1149
2001		0.0838	0.0147	0.0549	0.1127
2002		0.0875	0.0164	0.0552	0.1197
2003		0.0896	0.0170	0.0563	0.1229
2004		0.0876	0.0174	0.0536	0.1216
2005		0.0870	0.0182	0.0514	0.1226
2006		0.0887	0.0190	0.0515	0.1259
2007		0.0892	0.0195	0.0510	0.1275
2008		0.0886	0.0200	0.0494	0.1278
2009		0.0887	0.0207	0.0483	0.1292
2010		0.0895	0.0213	0.0478	0.1312
2011		0.0897	0.0218	0.0471	0.1324
2012		0.0897	0.0223	0.0460	0.1333
2013		0.0900	0.0228	0.0452	0.1347
2014		0.0904	0.0234	0.0446	0.1361
2015		0.0906	0.0238	0.0438	0.1373
2016		0.0907	0.0243	0.0430	0.1384
2017		0.0910	0.0248	0.0424	0.1396
2018		0.0913	0.0253	0.0417	0.1408
2019		0.0915	0.0257	0.0410	0.1419
2020		0.0917	0.0262	0.0404	0.1430

APPENDIX 7C: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, RURAL MALES, INDIA

Parameter D, rural males, India

Year		ARIMA model (1, 1, 1)			
	D	Est/ Forecast	Std errors	L95	U95
1970	0.00513				
1971	0.00440	0.00514	0.00168	0.00184	0.00844
1972	0.00663	0.00499	0.00168	0.00169	0.00828
1973	0.00323	0.00548	0.00168	0.00219	0.00878
1974	0.00484	0.00475	0.00168	0.00145	0.00804
1975	0.00158	0.00511	0.00168	0.00181	0.00840
1976	0.00359	0.00440	0.00168	0.00111	0.00770
1977	0.00484	0.00485	0.00168	0.00155	0.00814
1978	0.00766	0.00513	0.00168	0.00183	0.00842
1979	0.00446	0.00575	0.00168	0.00246	0.00905
1980	0.00291	0.00506	0.00168	0.00176	0.00836
1981	0.00375	0.00473	0.00168	0.00143	0.00803
1982	0.00347	0.00492	0.00168	0.00162	0.00822
1983	0.00843	0.00487	0.00168	0.00157	0.00816
1984	0.00569	0.00596	0.00168	0.00266	0.00925
1985	0.00505	0.00537	0.00168	0.00207	0.00866
1986	0.00431	0.00523	0.00168	0.00194	0.00853
1987	0.00439	0.00508	0.00168	0.00178	0.00838
1988	0.00389	0.00511	0.00168	0.00181	0.00840
1989	0.00619	0.00500	0.00168	0.00171	0.00830
1990	0.00479	0.00551	0.00168	0.00222	0.00881
1991	0.00707	0.00522	0.00168	0.00192	0.00851
1992	0.00509	0.00572	0.00168	0.00242	0.00902
1993	0.00560	0.00530	0.00168	0.00200	0.00859
1994	0.00759	0.00541	0.00168	0.00212	0.00871
1995	0.00742	0.00586	0.00168	0.00256	0.00915
1996	0.00442	0.00583	0.00168	0.00253	0.00912
1997	0.00522	0.00518	0.00168	0.00188	0.00848
1998		0.00536	0.00168	0.00207	0.00866
1999		0.00540	0.00172	0.00203	0.00877
2000		0.00542	0.00172	0.00204	0.00879
2001		0.00543	0.00172	0.00205	0.00880
2002		0.00544	0.00172	0.00206	0.00881
2003		0.00545	0.00172	0.00207	0.00882
2004		0.00546	0.00172	0.00208	0.00883
2005		0.00547	0.00172	0.00209	0.00884
2006		0.00548	0.00172	0.00210	0.00885
2007		0.00549	0.00172	0.00211	0.00886
2008		0.00549	0.00172	0.00212	0.00887
2009		0.00550	0.00172	0.00213	0.00888
2010		0.00551	0.00172	0.00214	0.00889
2011		0.00552	0.00172	0.00215	0.00890
2012		0.00553	0.00172	0.00216	0.00891
2013		0.00554	0.00172	0.00217	0.00892
2014		0.00555	0.00172	0.00218	0.00893
2015		0.00556	0.00172	0.00218	0.00894
2016		0.00557	0.00172	0.00219	0.00895
2017		0.00558	0.00172	0.00220	0.00896
2018		0.00559	0.00172	0.00221	0.00897
2019		0.00560	0.00172	0.00222	0.00898
2020		0.00561	0.00172	0.00223	0.00899

Parameter E, rural males, India

Year	ARIMA model (1, 1, 1)				
	E	Est/ Forecast	Std errors	L95	U95
1970	10.15				
1971	15.00	9.94	8.67	-7.05	26.94
1972	12.11	9.79	8.67	-7.21	26.78
1973	11.25	10.63	8.67	-6.36	27.63
1974	12.09	10.69	8.67	-6.30	27.69
1975	13.54	10.57	8.67	-6.42	27.57
1976	7.62	10.66	8.67	-6.33	27.66
1977	15.00	10.97	8.67	-6.02	27.97
1978	4.29	10.07	8.67	-6.92	27.07
1979	42.19	10.77	8.67	-6.22	27.77
1980	15.00	9.53	8.67	-7.46	26.53
1981	24.34	16.07	8.67	-0.93	33.06
1982	6.02	15.69	8.67	-1.31	32.68
1983	3.06	17.11	8.67	0.12	34.10
1984	18.90	14.42	8.67	-2.58	31.41
1985	14.95	11.88	8.67	-5.12	28.87
1986	12.84	13.10	8.67	-3.89	30.10
1987	12.64	13.23	8.67	-3.76	30.22
1988	18.87	12.89	8.67	-4.10	29.89
1989	4.01	12.65	8.67	-4.34	29.65
1990	13.71	13.59	8.67	-3.41	30.58
1991	6.87	11.36	8.67	-5.63	28.36
1992	5.19	11.54	8.67	-5.45	28.54
1993	6.72	10.26	8.67	-6.73	27.25
1994	2.59	8.92	8.67	-8.07	25.92
1995	2.79	8.15	8.67	-8.85	25.14
1996	3.83	6.69	8.67	-10.30	23.69
1997	11.64	5.60	8.67	-11.39	22.60
1998		5.05	8.67	-11.94	22.05
1999		6.16	8.67	-10.84	23.15
2000		5.68	8.87	-11.70	23.06
2001		5.53	8.99	-12.10	23.15
2002		5.30	9.13	-12.59	23.20
2003		5.10	9.26	-13.06	23.25
2004		4.89	9.39	-13.52	23.30
2005		4.68	9.52	-13.98	23.34
2006		4.47	9.65	-14.44	23.38
2007		4.26	9.77	-14.90	23.42
2008		4.05	9.90	-15.35	23.45
2009		3.84	10.02	-15.80	23.48
2010		3.63	10.14	-16.24	23.51
2011		3.43	10.26	-16.68	23.54
2012		3.22	10.38	-17.12	23.56
2013		3.01	10.49	-17.56	23.58
2014		2.80	10.61	-18.00	23.59
2015		2.59	10.72	-18.43	23.61
2016		2.38	10.84	-18.86	23.62
2017		2.17	10.95	-19.29	23.63
2018		1.96	11.06	-19.71	23.64
2019		1.75	11.17	-20.14	23.64
2020		1.54	11.28	-20.56	23.65

Parameter F, rural males, India

Year	ARIMA model (5, 1, 0)				
	F	Est/ Forecast	Std errors	L95	U95
1970	26.40				
1971	27.51	26.59	3.66	19.42	33.76
1972	26.31	26.81	3.66	19.64	33.97
1973	25.00	27.14	3.66	19.97	34.31
1974	26.13	26.97	3.66	19.80	34.13
1975	25.00	26.99	3.66	19.82	34.15
1976	26.84	27.76	3.66	20.59	34.93
1977	23.26	27.26	3.66	20.09	34.43
1978	34.56	26.65	3.66	19.48	33.82
1979	25.00	26.25	3.66	19.09	33.42
1980	23.24	28.32	3.66	21.15	35.48
1981	25.00	26.17	3.66	19.00	33.33
1982	25.00	25.71	3.66	18.54	32.88
1983	34.67	29.85	3.66	22.69	37.02
1984	25.00	29.03	3.66	21.86	36.20
1985	27.27	27.01	3.66	19.85	34.18
1986	25.04	24.97	3.66	17.80	32.13
1987	23.93	26.77	3.66	19.60	33.93
1988	24.43	30.23	3.66	23.06	37.40
1989	31.78	28.72	3.66	21.55	35.88
1990	25.00	27.07	3.66	19.90	34.24
1991	28.90	27.94	3.66	20.77	35.11
1992	27.15	25.18	3.66	18.01	32.35
1993	27.38	26.44	3.66	19.27	33.61
1994	33.01	29.17	3.66	22.00	36.34
1995	34.56	28.76	3.66	21.59	35.92
1996	32.91	29.57	3.66	22.41	36.74
1997	28.64	29.98	3.66	22.81	37.14
1998		29.26	3.66	22.10	36.43
1999		31.24	3.66	24.07	38.41
2000		34.00	3.73	26.68	41.31
2001		33.97	3.73	26.65	41.29
2002		31.98	3.75	24.64	39.32
2003		30.85	4.09	22.83	38.88
2004		31.65	4.30	23.22	40.08
2005		33.42	4.36	24.87	41.97
2006		34.43	4.38	25.84	43.02
2007		33.80	4.39	25.20	42.40
2008		32.81	4.49	24.00	41.62
2009		32.70	4.66	23.58	41.83
2010		33.64	4.76	24.31	42.97
2011		34.68	4.80	25.26	44.09
2012		34.87	4.82	25.42	44.33
2013		34.39	4.87	24.84	43.94
2014		34.06	4.97	24.31	43.80
2015		34.39	5.07	24.45	44.33
2016		35.13	5.14	25.06	45.19
2017		35.62	5.17	25.48	45.76
2018		35.59	5.21	25.37	45.81
2019		35.37	5.28	25.02	45.71
2020		35.41	5.36	24.91	45.92

Parameter G, rural males, India

Year		ARIMA model (0, 1, 2)			
	G	Est/ Forecast	Std errors	L95	U95
1970	0.00053				
1971	0.00058	0.00052	0.00013	0.00026	0.00078
1972	0.00052	0.00051	0.00013	0.00025	0.00077
1973	0.00057	0.00050	0.00013	0.00024	0.00076
1974	0.00051	0.00049	0.00013	0.00023	0.00075
1975	0.00081	0.00049	0.00013	0.00023	0.00074
1976	0.00050	0.00046	0.00013	0.00021	0.00072
1977	0.00067	0.00047	0.00013	0.00021	0.00073
1978	0.00030	0.00045	0.00013	0.00019	0.00071
1979	0.00053	0.00046	0.00013	0.00020	0.00071
1980	0.00048	0.00044	0.00013	0.00018	0.00070
1981	0.00051	0.00043	0.00013	0.00017	0.00069
1982	0.00054	0.00042	0.00013	0.00016	0.00068
1983	0.00016	0.00041	0.00013	0.00015	0.00067
1984	0.00047	0.00042	0.00013	0.00016	0.00067
1985	0.00042	0.00040	0.00013	0.00014	0.00065
1986	0.00053	0.00039	0.00013	0.00013	0.00065
1987	0.00042	0.00037	0.00013	0.00012	0.00063
1988	0.00045	0.00037	0.00013	0.00011	0.00063
1989	0.00024	0.00036	0.00013	0.00010	0.00062
1990	0.00052	0.00036	0.00013	0.00010	0.00062
1991	0.00030	0.00034	0.00013	0.00008	0.00060
1992	0.00033	0.00034	0.00013	0.00008	0.00060
1993	0.00029	0.00033	0.00013	0.00007	0.00059
1994	0.00018	0.00032	0.00013	0.00006	0.00058
1995	0.00019	0.00031	0.00013	0.00006	0.00057
1996	0.00037	0.00030	0.00013	0.00005	0.00056
1997	0.00048	0.00029	0.00013	0.00003	0.00055
1998		0.00028	0.00013	0.00002	0.00053
1999		0.00027	0.00013	0.00002	0.00053
2000		0.00026	0.00013	0.00001	0.00052
2001		0.00026	0.00013	0.00000	0.00051
2002		0.00025	0.00013	-0.00001	0.00051
2003		0.00024	0.00013	-0.00002	0.00050
2004		0.00023	0.00013	-0.00003	0.00049
2005		0.00022	0.00013	-0.00004	0.00048
2006		0.00021	0.00013	-0.00005	0.00047
2007		0.00020	0.00013	-0.00006	0.00046
2008		0.00019	0.00013	-0.00006	0.00045
2009		0.00019	0.00013	-0.00007	0.00044
2010		0.00018	0.00013	-0.00008	0.00044
2011		0.00017	0.00013	-0.00009	0.00043
2012		0.00016	0.00013	-0.00010	0.00042
2013		0.00015	0.00013	-0.00011	0.00041
2014		0.00014	0.00013	-0.00012	0.00040
2015		0.00013	0.00013	-0.00013	0.00039
2016		0.00012	0.00013	-0.00013	0.00038
2017		0.00011	0.00013	-0.00014	0.00037
2018		0.00011	0.00013	-0.00015	0.00036
2019		0.00010	0.00013	-0.00016	0.00036
2020		0.00009	0.00013	-0.00017	0.00035

Parameter H, rural males, India

Year	ARIMA model (4, 1, 0)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.0985				
1971	1.0960	1.0987	0.0063	1.0863	1.1111
1972	1.1006	1.0985	0.0063	1.0862	1.1109
1973	1.0975	1.0984	0.0063	1.0860	1.1108
1974	1.0989	1.1002	0.0063	1.0878	1.1125
1975	1.0913	1.0979	0.0063	1.0856	1.1103
1976	1.1014	1.0977	0.0063	1.0853	1.1101
1977	1.0942	1.0981	0.0063	1.0857	1.1105
1978	1.1082	1.1006	0.0063	1.0883	1.1130
1979	1.0953	1.0971	0.0063	1.0847	1.1094
1980	1.0979	1.0994	0.0063	1.0870	1.1118
1981	1.0970	1.0968	0.0063	1.0844	1.1091
1982	1.0945	1.0991	0.0063	1.0867	1.1115
1983	1.1156	1.1031	0.0063	1.0908	1.1155
1984	1.0975	1.0983	0.0063	1.0860	1.1107
1985	1.0998	1.1047	0.0063	1.0924	1.1171
1986	1.0947	1.0938	0.0063	1.0814	1.1061
1987	1.0985	1.1007	0.0063	1.0883	1.1131
1988	1.0973	1.1071	0.0063	1.0947	1.1194
1989	1.1073	1.0996	0.0063	1.0872	1.1119
1990	1.0938	1.0998	0.0063	1.0874	1.1121
1991	1.1038	1.0996	0.0063	1.0872	1.1120
1992	1.1023	1.0962	0.0063	1.0838	1.1086
1993	1.1051	1.1035	0.0063	1.0911	1.1159
1994	1.1118	1.1040	0.0063	1.0917	1.1164
1995	1.1086	1.1006	0.0063	1.0882	1.1129
1996	1.0973	1.1070	0.0063	1.0947	1.1194
1997	1.0959	1.1030	0.0063	1.0906	1.1154
1998		1.1029	0.0063	1.0905	1.1152
1999		1.1087	0.0064	1.0962	1.1212
2000		1.1077	0.0067	1.0946	1.1209
2001		1.1014	0.0067	1.0882	1.1146
2002		1.0998	0.0070	1.0862	1.1135
2003		1.1029	0.0077	1.0879	1.1180
2004		1.1072	0.0079	1.0917	1.1227
2005		1.1072	0.0081	1.0912	1.1231
2006		1.1040	0.0081	1.0880	1.1199
2007		1.1025	0.0084	1.0861	1.1188
2008		1.1039	0.0086	1.0871	1.1208
2009		1.1066	0.0089	1.0892	1.1241
2010		1.1071	0.0091	1.0893	1.1248
2011		1.1056	0.0091	1.0877	1.1234
2012		1.1044	0.0092	1.0863	1.1226
2013		1.1051	0.0094	1.0866	1.1236
2014		1.1067	0.0097	1.0878	1.1257
2015		1.1073	0.0098	1.0881	1.1265
2016		1.1067	0.0099	1.0873	1.1261
2017		1.1060	0.0100	1.0863	1.1256
2018		1.1063	0.0102	1.0864	1.1262
2019		1.1073	0.0104	1.0870	1.1276
2020		1.1078	0.0105	1.0873	1.1284

Parameter M, rural males, India

Year	Logistic ARIMA model (1, 1, 1)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.4179				
1971	0.3961	0.4089	0.0497	0.3145	0.5087
1972	0.4400	0.3952	0.0351	0.3280	0.4654
1973	0.3705	0.3911	0.0349	0.3244	0.4610
1974	0.3990	0.4018	0.0352	0.3343	0.4721
1975	0.4499	0.3653	0.0340	0.3007	0.4338
1976	0.3802	0.3924	0.0349	0.3256	0.4624
1977	0.4816	0.4107	0.0355	0.3426	0.4814
1978	0.3580	0.3921	0.0349	0.3253	0.4620
1979	0.4068	0.4251	0.0358	0.3561	0.4963
1980	0.3547	0.3612	0.0338	0.2969	0.4293
1981	0.3300	0.3738	0.0343	0.3084	0.4427
1982	0.3448	0.3358	0.0327	0.2740	0.4020
1983	0.3148	0.3201	0.0319	0.2600	0.3850
1984	0.3509	0.3207	0.0320	0.2605	0.3856
1985	0.3046	0.3103	0.0314	0.2513	0.3742
1986	0.3194	0.3210	0.0320	0.2608	0.3859
1987	0.2976	0.2964	0.0306	0.2391	0.3589
1988	0.2659	0.2981	0.0307	0.2406	0.3608
1989	0.2427	0.2767	0.0294	0.2219	0.3369
1990	0.2126	0.2491	0.0275	0.1982	0.3057
1991	0.2125	0.2245	0.0256	0.1774	0.2775
1992	0.2064	0.2035	0.0238	0.1598	0.2530
1993	0.1954	0.2000	0.0235	0.1569	0.2490
1994	0.2242	0.1932	0.0229	0.1513	0.2410
1995	0.1946	0.1923	0.0229	0.1505	0.2400
1996	0.1958	0.2032	0.0238	0.1596	0.2528
1997	0.2164	0.1862	0.0223	0.1455	0.2328
1998		0.1902	0.0227	0.1488	0.2374
1999		0.1969	0.0240	0.1531	0.2469
2000		0.1828	0.0279	0.1330	0.2419
2001		0.1822	0.0293	0.1302	0.2446
2002		0.1733	0.0311	0.1187	0.2402
2003		0.1699	0.0323	0.1138	0.2398
2004		0.1632	0.0334	0.1058	0.2361
2005		0.1589	0.0343	0.1005	0.2341
2006		0.1533	0.0350	0.0943	0.2305
2007		0.1488	0.0356	0.0893	0.2277
2008		0.1438	0.0360	0.0841	0.2242
2009		0.1393	0.0364	0.0795	0.2210
2010		0.1347	0.0366	0.0751	0.2174
2011		0.1304	0.0368	0.0710	0.2139
2012		0.1261	0.0369	0.0671	0.2103
2013		0.1220	0.0369	0.0635	0.2067
2014		0.1179	0.0368	0.0601	0.2030
2015		0.1141	0.0367	0.0569	0.1993
2016		0.1103	0.0366	0.0538	0.1955
2017		0.1066	0.0364	0.0509	0.1918
2018		0.1031	0.0361	0.0482	0.1880
2019		0.0996	0.0358	0.0457	0.1843
2020		0.0963	0.0355	0.0433	0.1806

Parameter R, rural males, India

Year	ARIMA model (1, 1, 1)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0639				
1971	0.0681	0.0639	0.0125	0.0395	0.0884
1972	0.0634	0.0636	0.0125	0.0392	0.0881
1973	0.0874	0.0657	0.0125	0.0412	0.0901
1974	0.0607	0.0629	0.0125	0.0384	0.0873
1975	0.0559	0.0739	0.0125	0.0494	0.0983
1976	0.0829	0.0697	0.0125	0.0453	0.0941
1977	0.0417	0.0626	0.0125	0.0381	0.0870
1978	0.0740	0.0733	0.0125	0.0489	0.0978
1979	0.0509	0.0595	0.0125	0.0350	0.0839
1980	0.0583	0.0666	0.0125	0.0422	0.0911
1981	0.0677	0.0605	0.0125	0.0361	0.0850
1982	0.0564	0.0590	0.0125	0.0346	0.0834
1983	0.0663	0.0631	0.0125	0.0387	0.0875
1984	0.0607	0.0599	0.0125	0.0355	0.0844
1985	0.0786	0.0627	0.0125	0.0383	0.0872
1986	0.0564	0.0605	0.0125	0.0361	0.0850
1987	0.0645	0.0689	0.0125	0.0444	0.0933
1988	0.0764	0.0638	0.0125	0.0394	0.0883
1989	0.0723	0.0631	0.0125	0.0387	0.0876
1990	0.0734	0.0682	0.0125	0.0438	0.0926
1991	0.0843	0.0696	0.0125	0.0452	0.0941
1992	0.0839	0.0701	0.0125	0.0456	0.0945
1993	0.0798	0.0752	0.0125	0.0507	0.0996
1994	0.0544	0.0787	0.0125	0.0542	0.1031
1995	0.0794	0.0813	0.0125	0.0569	0.1058
1996	0.0678	0.0697	0.0125	0.0453	0.0942
1997	0.0516	0.0742	0.0125	0.0498	0.0986
1998		0.0734	0.0125	0.0490	0.0979
1999		0.0639	0.0125	0.0394	0.0884
2000		0.0681	0.0134	0.0418	0.0945
2001		0.0663	0.0136	0.0397	0.0930
2002		0.0672	0.0140	0.0397	0.0947
2003		0.0669	0.0143	0.0388	0.0950
2004		0.0671	0.0147	0.0383	0.0958
2005		0.0671	0.0150	0.0377	0.0964
2006		0.0671	0.0153	0.0372	0.0971
2007		0.0672	0.0156	0.0366	0.0977
2008		0.0672	0.0159	0.0361	0.0983
2009		0.0672	0.0162	0.0356	0.0989
2010		0.0673	0.0165	0.0350	0.0995
2011		0.0673	0.0167	0.0345	0.1001
2012		0.0674	0.0170	0.0340	0.1007
2013		0.0674	0.0173	0.0335	0.1013
2014		0.0675	0.0176	0.0331	0.1019
2015		0.0675	0.0178	0.0326	0.1024
2016		0.0676	0.0181	0.0321	0.1030
2017		0.0676	0.0183	0.0317	0.1035
2018		0.0676	0.0186	0.0312	0.1041
2019		0.0677	0.0188	0.0308	0.1046
2020		0.0677	0.0191	0.0303	0.1051

APPENDIX 7D: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, RURAL FEMALES, INDIA

Parameter D, rural females, India

Year	ARIMA model (2, 1, 0)				
	D	Est/ Forecast	Std errors	L95	U95
1970	0.01632				
1971	0.01790	0.01632	0.00255	0.01132	0.02132
1972	0.01938	0.01723	0.00231	0.01270	0.02176
1973	0.02055	0.01825	0.00225	0.01384	0.02265
1974	0.01347	0.01960	0.00225	0.01519	0.02401
1975	0.01647	0.01690	0.00225	0.01249	0.02131
1976	0.01593	0.01652	0.00225	0.01212	0.02093
1977	0.01920	0.01552	0.00225	0.01112	0.01993
1978	0.01360	0.01762	0.00225	0.01321	0.02202
1979	0.01298	0.01578	0.00225	0.01137	0.02019
1980	0.01481	0.01459	0.00225	0.01018	0.01900
1981	0.01547	0.01399	0.00225	0.00959	0.01840
1982	0.01051	0.01470	0.00225	0.01030	0.01911
1983	0.01401	0.01295	0.00225	0.00854	0.01735
1984	0.01351	0.01332	0.00225	0.00891	0.01772
1985	0.01352	0.01297	0.00225	0.00856	0.01737
1986	0.01360	0.01363	0.00225	0.00922	0.01804
1987	0.01183	0.01355	0.00225	0.00915	0.01796
1988	0.01072	0.01273	0.00225	0.00833	0.01714
1989	0.01097	0.01170	0.00225	0.00730	0.01611
1990	0.01064	0.01109	0.00225	0.00669	0.01550
1991	0.01197	0.01075	0.00225	0.00635	0.01516
1992	0.01102	0.01135	0.00225	0.00694	0.01576
1993	0.01149	0.01121	0.00225	0.00680	0.01562
1994	0.00938	0.01146	0.00225	0.00705	0.01587
1995	0.00937	0.01037	0.00225	0.00597	0.01478
1996	0.00857	0.00985	0.00225	0.00545	0.01426
1997	0.01016	0.00899	0.00225	0.00458	0.01340
1998		0.00951	0.00225	0.00511	0.01392
1999		0.00949	0.00249	0.00461	0.01437
2000		0.00965	0.00275	0.00425	0.01505
2001		0.00957	0.00308	0.00353	0.01562
2002		0.00958	0.00333	0.00305	0.01610
2003		0.00959	0.00357	0.00260	0.01658
2004		0.00958	0.00379	0.00215	0.01702
2005		0.00958	0.00400	0.00174	0.01743
2006		0.00958	0.00420	0.00135	0.01782
2007		0.00958	0.00440	0.00097	0.01820
2008		0.00958	0.00458	0.00061	0.01856
2009		0.00958	0.00476	0.00026	0.01890
2010		0.00958	0.00493	-0.00007	0.01924
2011		0.00958	0.00509	-0.00039	0.01956
2012		0.00958	0.00525	-0.00070	0.01987
2013		0.00958	0.00540	-0.00101	0.02018
2014		0.00958	0.00555	-0.00130	0.02047
2015		0.00958	0.00570	-0.00159	0.02076
2016		0.00958	0.00584	-0.00187	0.02104
2017		0.00958	0.00598	-0.00214	0.02131
2018		0.00958	0.00612	-0.00241	0.02158
2019		0.00958	0.00625	-0.00267	0.02184
2020		0.00958	0.00638	-0.00293	0.02209

Parameter E, rural females, India

Year	ARIMA model (3, 1, 0)				
	E	Est/ Forecast	Std errors	L95	U95
1970	5.41				
1971	4.14	5.50	1.80	1.98	9.02
1972	4.40	5.35	1.80	1.83	8.87
1973	4.50	4.97	1.80	1.45	8.49
1974	10.91	4.94	1.80	1.42	8.46
1975	5.08	5.73	1.80	2.21	9.25
1976	7.35	7.15	1.80	3.63	10.67
1977	6.96	6.28	1.80	2.76	9.80
1978	6.56	8.38	1.80	4.86	11.90
1979	4.07	6.55	1.80	3.03	10.07
1980	5.01	6.66	1.80	3.14	10.18
1981	5.43	5.73	1.80	2.21	9.25
1982	5.69	5.70	1.80	2.18	9.22
1983	4.28	5.20	1.80	1.68	8.72
1984	5.19	5.41	1.80	1.89	8.93
1985	5.92	5.23	1.80	1.71	8.75
1986	4.79	5.59	1.80	2.07	9.11
1987	6.98	5.32	1.80	1.80	8.84
1988	5.46	5.68	1.80	2.16	9.20
1989	5.90	6.31	1.80	2.79	9.83
1990	5.43	5.74	1.80	2.22	9.26
1991	3.99	6.34	1.80	2.82	9.86
1992	7.25	5.48	1.80	1.96	9.00
1993	6.84	5.61	1.80	2.09	9.13
1994	8.50	6.39	1.80	2.87	9.91
1995	5.00	6.48	1.80	2.96	10.00
1996	8.05	7.49	1.80	3.97	11.01
1997	7.17	6.82	1.80	3.30	10.34
1998		7.87	1.80	4.35	11.39
1999		6.93	1.82	3.35	10.50
2000		7.90	1.96	4.06	11.74
2001		7.53	2.02	3.58	11.48
2002		7.93	2.24	3.54	12.32
2003		7.68	2.30	3.17	12.19
2004		8.05	2.42	3.31	12.80
2005		7.96	2.50	3.06	12.85
2006		8.18	2.61	3.05	13.30
2007		8.15	2.69	2.87	13.42
2008		8.33	2.78	2.87	13.79
2009		8.35	2.86	2.74	13.96
2010		8.49	2.95	2.71	14.27
2011		8.53	3.03	2.61	14.46
2012		8.66	3.11	2.57	14.74
2013		8.72	3.18	2.49	14.95
2014		8.83	3.25	2.45	15.20
2015		8.90	3.32	2.38	15.41
2016		9.00	3.40	2.34	15.65
2017		9.08	3.46	2.29	15.87
2018		9.17	3.53	2.25	16.10
2019		9.25	3.60	2.20	16.31
2020		9.35	3.66	2.16	16.53

Parameter F, rural females, India

Year	ARIMA model (2, 1, 2)				
	F	Est/ Forecast	Std errors	L95	U95
1970	29.36				
1971	33.09	29.26	1.89	25.56	32.96
1972	31.47	29.73	1.89	26.03	33.44
1973	32.27	30.34	1.89	26.64	34.04
1974	25.83	29.64	1.89	25.94	33.34
1975	29.24	29.43	1.89	25.73	33.13
1976	29.99	27.95	1.89	24.25	31.65
1977	29.47	29.85	1.89	26.15	33.55
1978	28.42	28.68	1.89	24.97	32.38
1979	32.01	29.05	1.89	25.35	32.76
1980	29.66	28.90	1.89	25.20	32.60
1981	29.93	29.73	1.89	26.02	33.43
1982	28.92	28.51	1.89	24.81	32.22
1983	30.31	29.18	1.89	25.48	32.89
1984	30.16	28.56	1.89	24.86	32.27
1985	28.99	29.31	1.89	25.61	33.02
1986	30.17	28.59	1.89	24.88	32.29
1987	28.22	28.82	1.89	25.11	32.52
1988	27.58	28.68	1.89	24.98	32.39
1989	27.26	28.00	1.89	24.30	31.70
1990	27.24	28.04	1.89	24.34	31.75
1991	29.01	27.73	1.89	24.03	31.44
1992	25.00	28.01	1.89	24.31	31.71
1993	27.07	27.67	1.89	23.96	31.37
1994	25.00	26.78	1.89	23.08	30.49
1995	29.38	27.46	1.89	23.75	31.16
1996	27.10	26.81	1.89	23.11	30.51
1997	26.90	28.11	1.89	24.40	31.81
1998		26.47	1.89	22.77	30.17
1999		27.28	1.91	23.53	31.02
2000		26.59	1.97	22.72	30.46
2001		27.05	1.97	23.18	30.93
2002		26.49	2.00	22.57	30.41
2003		26.80	2.00	22.88	30.72
2004		26.35	2.02	22.40	30.31
2005		26.56	2.02	22.60	30.51
2006		26.20	2.03	22.22	30.18
2007		26.32	2.03	22.34	30.30
2008		26.03	2.04	22.03	30.03
2009		26.10	2.04	22.10	30.10
2010		25.86	2.05	21.84	29.88
2011		25.89	2.05	21.86	29.91
2012		25.68	2.06	21.65	29.72
2013		25.68	2.06	21.64	29.72
2014		25.50	2.07	21.45	29.55
2015		25.47	2.07	21.41	29.53
2016		25.32	2.08	21.25	29.38
2017		25.27	2.08	21.19	29.34
2018		25.13	2.08	21.04	29.21
2019		25.07	2.09	20.98	29.16
2020		24.94	2.09	20.84	29.04

Parameter G, rural females, India

Year		ARIMA model (1, 0, 1)			
	G	Est/ Forecast	Std errors	L95	U95
1970	0.00024				
1971	0.00011	0.00019	0.00012	-0.00005	0.00043
1972	0.00013	0.00015	0.00011	-0.00006	0.00037
1973	0.00008	0.00015	0.00010	-0.00006	0.00035
1974	0.00045	0.00013	0.00010	-0.00007	0.00033
1975	0.00024	0.00019	0.00010	0.00000	0.00038
1976	0.00013	0.00020	0.00010	0.00001	0.00039
1977	0.00015	0.00019	0.00010	0.00000	0.00038
1978	0.00020	0.00018	0.00010	0.00000	0.00037
1979	0.00013	0.00019	0.00010	0.00000	0.00037
1980	0.00013	0.00018	0.00010	-0.00001	0.00037
1981	0.00011	0.00018	0.00009	-0.00001	0.00036
1982	0.00022	0.00017	0.00009	-0.00001	0.00036
1983	0.00010	0.00017	0.00009	-0.00001	0.00036
1984	0.00009	0.00017	0.00009	-0.00001	0.00035
1985	0.00015	0.00016	0.00009	-0.00002	0.00035
1986	0.00007	0.00016	0.00009	-0.00002	0.00035
1987	0.00016	0.00016	0.00009	-0.00003	0.00034
1988	0.00014	0.00016	0.00009	-0.00003	0.00034
1989	0.00013	0.00016	0.00009	-0.00003	0.00034
1990	0.00017	0.00016	0.00009	-0.00003	0.00034
1991	0.00008	0.00016	0.00009	-0.00003	0.00034
1992	0.00021	0.00015	0.00009	-0.00003	0.00033
1993	0.00013	0.00015	0.00009	-0.00003	0.00034
1994	0.00039	0.00015	0.00009	-0.00003	0.00034
1995	0.00012	0.00016	0.00009	-0.00002	0.00034
1996	0.00024	0.00016	0.00009	-0.00002	0.00034
1997	0.00018	0.00016	0.00009	-0.00002	0.00035
1998		0.00016	0.00009	-0.00001	0.00034
1999		0.00016	0.00009	-0.00001	0.00034
2000		0.00016	0.00009	-0.00001	0.00034
2001		0.00016	0.00009	-0.00001	0.00034
2002		0.00016	0.00009	-0.00001	0.00034
2003		0.00016	0.00009	-0.00001	0.00034
2004		0.00016	0.00009	-0.00001	0.00034
2005		0.00016	0.00009	-0.00001	0.00034
2006		0.00016	0.00009	-0.00001	0.00034
2007		0.00016	0.00009	-0.00001	0.00034
2008		0.00016	0.00009	-0.00001	0.00034
2009		0.00016	0.00009	-0.00001	0.00034
2010		0.00016	0.00009	-0.00001	0.00034
2011		0.00016	0.00009	-0.00001	0.00034
2012		0.00016	0.00009	-0.00001	0.00034
2013		0.00016	0.00009	-0.00001	0.00034
2014		0.00016	0.00009	-0.00001	0.00034
2015		0.00016	0.00009	-0.00001	0.00034
2016		0.00016	0.00009	-0.00001	0.00034
2017		0.00016	0.00009	-0.00001	0.00034
2018		0.00016	0.00009	-0.00001	0.00034
2019		0.00016	0.00009	-0.00001	0.00034
2020		0.00016	0.00009	-0.00001	0.00034

Parameter H, rural females, India

Year	ARIMA model (3, 1, 0)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.1076				
1971	1.1196	1.1072	0.0091	1.0893	1.1252
1972	1.1222	1.1094	0.0091	1.0914	1.1273
1973	1.1276	1.1142	0.0091	1.0962	1.1321
1974	1.0976	1.1168	0.0091	1.0988	1.1347
1975	1.1103	1.1173	0.0091	1.0994	1.1353
1976	1.1166	1.1101	0.0091	1.0921	1.1280
1977	1.1165	1.1163	0.0091	1.0984	1.1342
1978	1.1108	1.1082	0.0091	1.0903	1.1262
1979	1.1151	1.1121	0.0091	1.0942	1.1300
1980	1.1151	1.1132	0.0091	1.0953	1.1311
1981	1.1184	1.1144	0.0091	1.0965	1.1324
1982	1.1053	1.1132	0.0091	1.0953	1.1312
1983	1.1196	1.1134	0.0091	1.0955	1.1313
1984	1.1218	1.1116	0.0091	1.0937	1.1295
1985	1.1127	1.1178	0.0091	1.0999	1.1358
1986	1.1239	1.1128	0.0091	1.0949	1.1307
1987	1.1108	1.1171	0.0091	1.0992	1.1350
1988	1.1127	1.1189	0.0091	1.1010	1.1368
1989	1.1125	1.1117	0.0091	1.0938	1.1296
1990	1.1082	1.1158	0.0091	1.0978	1.1337
1991	1.1217	1.1101	0.0091	1.0921	1.1280
1992	1.1054	1.1119	0.0091	1.0940	1.1298
1993	1.1145	1.1133	0.0091	1.0954	1.1312
1994	1.0922	1.1083	0.0091	1.0903	1.1262
1995	1.1114	1.1112	0.0091	1.0933	1.1291
1996	1.1001	1.1013	0.0091	1.0834	1.1192
1997	1.1066	1.1082	0.0091	1.0903	1.1261
1998		1.0982	0.0091	1.0803	1.1161
1999		1.1054	0.0093	1.0871	1.1237
2000		1.0999	0.0101	1.0802	1.1196
2001		1.1034	0.0102	1.0833	1.1234
2002		1.0993	0.0115	1.0767	1.1219
2003		1.1021	0.0118	1.0789	1.1254
2004		1.0994	0.0125	1.0749	1.1239
2005		1.1009	0.0128	1.0758	1.1260
2006		1.0989	0.0135	1.0725	1.1253
2007		1.0999	0.0138	1.0728	1.1270
2008		1.0985	0.0144	1.0703	1.1266
2009		1.0990	0.0147	1.0702	1.1278
2010		1.0979	0.0152	1.0681	1.1276
2011		1.0981	0.0155	1.0677	1.1286
2012		1.0973	0.0160	1.0659	1.1286
2013		1.0973	0.0163	1.0653	1.1293
2014		1.0966	0.0167	1.0638	1.1294
2015		1.0965	0.0171	1.0631	1.1300
2016		1.0959	0.0175	1.0617	1.1301
2017		1.0958	0.0178	1.0609	1.1306
2018		1.0952	0.0181	1.0597	1.1308
2019		1.0950	0.0185	1.0588	1.1312
2020		1.0945	0.0188	1.0576	1.1314

Parameter M, rural females, India

Year	ARIMA model (1, 0, 1)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.4290	0.4019	0.0264	0.3501	0.4537
1971	0.4210	0.4233	0.0264	0.3715	0.4751
1972	0.4534	0.4215	0.0264	0.3697	0.4733
1973	0.4145	0.4467	0.0264	0.3949	0.4985
1974	0.4154	0.4213	0.0264	0.3695	0.4731
1975	0.4520	0.4166	0.0264	0.3648	0.4685
1976	0.3836	0.4445	0.0264	0.3927	0.4963
1977	0.4347	0.3965	0.0264	0.3447	0.4483
1978	0.4151	0.4266	0.0264	0.3748	0.4784
1979	0.3973	0.4175	0.0264	0.3657	0.4693
1980	0.3605	0.4016	0.0264	0.3498	0.4534
1981	0.3481	0.3692	0.0264	0.3174	0.4210
1982	0.3385	0.3525	0.0264	0.3007	0.4044
1983	0.3155	0.3415	0.0264	0.2897	0.3933
1984	0.3398	0.3210	0.0264	0.2692	0.3728
1985	0.3227	0.3358	0.0264	0.2840	0.3876
1986	0.3201	0.3255	0.0264	0.2737	0.3773
1987	0.3167	0.3212	0.0264	0.2694	0.3730
1988	0.2884	0.3177	0.0264	0.2658	0.3695
1989	0.2561	0.2946	0.0264	0.2428	0.3464
1990	0.2338	0.2642	0.0264	0.2124	0.3160
1991	0.2233	0.2402	0.0264	0.1884	0.2920
1992	0.2288	0.2269	0.0264	0.1751	0.2787
1993	0.2065	0.2284	0.0264	0.1766	0.2802
1994	0.2109	0.2111	0.0264	0.1593	0.2629
1995	0.2076	0.2109	0.0264	0.1591	0.2628
1996	0.2151	0.2083	0.0264	0.1565	0.2601
1997	0.2115	0.2137	0.0264	0.1619	0.2655
1998		0.2120	0.0264	0.1601	0.2638
1999		0.2120	0.0337	0.1460	0.2779
2000		0.2120	0.0396	0.1343	0.2896
2001		0.2120	0.0448	0.1242	0.2997
2002		0.2120	0.0494	0.1152	0.3087
2003		0.2120	0.0536	0.1069	0.3170
2004		0.2120	0.0575	0.0992	0.3247
2005		0.2120	0.0612	0.0921	0.3319
2006		0.2120	0.0646	0.0853	0.3386
2007		0.2120	0.0679	0.0788	0.3451
2008		0.2120	0.0710	0.0727	0.3512
2009		0.2120	0.0740	0.0668	0.3571
2010		0.2120	0.0769	0.0612	0.3627
2011		0.2120	0.0797	0.0558	0.3682
2012		0.2120	0.0824	0.0505	0.3734
2013		0.2120	0.0850	0.0454	0.3785
2014		0.2120	0.0875	0.0405	0.3835
2015		0.2120	0.0900	0.0357	0.3883
2016		0.2120	0.0923	0.0310	0.3929
2017		0.2120	0.0947	0.0264	0.3975
2018		0.2120	0.0969	0.0220	0.4019
2019		0.2120	0.0992	0.0176	0.4063
2020		0.2120	0.1013	0.0134	0.4105

Parameter R, rural females, India

Year	ARIMA model (6, 1, 0)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0773				
1971	0.0764	0.0776	0.0130	0.0521	0.1030
1972	0.0876	0.0774	0.0130	0.0520	0.1029
1973	0.0929	0.0811	0.0130	0.0557	0.1065
1974	0.0768	0.0851	0.0130	0.0596	0.1105
1975	0.0719	0.0814	0.0130	0.0559	0.1068
1976	0.0975	0.0805	0.0130	0.0551	0.1059
1977	0.0720	0.0857	0.0130	0.0603	0.1111
1978	0.0748	0.0755	0.0130	0.0500	0.1009
1979	0.0654	0.0801	0.0130	0.0547	0.1055
1980	0.0696	0.0863	0.0130	0.0609	0.1117
1981	0.0791	0.0667	0.0130	0.0413	0.0922
1982	0.0748	0.0745	0.0130	0.0491	0.0999
1983	0.0810	0.0822	0.0130	0.0568	0.1077
1984	0.0970	0.0778	0.0130	0.0524	0.1032
1985	0.0932	0.0869	0.0130	0.0615	0.1123
1986	0.0782	0.0820	0.0130	0.0566	0.1074
1987	0.0728	0.0817	0.0130	0.0563	0.1071
1988	0.0830	0.0834	0.0130	0.0580	0.1088
1989	0.0996	0.0779	0.0130	0.0525	0.1034
1990	0.0831	0.0838	0.0130	0.0584	0.1092
1991	0.0978	0.0906	0.0130	0.0652	0.1160
1992	0.1006	0.0965	0.0130	0.0711	0.1219
1993	0.1069	0.0956	0.0130	0.0702	0.1210
1994	0.0696	0.0885	0.0130	0.0630	0.1139
1995	0.0958	0.0891	0.0130	0.0637	0.1145
1996	0.0831	0.0951	0.0130	0.0696	0.1205
1997	0.0764	0.0875	0.0130	0.0621	0.1129
1998		0.0785	0.0130	0.0531	0.1039
1999		0.0942	0.0137	0.0674	0.1209
2000		0.0926	0.0142	0.0647	0.1205
2001		0.0816	0.0146	0.0531	0.1101
2002		0.0914	0.0156	0.0607	0.1220
2003		0.0915	0.0157	0.0608	0.1222
2004		0.0858	0.0157	0.0549	0.1166
2005		0.0824	0.0168	0.0495	0.1153
2006		0.0916	0.0174	0.0574	0.1258
2007		0.0909	0.0177	0.0562	0.1256
2008		0.0861	0.0181	0.0507	0.1216
2009		0.0893	0.0189	0.0522	0.1264
2010		0.0929	0.0192	0.0554	0.1305
2011		0.0898	0.0193	0.0519	0.1277
2012		0.0871	0.0198	0.0483	0.1259
2013		0.0914	0.0203	0.0516	0.1312
2014		0.0922	0.0205	0.0519	0.1325
2015		0.0895	0.0208	0.0486	0.1303
2016		0.0902	0.0214	0.0483	0.1321
2017		0.0932	0.0217	0.0507	0.1358
2018		0.0922	0.0219	0.0492	0.1352
2019		0.0904	0.0223	0.0468	0.1341
2020		0.0924	0.0227	0.0479	0.1369

APPENDIX 7E: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, URBAN MALES, INDIA

Parameter D, urban males, India

Year	ARIMA model (3, 1, 0)				
	D	Est/ Forecast	Std errors	L95	U95
1970	0.00298				
1971	0.00159	0.00303	0.00125	0.00058	0.00547
1972	0.00052	0.00264	0.00125	0.00020	0.00509
1973	0.00045	0.00192	0.00125	-0.00053	0.00436
1974	0.00336	0.00122	0.00125	-0.00123	0.00366
1975	0.00307	0.00160	0.00125	-0.00085	0.00404
1976	0.00299	0.00222	0.00125	-0.00022	0.00467
1977	0.00291	0.00285	0.00125	0.00040	0.00529
1978	0.00536	0.00314	0.00125	0.00069	0.00558
1979	0.00320	0.00379	0.00125	0.00134	0.00623
1980	0.00291	0.00385	0.00125	0.00141	0.00630
1981	0.00397	0.00373	0.00125	0.00128	0.00617
1982	0.00416	0.00375	0.00125	0.00130	0.00619
1983	0.00344	0.00375	0.00125	0.00130	0.00619
1984	0.00437	0.00382	0.00125	0.00137	0.00626
1985	0.00274	0.00408	0.00125	0.00163	0.00652
1986	0.00506	0.00371	0.00125	0.00127	0.00616
1987	0.00397	0.00405	0.00125	0.00161	0.00650
1988	0.00367	0.00414	0.00125	0.00169	0.00658
1989	0.00539	0.00408	0.00125	0.00164	0.00653
1990	0.00409	0.00456	0.00125	0.00211	0.00700
1991	0.00424	0.00446	0.00125	0.00201	0.00690
1992	0.00563	0.00451	0.00125	0.00206	0.00695
1993	0.00280	0.00488	0.00125	0.00243	0.00732
1994	0.00422	0.00431	0.00125	0.00186	0.00675
1995	0.00326	0.00426	0.00125	0.00181	0.00670
1996	0.00409	0.00387	0.00125	0.00142	0.00631
1997	0.00310	0.00379	0.00125	0.00135	0.00624
1998		0.00370	0.00125	0.00126	0.00615
1999		0.00366	0.00130	0.00111	0.00621
2000		0.00369	0.00139	0.00096	0.00642
2001		0.00370	0.00151	0.00074	0.00665
2002		0.00378	0.00162	0.00061	0.00696
2003		0.00381	0.00170	0.00047	0.00716
2004		0.00386	0.00179	0.00034	0.00737
2005		0.00390	0.00188	0.00022	0.00759
2006		0.00395	0.00196	0.00011	0.00779
2007		0.00399	0.00204	0.00000	0.00798
2008		0.00404	0.00211	-0.00010	0.00817
2009		0.00408	0.00218	-0.00020	0.00836
2010		0.00412	0.00225	-0.00029	0.00854
2011		0.00417	0.00232	-0.00038	0.00872
2012		0.00421	0.00239	-0.00047	0.00889
2013		0.00426	0.00245	-0.00055	0.00906
2014		0.00430	0.00251	-0.00063	0.00923
2015		0.00434	0.00258	-0.00070	0.00939
2016		0.00439	0.00263	-0.00078	0.00955
2017		0.00443	0.00269	-0.00085	0.00971
2018		0.00448	0.00275	-0.00091	0.00987
2019		0.00452	0.00281	-0.00098	0.01002
2020		0.00456	0.00286	-0.00104	0.01017

Parameter E, urban males, India

Year	ARIMA model (5, 1, 0)				
	E	Est/ Forecast	Std errors	L95	U95
1970	8.88				
1971	12.35	8.81	9.62	-10.05	27.68
1972	17.60	9.55	9.62	-9.31	28.41
1973	15.00	10.88	9.62	-7.99	29.74
1974	15.84	12.48	9.62	-6.38	31.34
1975	22.41	15.58	9.62	-3.29	34.44
1976	21.19	16.84	9.62	-2.03	35.70
1977	31.96	18.20	9.62	-0.66	37.06
1978	17.58	22.77	9.62	3.90	41.63
1979	25.00	20.25	9.62	1.39	39.12
1980	39.80	28.82	9.62	9.96	47.68
1981	13.84	23.66	9.62	4.80	42.52
1982	10.43	25.27	9.62	6.41	44.13
1983	37.89	26.06	9.62	7.20	44.93
1984	9.01	22.46	9.62	3.59	41.32
1985	15.00	17.41	9.62	-1.45	36.28
1986	9.00	23.02	9.62	4.16	41.89
1987	15.00	9.91	9.62	-8.96	28.77
1988	19.40	21.28	9.62	2.42	40.14
1989	9.51	7.39	9.62	-11.47	26.26
1990	6.64	14.47	9.62	-4.39	33.34
1991	4.47	13.42	9.62	-5.44	32.29
1992	11.49	10.00	9.62	-8.86	28.86
1993	26.43	9.82	9.62	-9.04	28.69
1994	9.48	8.88	9.62	-9.98	27.75
1995	11.36	10.33	9.62	-8.53	29.19
1996	10.37	18.28	9.62	-0.58	37.14
1997	3.98	11.55	9.62	-7.31	30.42
1998		13.52	9.62	-5.35	32.38
1999		7.95	9.87	-11.39	27.30
2000		6.66	9.92	-12.79	26.10
2001		10.58	11.44	-11.84	33.00
2002		7.06	11.81	-16.09	30.20
2003		9.12	12.56	-15.49	33.73
2004		8.57	13.03	-16.97	34.11
2005		7.22	13.33	-18.91	33.35
2006		9.17	14.15	-18.55	36.90
2007		7.76	14.52	-20.70	36.22
2008		7.86	14.92	-21.38	37.11
2009		8.39	15.45	-21.89	38.67
2010		7.51	15.81	-23.47	38.49
2011		8.09	16.28	-23.82	39.99
2012		7.78	16.67	-24.90	40.47
2013		7.52	17.04	-25.87	40.91
2014		7.86	17.47	-26.38	42.10
2015		7.45	17.83	-27.49	42.39
2016		7.51	18.20	-28.16	43.18
2017		7.51	18.57	-28.89	43.91
2018		7.27	18.91	-29.80	44.34
2019		7.37	19.28	-30.41	45.15
2020		7.22	19.62	-31.23	45.67

Table 7.45: Empirical, estimated and forecasted values for parameter F, urban males, India

Year	ARIMA model (1, 1, 1)				
	F	Est/ Forecast	Std errors	L95	U95
1970	28.84				
1971	30.00	28.74	3.95	21.00	36.47
1972	30.00	28.98	3.95	21.25	36.72
1973	25.00	29.12	3.95	21.39	36.86
1974	19.26	27.85	3.95	20.11	35.58
1975	24.37	25.48	3.95	17.75	33.21
1976	23.07	25.32	3.95	17.59	33.06
1977	25.00	24.64	3.95	16.90	32.37
1978	25.00	24.70	3.95	16.97	32.44
1979	22.03	24.67	3.95	16.94	32.41
1980	20.93	23.83	3.95	16.09	31.56
1981	17.31	23.00	3.95	15.26	30.73
1982	27.78	21.40	3.95	13.67	29.14
1983	22.92	23.24	3.95	15.50	30.97
1984	23.33	22.87	3.95	15.13	30.60
1985	24.72	22.89	3.95	15.16	30.63
1986	25.85	23.28	3.95	15.55	31.02
1987	25.00	23.84	3.95	16.10	31.57
1988	25.00	23.98	3.95	16.24	31.71
1989	25.93	24.12	3.95	16.39	31.86
1990	26.33	24.49	3.95	16.75	32.22
1991	28.20	24.84	3.95	17.11	32.58
1992	27.15	25.61	3.95	17.88	33.35
1993	15.00	25.84	3.95	18.10	33.57
1994	27.62	22.68	3.95	14.94	30.41
1995	26.45	24.26	3.95	16.52	31.99
1996	27.05	24.63	3.95	16.90	32.37
1997	25.00	25.13	3.95	17.39	32.86
1998		24.92	3.95	17.18	32.65
1999		24.81	4.09	16.79	32.84
2000		24.71	4.21	16.46	32.96
2001		24.61	4.32	16.14	33.08
2002		24.50	4.43	15.82	33.19
2003		24.40	4.54	15.51	33.29
2004		24.30	4.64	15.20	33.39
2005		24.19	4.74	14.90	33.48
2006		24.09	4.84	14.60	33.57
2007		23.98	4.94	14.31	33.66
2008		23.88	5.03	14.02	33.74
2009		23.78	5.13	13.73	33.82
2010		23.67	5.22	13.45	33.90
2011		23.57	5.31	13.17	33.97
2012		23.47	5.40	12.89	34.04
2013		23.36	5.48	12.62	34.11
2014		23.26	5.57	12.34	34.17
2015		23.16	5.65	12.07	34.24
2016		23.05	5.74	11.81	34.30
2017		22.95	5.82	11.54	34.35
2018		22.84	5.90	11.28	34.41
2019		22.74	5.98	11.02	34.46
2020		22.64	6.06	10.76	34.51

Parameter G, urban males, India

Year	Logistic ARIMA model (2, 1, 1)				
	G	Est/ Forecast	Std errors	L95	U95
1970	0.00028				
1971	0.00044	0.00029	0.00011	0.00011	0.00054
1972	0.00048	0.00038	0.00012	0.00018	0.00062
1973	0.00051	0.00038	0.00011	0.00019	0.00060
1974	0.00030	0.00043	0.00011	0.00023	0.00064
1975	0.00047	0.00038	0.00010	0.00020	0.00060
1976	0.00047	0.00045	0.00011	0.00025	0.00067
1977	0.00037	0.00041	0.00010	0.00022	0.00062
1978	0.00038	0.00040	0.00010	0.00021	0.00061
1979	0.00045	0.00042	0.00010	0.00023	0.00063
1980	0.00062	0.00042	0.00010	0.00022	0.00063
1981	0.00041	0.00046	0.00011	0.00026	0.00067
1982	0.00027	0.00042	0.00010	0.00023	0.00063
1983	0.00045	0.00042	0.00011	0.00023	0.00064
1984	0.00035	0.00045	0.00011	0.00025	0.00066
1985	0.00047	0.00037	0.00010	0.00019	0.00058
1986	0.00029	0.00042	0.00011	0.00023	0.00063
1987	0.00019	0.00035	0.00010	0.00018	0.00056
1988	0.00035	0.00034	0.00010	0.00017	0.00055
1989	0.00024	0.00035	0.00010	0.00018	0.00056
1990	0.00028	0.00027	0.00009	0.00013	0.00046
1991	0.00028	0.00030	0.00009	0.00014	0.00050
1992	0.00023	0.00028	0.00009	0.00013	0.00047
1993	0.00029	0.00027	0.00009	0.00013	0.00046
1994	0.00028	0.00028	0.00009	0.00013	0.00047
1995	0.00033	0.00027	0.00009	0.00013	0.00046
1996	0.00043	0.00029	0.00009	0.00014	0.00048
1997	0.00034	0.00032	0.00009	0.00016	0.00052
1998		0.00031	0.00009	0.00015	0.00051
1999		0.00034	0.00010	0.00016	0.00055
2000		0.00034	0.00010	0.00016	0.00056
2001		0.00033	0.00010	0.00015	0.00055
2002		0.00033	0.00011	0.00014	0.00056
2003		0.00033	0.00011	0.00014	0.00057
2004		0.00033	0.00011	0.00013	0.00058
2005		0.00032	0.00012	0.00013	0.00058
2006		0.00032	0.00012	0.00012	0.00059
2007		0.00032	0.00012	0.00012	0.00059
2008		0.00032	0.00013	0.00012	0.00059
2009		0.00032	0.00013	0.00011	0.00060
2010		0.00032	0.00013	0.00011	0.00060
2011		0.00031	0.00013	0.00010	0.00061
2012		0.00031	0.00013	0.00010	0.00061
2013		0.00031	0.00014	0.00010	0.00061
2014		0.00031	0.00014	0.00009	0.00062
2015		0.00031	0.00014	0.00009	0.00062
2016		0.00031	0.00014	0.00009	0.00062
2017		0.00030	0.00014	0.00009	0.00063
2018		0.00030	0.00015	0.00008	0.00063
2019		0.00030	0.00015	0.00008	0.00063
2020		0.00030	0.00015	0.00008	0.00064

Parameter H, urban males, India

Year	ARIMA model (5, 1, 1)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.1058				
1971	1.0983	1.1055	0.0053	1.0951	1.1159
1972	1.0982	1.1018	0.0053	1.0914	1.1122
1973	1.0955	1.1008	0.0053	1.0904	1.1112
1974	1.1060	1.0969	0.0053	1.0865	1.1073
1975	1.0986	1.1015	0.0053	1.0911	1.1119
1976	1.0973	1.0951	0.0053	1.0847	1.1055
1977	1.1026	1.1002	0.0053	1.0897	1.1106
1978	1.1026	1.0988	0.0053	1.0884	1.1092
1979	1.0964	1.1043	0.0053	1.0939	1.1147
1980	1.0900	1.0957	0.0053	1.0853	1.1061
1981	1.0978	1.0950	0.0053	1.0845	1.1054
1982	1.1033	1.0983	0.0053	1.0879	1.1087
1983	1.0965	1.0973	0.0053	1.0869	1.1078
1984	1.1025	1.0949	0.0053	1.0845	1.1053
1985	1.0954	1.0987	0.0053	1.0883	1.1091
1986	1.1039	1.0993	0.0053	1.0889	1.1097
1987	1.1112	1.1035	0.0053	1.0931	1.1139
1988	1.1014	1.1009	0.0053	1.0905	1.1113
1989	1.1076	1.1052	0.0053	1.0948	1.1156
1990	1.1037	1.1041	0.0053	1.0937	1.1145
1991	1.1041	1.1063	0.0053	1.0959	1.1167
1992	1.1063	1.1075	0.0053	1.0970	1.1179
1993	1.1001	1.1007	0.0053	1.0903	1.1111
1994	1.1019	1.1048	0.0053	1.0944	1.1152
1995	1.0978	1.1012	0.0053	1.0907	1.1116
1996	1.0931	1.0994	0.0053	1.0889	1.1098
1997	1.0997	1.0977	0.0053	1.0873	1.1081
1998		1.0956	0.0053	1.0852	1.1060
1999		1.0960	0.0059	1.0844	1.1075
2000		1.0952	0.0062	1.0831	1.1073
2001		1.0934	0.0068	1.0800	1.1067
2002		1.0969	0.0073	1.0826	1.1112
2003		1.0936	0.0085	1.0770	1.1103
2004		1.0939	0.0088	1.0766	1.1112
2005		1.0942	0.0092	1.0762	1.1121
2006		1.0928	0.0098	1.0736	1.1121
2007		1.0944	0.0102	1.0744	1.1145
2008		1.0923	0.0108	1.0711	1.1135
2009		1.0926	0.0111	1.0708	1.1144
2010		1.0929	0.0115	1.0703	1.1154
2011		1.0917	0.0120	1.0681	1.1152
2012		1.0924	0.0123	1.0682	1.1166
2013		1.0912	0.0128	1.0662	1.1162
2014		1.0913	0.0131	1.0657	1.1169
2015		1.0913	0.0134	1.0651	1.1176
2016		1.0904	0.0138	1.0633	1.1175
2017		1.0908	0.0141	1.0631	1.1184
2018		1.0900	0.0144	1.0617	1.1183
2019		1.0899	0.0148	1.0610	1.1188
2020		1.0898	0.0151	1.0603	1.1193

Parameter M, urban males, India

Year	ARIMA model (5, 1, 0)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.2371				
1971	0.2575	0.2339	0.0712	0.0942	0.3735
1972	0.3831	0.2352	0.0712	0.0956	0.3748
1973	0.2475	0.2571	0.0712	0.1175	0.3967
1974	0.2047	0.2319	0.0712	0.0923	0.3716
1975	0.2874	0.2537	0.0712	0.1140	0.3933
1976	0.3348	0.2873	0.0712	0.1477	0.4269
1977	0.2091	0.2211	0.0712	0.0815	0.3607
1978	0.4605	0.2788	0.0712	0.1392	0.4184
1979	0.3000	0.3161	0.0712	0.1764	0.4557
1980	0.2243	0.2433	0.0712	0.1037	0.3829
1981	0.1845	0.2545	0.0712	0.1149	0.3942
1982	0.2982	0.3595	0.0712	0.2199	0.4991
1983	0.2053	0.2004	0.0712	0.0608	0.3400
1984	0.2428	0.2910	0.0712	0.1514	0.4307
1985	0.1948	0.2475	0.0712	0.1079	0.3871
1986	0.2037	0.2383	0.0712	0.0987	0.3779
1987	0.1496	0.1711	0.0712	0.0315	0.3107
1988	0.2605	0.2356	0.0712	0.0960	0.3752
1989	0.1524	0.1917	0.0712	0.0521	0.3313
1990	0.1352	0.1903	0.0712	0.0506	0.3299
1991	0.1178	0.1636	0.0712	0.0239	0.3032
1992	0.1900	0.1974	0.0712	0.0578	0.3370
1993	0.2000	0.1210	0.0712	-0.0186	0.2606
1994	0.1498	0.1845	0.0712	0.0448	0.3241
1995	0.1118	0.1362	0.0712	-0.0034	0.2758
1996	0.2775	0.1538	0.0712	0.0142	0.2935
1997	0.1639	0.1587	0.0712	0.0191	0.2983
1998		0.1405	0.0712	0.0009	0.2801
1999		0.1665	0.0726	0.0242	0.3088
2000		0.1989	0.0726	0.0566	0.3413
2001		0.1075	0.0739	-0.0372	0.2523
2002		0.1826	0.0805	0.0247	0.3404
2003		0.1670	0.0806	0.0091	0.3249
2004		0.1437	0.0867	-0.0262	0.3136
2005		0.1233	0.0897	-0.0524	0.2991
2006		0.1779	0.0908	-0.0001	0.3560
2007		0.1225	0.0911	-0.0561	0.3011
2008		0.1357	0.0967	-0.0539	0.3253
2009		0.1384	0.0973	-0.0522	0.3291
2010		0.1431	0.0990	-0.0509	0.3372
2011		0.1054	0.1009	-0.0924	0.3031
2012		0.1391	0.1035	-0.0638	0.3419
2013		0.1201	0.1038	-0.0834	0.3236
2014		0.1167	0.1067	-0.0925	0.3258
2015		0.1081	0.1081	-0.1038	0.3200
2016		0.1255	0.1096	-0.0892	0.3402
2017		0.0974	0.1106	-0.1194	0.3142
2018		0.1079	0.1132	-0.1139	0.3298
2019		0.1022	0.1140	-0.1212	0.3255
2020		0.1027	0.1156	-0.1240	0.3293

Parameter R, urban males, India

Year	ARIMA model (2, 1, 1)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0814				
1971	0.0566	0.0804	0.0238	0.0337	0.1272
1972	0.0263	0.0745	0.0238	0.0278	0.1212
1973	0.0562	0.0689	0.0238	0.0221	0.1156
1974	0.0627	0.0664	0.0238	0.0197	0.1131
1975	0.0455	0.0483	0.0238	0.0016	0.0951
1976	0.0316	0.0470	0.0238	0.0003	0.0938
1977	0.0656	0.0492	0.0238	0.0025	0.0959
1978	0.0140	0.0522	0.0238	0.0055	0.0990
1979	0.0220	0.0315	0.0238	-0.0152	0.0783
1980	0.0377	0.0455	0.0238	-0.0012	0.0923
1981	0.0484	0.0326	0.0238	-0.0142	0.0793
1982	0.0246	0.0287	0.0238	-0.0180	0.0754
1983	0.0437	0.0267	0.0238	-0.0200	0.0735
1984	0.0363	0.0383	0.0238	-0.0085	0.0850
1985	0.0409	0.0293	0.0238	-0.0174	0.0761
1986	0.0415	0.0351	0.0238	-0.0116	0.0818
1987	0.0629	0.0342	0.0238	-0.0125	0.0810
1988	0.0255	0.0401	0.0238	-0.0066	0.0868
1989	0.0496	0.0322	0.0238	-0.0146	0.0789
1990	0.0511	0.0484	0.0238	0.0017	0.0952
1991	0.0845	0.0372	0.0238	-0.0095	0.0839
1992	0.0259	0.0483	0.0238	0.0015	0.0950
1993	0.0202	0.0371	0.0238	-0.0096	0.0838
1994	0.0475	0.0543	0.0238	0.0075	0.1010
1995	0.0903	0.0447	0.0238	-0.0021	0.0914
1996	0.0159	0.0413	0.0238	-0.0054	0.0881
1997	0.0280	0.0288	0.0238	-0.0179	0.0756
1998		0.0557	0.0238	0.0090	0.1024
1999		0.0415	0.0243	-0.0062	0.0892
2000		0.0316	0.0243	-0.0162	0.0793
2001		0.0386	0.0260	-0.0124	0.0896
2002		0.0394	0.0275	-0.0146	0.0933
2003		0.0344	0.0279	-0.0204	0.0892
2004		0.0337	0.0286	-0.0223	0.0898
2005		0.0343	0.0296	-0.0236	0.0923
2006		0.0328	0.0303	-0.0266	0.0922
2007		0.0312	0.0309	-0.0294	0.0918
2008		0.0306	0.0316	-0.0314	0.0926
2009		0.0298	0.0324	-0.0337	0.0932
2010		0.0285	0.0330	-0.0361	0.0932
2011		0.0275	0.0337	-0.0384	0.0935
2012		0.0266	0.0343	-0.0406	0.0939
2013		0.0256	0.0349	-0.0429	0.0941
2014		0.0246	0.0356	-0.0451	0.0943
2015		0.0236	0.0362	-0.0473	0.0945
2016		0.0226	0.0368	-0.0494	0.0947
2017		0.0216	0.0374	-0.0516	0.0948
2018		0.0206	0.0379	-0.0537	0.0949
2019		0.0196	0.0385	-0.0559	0.0951
2020		0.0186	0.0391	-0.0580	0.0952

APPENDIX 7F: EMPIRICAL, ESTIMATED/ FORECASTED VALUES OF PARAMETERS, URBAN FEMALES, INDIA

Parameter D, urban females, India

Year	ARIMA model (5, 1, 2)				
	D	Est/ Forecast	Std errors	L95	U95
1970	0.00431				
1971	0.00862	0.00436	0.00151	0.00141	0.00732
1972	0.00958	0.00733	0.00151	0.00437	0.01028
1973	0.00845	0.00929	0.00151	0.00633	0.01225
1974	0.00878	0.00733	0.00151	0.00438	0.01029
1975	0.00480	0.00649	0.00151	0.00353	0.00944
1976	0.00805	0.00745	0.00151	0.00450	0.01041
1977	0.00771	0.00828	0.00151	0.00532	0.01123
1978	0.00889	0.00829	0.00151	0.00534	0.01125
1979	0.00806	0.00909	0.00151	0.00613	0.01204
1980	0.00646	0.00618	0.00151	0.00322	0.00913
1981	0.00706	0.00740	0.00151	0.00445	0.01036
1982	0.00763	0.00735	0.00151	0.00439	0.01030
1983	0.00779	0.00844	0.00151	0.00549	0.01140
1984	0.00873	0.00805	0.00151	0.00510	0.01101
1985	0.00738	0.00748	0.00151	0.00452	0.01044
1986	0.00770	0.00771	0.00151	0.00475	0.01066
1987	0.00680	0.00774	0.00151	0.00478	0.01070
1988	0.00824	0.00722	0.00151	0.00426	0.01018
1989	0.00815	0.00855	0.00151	0.00559	0.01150
1990	0.00648	0.00812	0.00151	0.00517	0.01108
1991	0.00815	0.00686	0.00151	0.00391	0.00982
1992	0.00782	0.00702	0.00151	0.00406	0.00997
1993	0.00782	0.00900	0.00151	0.00604	0.01195
1994	0.00720	0.00823	0.00151	0.00528	0.01119
1995	0.00501	0.00613	0.00151	0.00317	0.00908
1996	0.00658	0.00628	0.00151	0.00333	0.00924
1997	0.00725	0.00678	0.00151	0.00382	0.00973
1998		0.00791	0.00151	0.00496	0.01087
1999		0.00795	0.00183	0.00436	0.01153
2000		0.00642	0.00217	0.00217	0.01067
2001		0.00681	0.00225	0.00241	0.01122
2002		0.00710	0.00226	0.00267	0.01153
2003		0.00768	0.00248	0.00282	0.01253
2004		0.00802	0.00264	0.00284	0.01319
2005		0.00724	0.00287	0.00162	0.01286
2006		0.00724	0.00298	0.00140	0.01309
2007		0.00728	0.00302	0.00135	0.01320
2008		0.00763	0.00313	0.00149	0.01377
2009		0.00801	0.00324	0.00166	0.01436
2010		0.00770	0.00339	0.00106	0.01435
2011		0.00765	0.00351	0.00077	0.01453
2012		0.00758	0.00357	0.00058	0.01459
2013		0.00775	0.00366	0.00058	0.01492
2014		0.00805	0.00374	0.00072	0.01538
2015		0.00799	0.00385	0.00045	0.01553
2016		0.00798	0.00395	0.00023	0.01572
2017		0.00791	0.00403	0.00001	0.01580
2018		0.00797	0.00411	-0.00008	0.01601
2019		0.00817	0.00418	-0.00002	0.01635
2020		0.00821	0.00426	-0.00015	0.01656

Parameter E, urban females, India

Year	ARIMA model (0, 0, 1)				
	E	Est/ Forecast	Std errors	L95	U95
1970	31.50	9.41	7.23	-4.77	23.59
1971	6.22	11.53	7.23	-2.65	25.71
1972	6.22	8.90	7.23	-5.28	23.08
1973	7.71	9.15	7.23	-5.03	23.33
1974	6.01	9.27	7.23	-4.91	23.45
1975	5.38	9.10	7.23	-5.08	23.27
1976	12.04	9.05	7.23	-5.13	23.23
1977	26.77	9.70	7.23	-4.48	23.87
1978	5.73	11.05	7.23	-3.13	25.23
1979	7.15	8.90	7.23	-5.28	23.08
1980	6.49	9.24	7.23	-4.94	23.42
1981	2.30	9.14	7.23	-5.03	23.32
1982	6.01	8.75	7.23	-5.43	22.93
1983	5.96	9.15	7.23	-5.03	23.32
1984	7.34	9.10	7.23	-5.08	23.28
1985	17.27	9.24	7.23	-4.94	23.42
1986	5.49	10.18	7.23	-4.00	24.36
1987	7.78	8.96	7.23	-5.22	23.14
1988	23.16	9.30	7.23	-4.88	23.47
1989	13.70	10.74	7.23	-3.44	24.92
1990	11.87	9.69	7.23	-4.49	23.87
1991	4.51	9.62	7.23	-4.56	23.80
1992	5.66	8.92	7.23	-5.26	23.10
1993	3.29	9.10	7.23	-5.08	23.27
1994	4.02	8.85	7.23	-5.33	23.03
1995	6.57	8.94	7.23	-5.23	23.12
1996	5.90	9.18	7.23	-5.00	23.36
1997	9.03	9.09	7.23	-5.09	23.27
1998		9.40	7.23	-4.78	23.58
1999		9.41	7.27	-4.84	23.65
2000		9.41	7.27	-4.84	23.65
2001		9.41	7.27	-4.84	23.65
2002		9.41	7.27	-4.84	23.65
2003		9.41	7.27	-4.84	23.65
2004		9.41	7.27	-4.84	23.65
2005		9.41	7.27	-4.84	23.65
2006		9.41	7.27	-4.84	23.65
2007		9.41	7.27	-4.84	23.65
2008		9.41	7.27	-4.84	23.65
2009		9.41	7.27	-4.84	23.65
2010		9.41	7.27	-4.84	23.65
2011		9.41	7.27	-4.84	23.65
2012		9.41	7.27	-4.84	23.65
2013		9.41	7.27	-4.84	23.65
2014		9.41	7.27	-4.84	23.65
2015		9.41	7.27	-4.84	23.65
2016		9.41	7.27	-4.84	23.65
2017		9.41	7.27	-4.84	23.65
2018		9.41	7.27	-4.84	23.65
2019		9.41	7.27	-4.84	23.65
2020		9.41	7.27	-4.84	23.65

Parameter F, urban females, India

Year	ARIMA model (6, 1, 1)				
	F	Est/ Forecast	Std errors	L95	U95
1970	20.00				
1971	29.03	20.17	3.30	13.69	26.64
1972	29.24	24.74	3.30	18.26	31.21
1973	26.91	27.91	3.30	21.44	34.39
1974	29.92	25.64	3.30	19.16	32.11
1975	25.00	27.66	3.30	21.19	34.14
1976	26.80	25.84	3.30	19.37	32.32
1977	20.00	23.13	3.30	16.65	29.60
1978	27.25	24.84	3.30	18.36	31.32
1979	27.12	25.56	3.30	19.08	32.03
1980	27.56	28.35	3.30	21.87	34.83
1981	25.00	28.05	3.30	21.58	34.53
1982	27.53	26.69	3.30	20.22	33.17
1983	25.38	27.82	3.30	21.35	34.30
1984	24.77	24.67	3.30	18.20	31.15
1985	23.87	25.28	3.30	18.81	31.76
1986	26.56	25.59	3.30	19.12	32.07
1987	26.01	26.69	3.30	20.22	33.17
1988	24.32	26.31	3.30	19.83	32.79
1989	24.52	25.96	3.30	19.48	32.43
1990	24.86	25.42	3.30	18.95	31.90
1991	29.91	25.43	3.30	18.96	31.91
1992	27.77	27.12	3.30	20.64	33.60
1993	26.56	28.89	3.30	22.41	35.36
1994	28.76	27.01	3.30	20.54	33.49
1995	25.34	28.07	3.30	21.59	34.54
1996	26.94	26.50	3.30	20.03	32.98
1997	25.80	25.28	3.30	18.80	31.75
1998		28.23	3.30	21.76	34.71
1999		27.70	3.69	20.47	34.93
2000		28.16	4.16	20.00	36.32
2001		28.78	4.29	20.36	37.20
2002		28.40	4.53	19.53	37.27
2003		28.70	4.59	19.71	37.70
2004		28.05	4.61	19.02	37.08
2005		28.84	4.76	19.52	38.16
2006		28.67	4.89	19.08	38.25
2007		29.09	5.11	19.07	39.10
2008		29.29	5.24	19.01	39.57
2009		29.57	5.46	18.87	40.26
2010		29.81	5.60	18.84	40.78
2011		29.74	5.70	18.56	40.92
2012		30.03	5.80	18.66	41.40
2013		30.04	5.90	18.47	41.61
2014		30.28	6.02	18.48	42.08
2015		30.38	6.12	18.38	42.38
2016		30.62	6.25	18.37	42.87
2017		30.82	6.37	18.34	43.31
2018		30.97	6.49	18.25	43.69
2019		31.17	6.60	18.24	44.10
2020		31.30	6.70	18.17	44.43

Parameter G, urban females, India

Year	ARIMA model (1, 1, 2)				
	G	Est/ Forecast	Std errors	L95	U95
1970					
1971	0.00023				
1972	0.00021	0.00022	0.00004	0.00015	0.00031
1973	0.00024	0.00021	0.00003	0.00014	0.00028
1974	0.00015	0.00023	0.00004	0.00017	0.00031
1975	0.00035	0.00021	0.00003	0.00015	0.00029
1976	0.00041	0.00022	0.00004	0.00016	0.00030
1977	0.00041	0.00031	0.00004	0.00023	0.00040
1978	0.00019	0.00037	0.00005	0.00028	0.00047
1979	0.00013	0.00034	0.00005	0.00026	0.00043
1980	0.00017	0.00025	0.00004	0.00018	0.00033
1981	0.00012	0.00018	0.00003	0.00013	0.00025
1982	0.00010	0.00013	0.00002	0.00009	0.00018
1983	0.00015	0.00011	0.00002	0.00008	0.00015
1984	0.00011	0.00013	0.00002	0.00009	0.00018
1985	0.00027	0.00013	0.00002	0.00009	0.00018
1986	0.00012	0.00015	0.00003	0.00010	0.00021
1987	0.00012	0.00018	0.00003	0.00013	0.00024
1988	0.00018	0.00016	0.00003	0.00011	0.00021
1989	0.00013	0.00016	0.00003	0.00011	0.00022
1990	0.00009	0.00014	0.00002	0.00010	0.00019
1991	0.00004	0.00012	0.00002	0.00008	0.00016
1992	0.00007	0.00010	0.00002	0.00007	0.00014
1993	0.00004	0.00008	0.00001	0.00005	0.00011
1994	0.00004	0.00005	0.00001	0.00003	0.00007
1995	0.00010	0.00004	0.00001	0.00003	0.00006
1996	0.00005	0.00007	0.00001	0.00004	0.00009
1997	0.00015	0.00007	0.00001	0.00005	0.00010
1998		0.00008	0.00002	0.00006	0.00012
1999		0.00009	0.00003	0.00005	0.00015
2000		0.00009	0.00004	0.00003	0.00020
2001		0.00009	0.00005	0.00003	0.00023
2002		0.00010	0.00007	0.00002	0.00027
2003		0.00010	0.00007	0.00002	0.00030
2004		0.00010	0.00008	0.00001	0.00032
2005		0.00010	0.00009	0.00001	0.00035
2006		0.00010	0.00010	0.00001	0.00037
2007		0.00010	0.00010	0.00001	0.00039
2008		0.00010	0.00011	0.00001	0.00041
2009		0.00010	0.00011	0.00000	0.00043
2010		0.00010	0.00012	0.00000	0.00044
2011		0.00010	0.00012	0.00000	0.00046
2012		0.00010	0.00012	0.00000	0.00047
2013		0.00010	0.00013	0.00000	0.00048
2014		0.00010	0.00013	0.00000	0.00050
2015		0.00010	0.00013	0.00000	0.00051
2016		0.00010	0.00014	0.00000	0.00052
2017		0.00009	0.00014	0.00000	0.00053
2018		0.00009	0.00014	0.00000	0.00054
2019		0.00009	0.00014	0.00000	0.00055
2020		0.00009	0.00014	0.00000	0.00056

Parameter H, urban females, India

Year	ARIMA model (0, 1, 0)				
	H	Est/ Forecast	Std errors	L95	U95
1970	1.0799				
1971	1.1038	1.0799	0.0097	1.0608	1.0990
1972	1.1097	1.1038	0.0097	1.0847	1.1229
1973	1.1035	1.1097	0.0097	1.0906	1.1288
1974	1.1100	1.1035	0.0097	1.0844	1.1226
1975	1.0993	1.1100	0.0097	1.0910	1.1291
1976	1.0946	1.0993	0.0097	1.0802	1.1184
1977	1.0958	1.0946	0.0097	1.0756	1.1137
1978	1.1085	1.0958	0.0097	1.0767	1.1149
1979	1.1122	1.1085	0.0097	1.0894	1.1276
1980	1.1063	1.1122	0.0097	1.0932	1.1313
1981	1.1130	1.1063	0.0097	1.0872	1.1254
1982	1.1156	1.1130	0.0097	1.0940	1.1321
1983	1.1098	1.1156	0.0097	1.0965	1.1347
1984	1.1159	1.1098	0.0097	1.0907	1.1289
1985	1.0994	1.1159	0.0097	1.0968	1.1349
1986	1.1135	1.0994	0.0097	1.0803	1.1184
1987	1.1121	1.1135	0.0097	1.0944	1.1326
1988	1.1064	1.1121	0.0097	1.0931	1.1312
1989	1.1093	1.1064	0.0097	1.0873	1.1255
1990	1.1174	1.1093	0.0097	1.0902	1.1284
1991	1.1289	1.1174	0.0097	1.0983	1.1365
1992	1.1200	1.1289	0.0097	1.1098	1.1480
1993	1.1256	1.1200	0.0097	1.1009	1.1390
1994	1.1259	1.1256	0.0097	1.1065	1.1447
1995	1.1131	1.1259	0.0097	1.1068	1.1450
1996	1.1235	1.1131	0.0097	1.0941	1.1322
1997	1.1060	1.1235	0.0097	1.1044	1.1426
1998		1.1060	0.0097	1.0869	1.1251
1999		1.1060	0.0138	1.0790	1.1330
2000		1.1060	0.0169	1.0729	1.1390
2001		1.1060	0.0195	1.0678	1.1441
2002		1.1060	0.0218	1.0633	1.1486
2003		1.1060	0.0239	1.0592	1.1527
2004		1.1060	0.0258	1.0555	1.1565
2005		1.1060	0.0275	1.0520	1.1600
2006		1.1060	0.0292	1.0487	1.1632
2007		1.1060	0.0308	1.0456	1.1663
2008		1.1060	0.0323	1.0427	1.1693
2009		1.1060	0.0337	1.0399	1.1721
2010		1.1060	0.0351	1.0372	1.1748
2011		1.1060	0.0364	1.0346	1.1774
2012		1.1060	0.0377	1.0321	1.1799
2013		1.1060	0.0390	1.0296	1.1823
2014		1.1060	0.0401	1.0273	1.1847
2015		1.1060	0.0413	1.0250	1.1869
2016		1.1060	0.0424	1.0228	1.1892
2017		1.1060	0.0435	1.0206	1.1913
2018		1.1060	0.0446	1.0185	1.1934
2019		1.1060	0.0457	1.0165	1.1955
2020		1.1060	0.0467	1.0144	1.1975

Parameter M, urban females, India

Year	ARIMA model (2, 1, 2)				
	M	Est/ Forecast	Std errors	L95	U95
1970	0.2339				
1971	0.2925	0.2300	0.0911	0.0514	0.4087
1972	0.3072	0.2341	0.0911	0.0555	0.4127
1973	0.2509	0.2387	0.0911	0.0601	0.4173
1974	0.4335	0.2513	0.0911	0.0727	0.4299
1975	0.2750	0.2978	0.0911	0.1192	0.4764
1976	0.2211	0.3017	0.0911	0.1230	0.4803
1977	0.2277	0.3331	0.0911	0.1545	0.5117
1978	0.5876	0.3320	0.0911	0.1533	0.5106
1979	0.2337	0.3358	0.0911	0.1572	0.5145
1980	0.1964	0.2698	0.0911	0.0912	0.4484
1981	0.2508	0.3064	0.0911	0.1277	0.4850
1982	0.1654	0.3109	0.0911	0.1322	0.4895
1983	0.1798	0.2632	0.0911	0.0846	0.4419
1984	0.1850	0.2181	0.0911	0.0395	0.3968
1985	0.2008	0.1644	0.0911	-0.0142	0.3430
1986	0.1787	0.1287	0.0911	-0.0499	0.3073
1987	0.1420	0.1164	0.0911	-0.0623	0.2950
1988	0.1377	0.1277	0.0911	-0.0509	0.3063
1989	0.1395	0.1486	0.0911	-0.0300	0.3273
1990	0.1155	0.1591	0.0911	-0.0195	0.3377
1991	0.1317	0.1536	0.0911	-0.0250	0.3322
1992	0.1270	0.1426	0.0911	-0.0361	0.3212
1993	0.1200	0.1221	0.0911	-0.0565	0.3007
1994	0.1256	0.1060	0.0911	-0.0727	0.2846
1995	0.1144	0.0986	0.0911	-0.0800	0.2772
1996	0.1118	0.0963	0.0911	-0.0823	0.2749
1997	0.1206	0.1009	0.0911	-0.0777	0.2796
1998		0.1071	0.0911	-0.0715	0.2857
1999		0.1090	0.0919	-0.0710	0.2891
2000		0.1112	0.0925	-0.0700	0.2924
2001		0.1087	0.0984	-0.0841	0.3015
2002		0.1038	0.1087	-0.1092	0.3169
2003		0.0990	0.1181	-0.1324	0.3304
2004		0.0949	0.1253	-0.1508	0.3406
2005		0.0912	0.1316	-0.1667	0.3492
2006		0.0875	0.1376	-0.1822	0.3572
2007		0.0837	0.1435	-0.1977	0.3650
2008		0.0798	0.1493	-0.2129	0.3725
2009		0.0759	0.1549	-0.2277	0.3795
2010		0.0720	0.1603	-0.2421	0.3861
2011		0.0681	0.1654	-0.2561	0.3923
2012		0.0643	0.1704	-0.2697	0.3983
2013		0.0604	0.1753	-0.2831	0.4040
2014		0.0565	0.1800	-0.2963	0.4094
2015		0.0527	0.1846	-0.3092	0.4146
2016		0.0488	0.1891	-0.3219	0.4195
2017		0.0449	0.1935	-0.3344	0.4243
2018		0.0411	0.1978	-0.3467	0.4288
2019		0.0372	0.2020	-0.3588	0.4332
2020		0.0333	0.2062	-0.3708	0.4374

Parameter R, urban females, India

Year	ARIMA model (3, 1, 0)				
	R	Est/ Forecast	Std errors	L95	U95
1970	0.0823				
1971	0.0502	0.0825	0.0245	0.0345	0.1305
1972	0.0511	0.0712	0.0245	0.0232	0.1192
1973	0.0746	0.0710	0.0245	0.0230	0.1191
1974	0.0230	0.0718	0.0245	0.0237	0.1198
1975	0.0536	0.0418	0.0245	-0.0063	0.0898
1976	0.0852	0.0577	0.0245	0.0097	0.1057
1977	0.0591	0.0661	0.0245	0.0181	0.1141
1978	0.0140	0.0450	0.0245	-0.0030	0.0930
1979	0.0467	0.0478	0.0245	-0.0002	0.0958
1980	0.0563	0.0644	0.0245	0.0164	0.1124
1981	0.0310	0.0475	0.0245	-0.0005	0.0956
1982	0.0670	0.0293	0.0245	-0.0187	0.0773
1983	0.0613	0.0565	0.0245	0.0085	0.1045
1984	0.0726	0.0527	0.0245	0.0047	0.1008
1985	0.0497	0.0556	0.0245	0.0076	0.1037
1986	0.0611	0.0601	0.0245	0.0121	0.1081
1987	0.0775	0.0643	0.0245	0.0163	0.1123
1988	0.0967	0.0692	0.0245	0.0211	0.1172
1989	0.0680	0.0703	0.0245	0.0223	0.1183
1990	0.0949	0.0688	0.0245	0.0207	0.1168
1991	0.0736	0.0887	0.0245	0.0407	0.1367
1992	0.0732	0.0821	0.0245	0.0341	0.1301
1993	0.0254	0.0770	0.0245	0.0290	0.1250
1994	0.0707	0.0652	0.0245	0.0171	0.1132
1995	0.0955	0.0721	0.0245	0.0241	0.1201
1996	0.0954	0.0701	0.0245	0.0221	0.1181
1997	0.0572	0.0632	0.0245	0.0151	0.1112
1998		0.0729	0.0245	0.0249	0.1209
1999		0.0872	0.0260	0.0363	0.1382
2000		0.0834	0.0263	0.0320	0.1349
2001		0.0715	0.0273	0.0180	0.1249
2002		0.0766	0.0307	0.0164	0.1368
2003		0.0828	0.0321	0.0198	0.1458
2004		0.0807	0.0328	0.0164	0.1451
2005		0.0768	0.0340	0.0101	0.1434
2006		0.0788	0.0357	0.0088	0.1488
2007		0.0813	0.0369	0.0089	0.1537
2008		0.0805	0.0378	0.0063	0.1547
2009		0.0792	0.0389	0.0029	0.1555
2010		0.0801	0.0402	0.0014	0.1589
2011		0.0812	0.0412	0.0004	0.1620
2012		0.0810	0.0422	-0.0016	0.1636
2013		0.0807	0.0432	-0.0039	0.1653
2014		0.0812	0.0442	-0.0054	0.1678
2015		0.0817	0.0451	-0.0068	0.1702
2016		0.0817	0.0461	-0.0085	0.1720
2017		0.0817	0.0470	-0.0103	0.1738
2018		0.0821	0.0479	-0.0118	0.1759
2019		0.0824	0.0488	-0.0132	0.1780
2020		0.0825	0.0496	-0.0147	0.1798